Khemis Miliana University – Djilali BOUNAAMA
Faculty of Science and Technology
Department of Physics

Electromagnetism

L2 Fundamental Physics

By:
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Content

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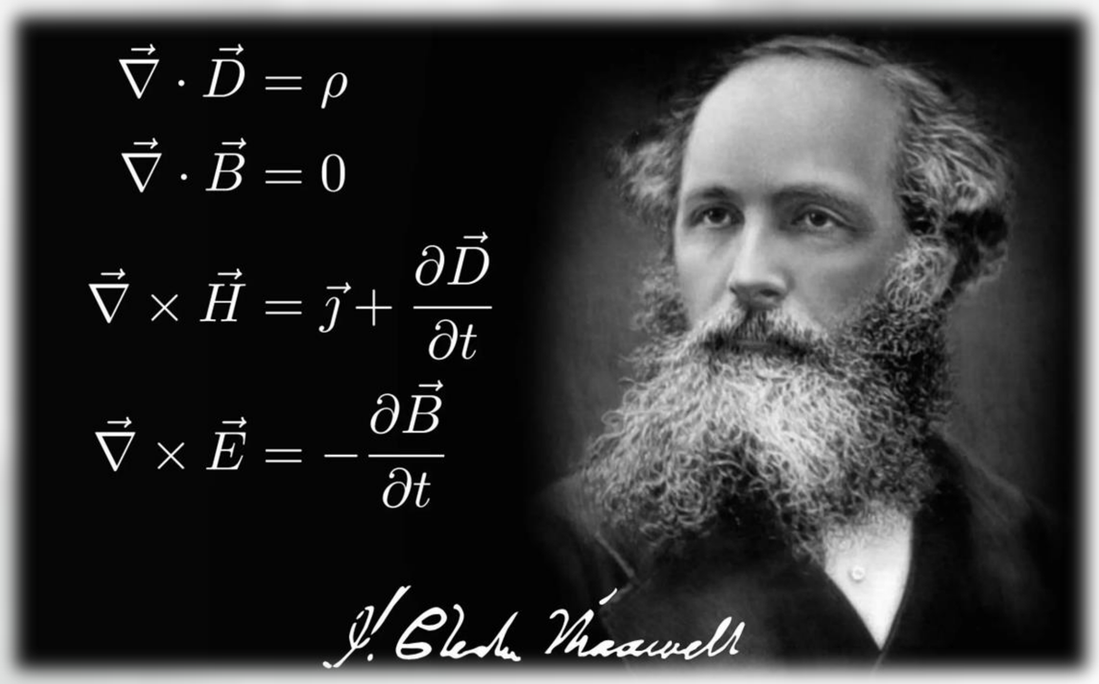
Chapter 02

Maxwell Equations

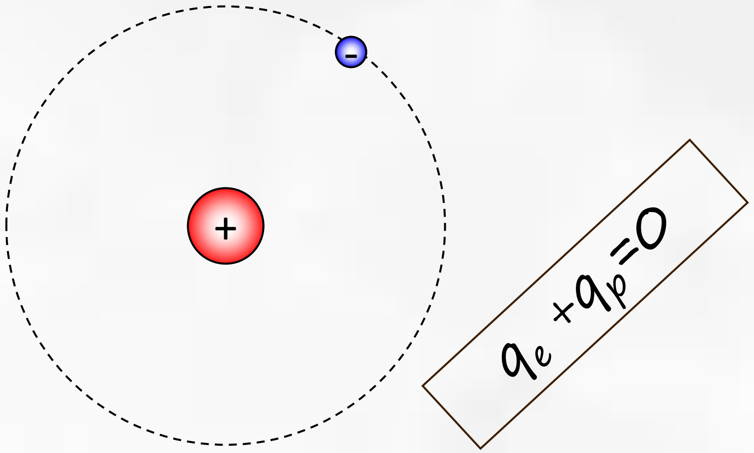


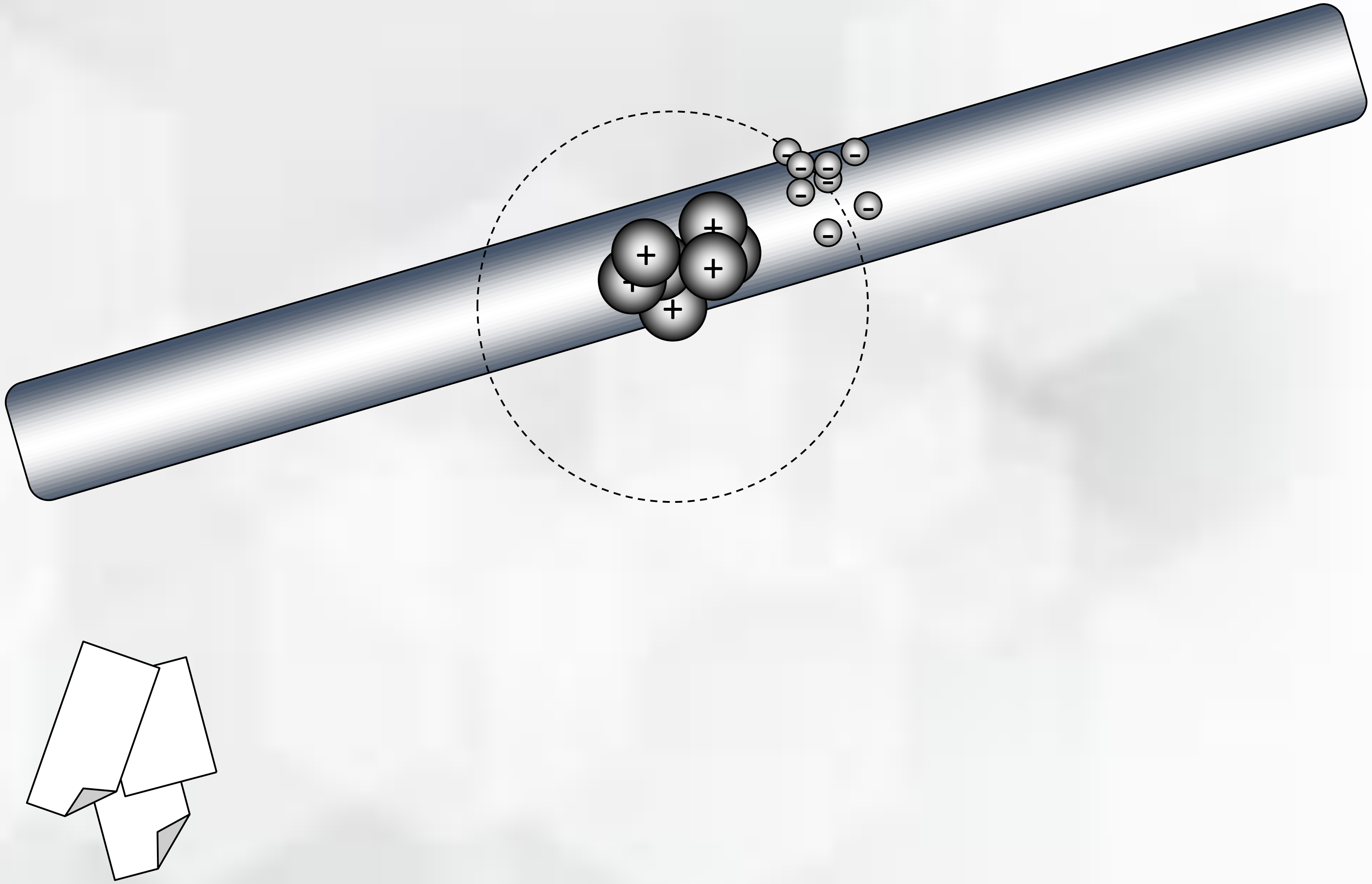
II. Maxwell Equations

1. Electrical charge:

$$q_e = -e = -1.6 \times 10^{-19} [C]$$

$$q_p = +e = +1.6 \times 10^{-19} [C]$$





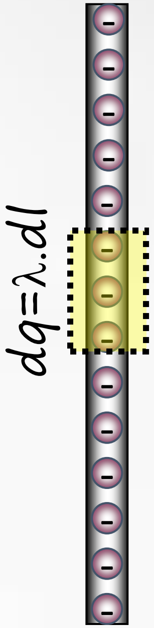
$$Q^- = N.qe = -N.e$$

$$Q^+ = M.qp = M.e$$

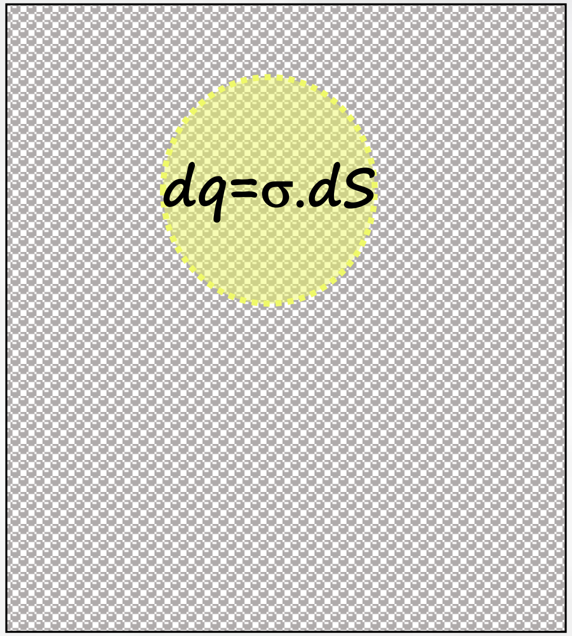
Neutral system: N=M

II. Maxwell Equations

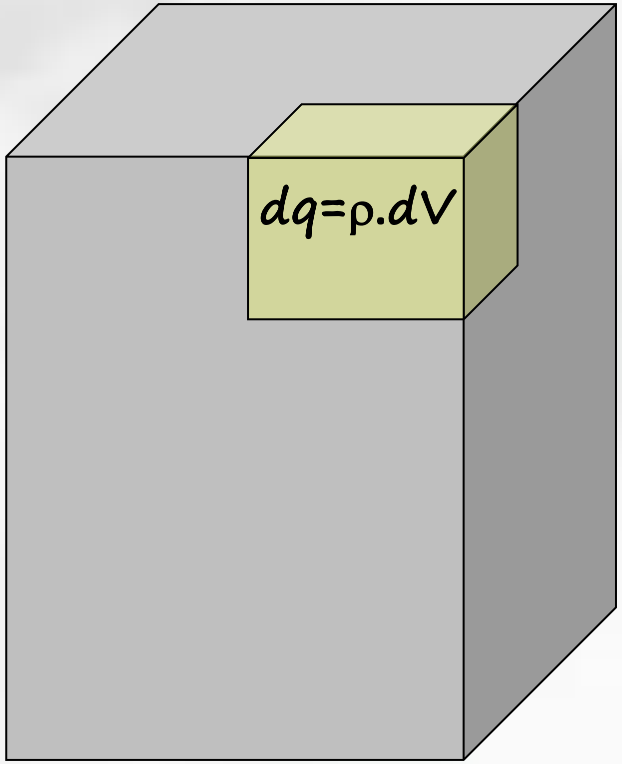
2. Distribution of electrical charge:



$$\lambda = \frac{dq}{dl}\,[C/m]$$



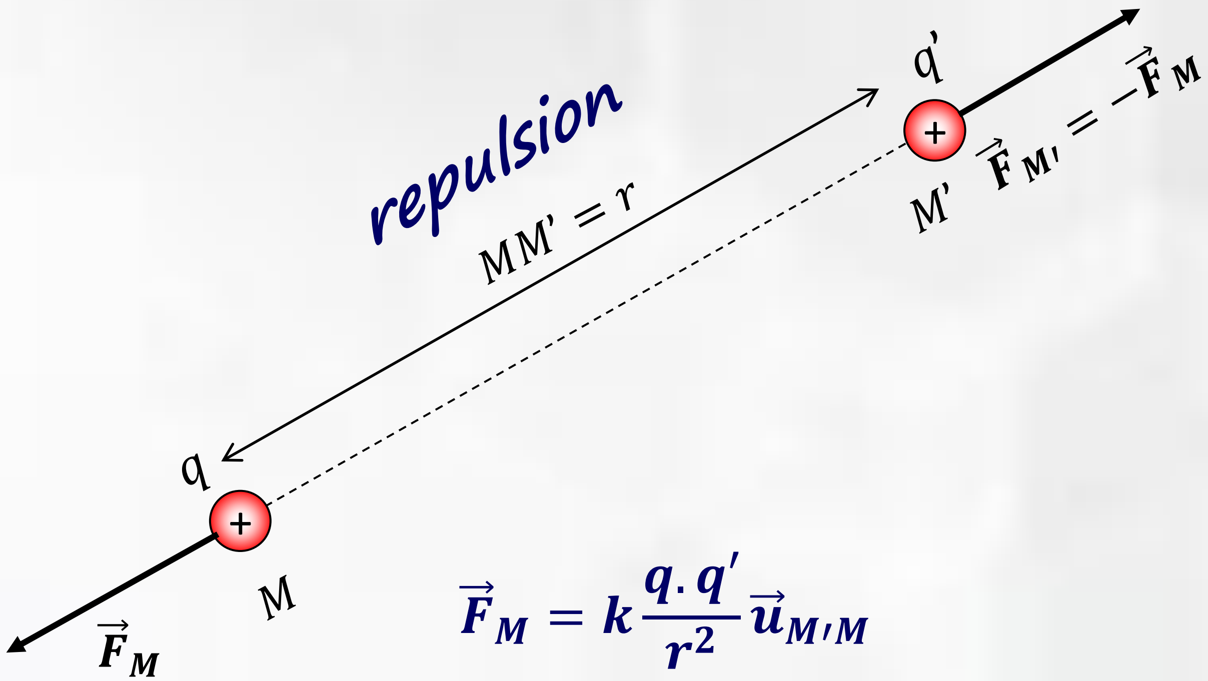
$$\sigma = \frac{dq}{dS}\,[C/m^2]$$

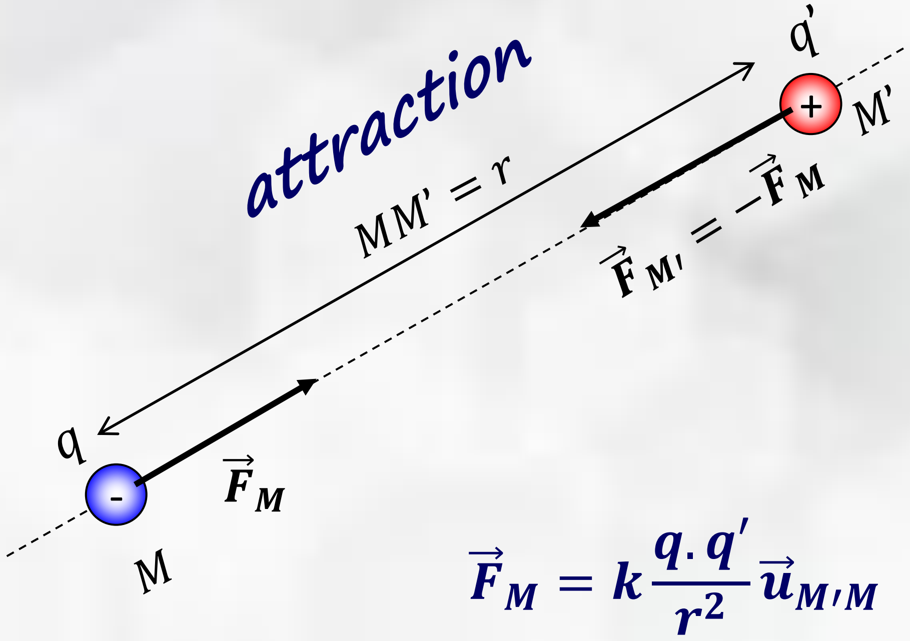


$$\rho = \frac{dq}{dV}\,[C/m^3]$$

II. Maxwell Equations

3. Coulomb Law (Electrical force):

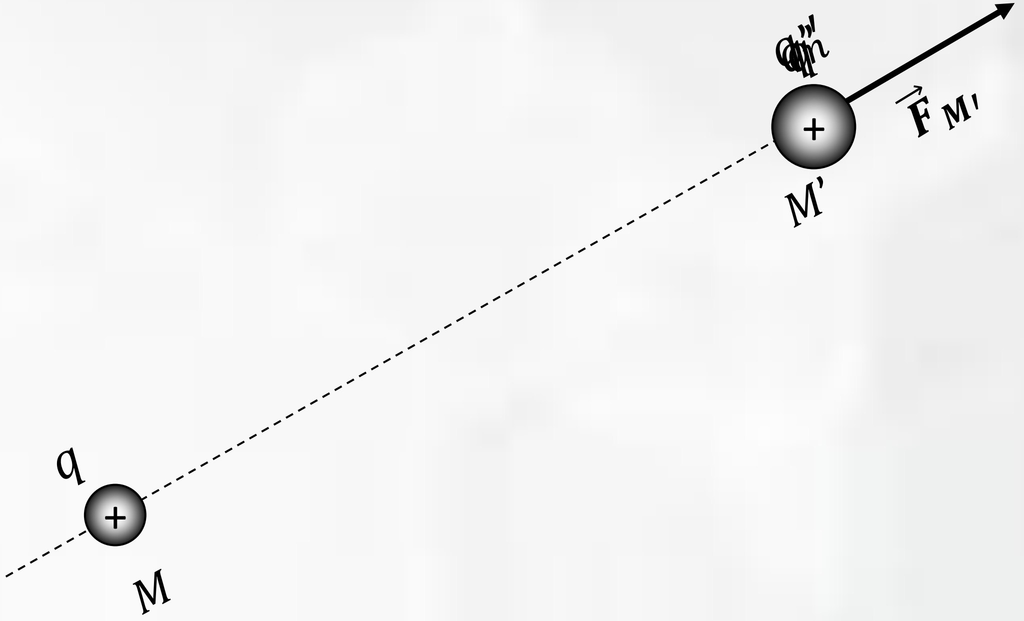




Electrical permittivity: $k = \frac{1}{4\pi\varepsilon_0}$; $\varepsilon_0 \cong 1/(36\pi \times 10^9) = 8.85 \times 10^{-12} [C^2.N^{-1}.m^{-2}]$

II. Maxwell Equations

4. Electric Field:



$$\vec{F}_{M'} = k\frac{q.q'}{r^2}\vec{u}_{M'M}$$

$$\vec{F}_{M'} = k\frac{q.q''}{r^2}\vec{u}_{M'M}$$

$$\vdots$$

$$\vec{F}_{M'} = k\frac{q.q_n}{r^2}\vec{u}_{M'M}$$

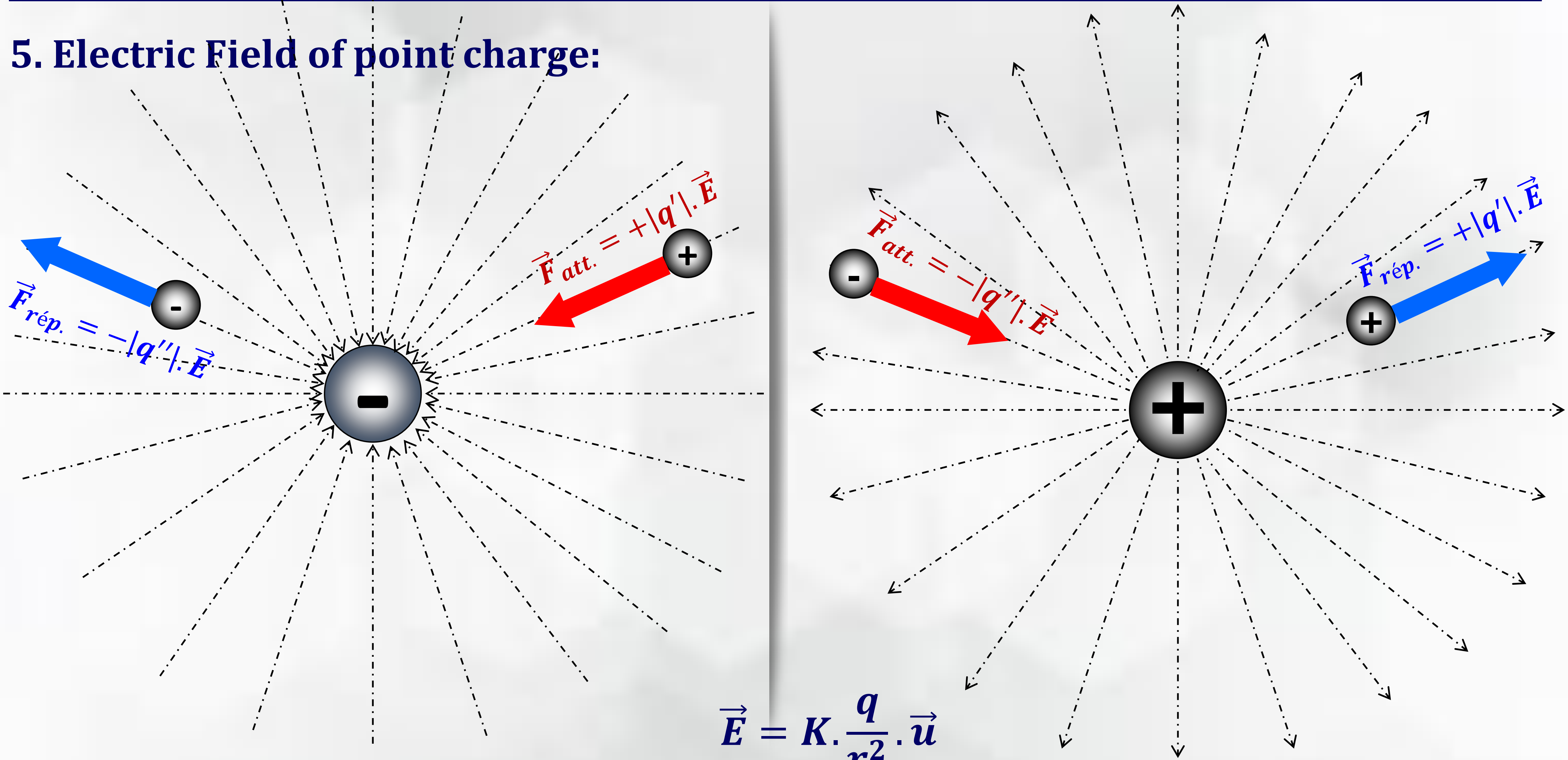
$$\vec{F}_{M'} = \left(K\frac{q}{r^2}\vec{u}_{M'M}\right).q_n = q_n.\vec{E}$$

The electric field

$$\vec{E} = K.\frac{q}{r^2}.\vec{u}$$

II. Maxwell Equations

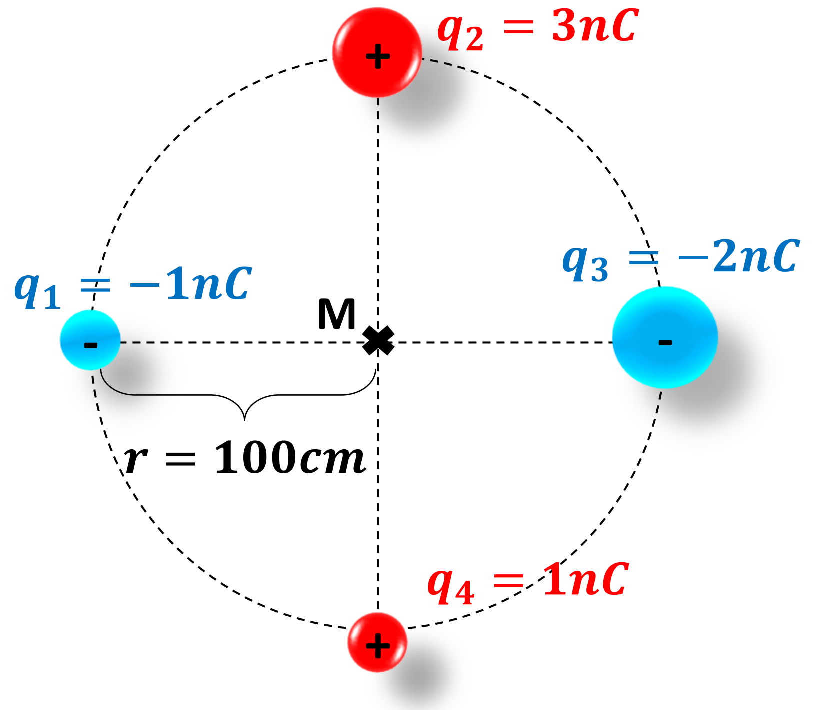
5. Electric Field of point charge:



$$\vec{E} = K.\frac{q}{r^2}.\vec{u}$$

- **Example 01**

1. Calculate the electrical field in the point M, created by the group of charges as shown in the figure below.



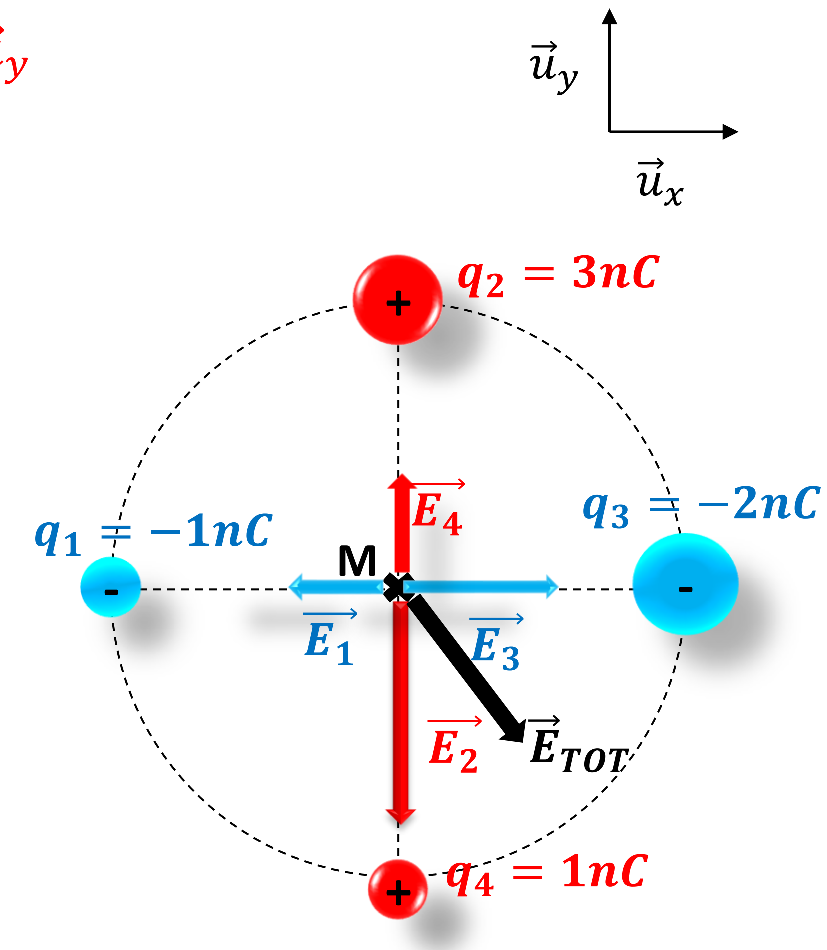
- ## Solution of the Example 01:

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\vec{u}_{q1M} = \frac{9\times10^9\times(-10^{-9})}{1^2}\vec{u}_x = -9[V/m]\vec{u}_x$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\vec{u}_{q2M} = 27(-\vec{u}_y) = -27[V/m]\vec{u}_y$$

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r^2}\vec{u}_{q3M} = 18[V/m]\vec{u}_x$$

$$\vec{E}_4 = \frac{1}{4\pi\varepsilon_0}\frac{q_4}{r^2}\vec{u}_{q4M} = 9[V/m]\vec{u}_y$$



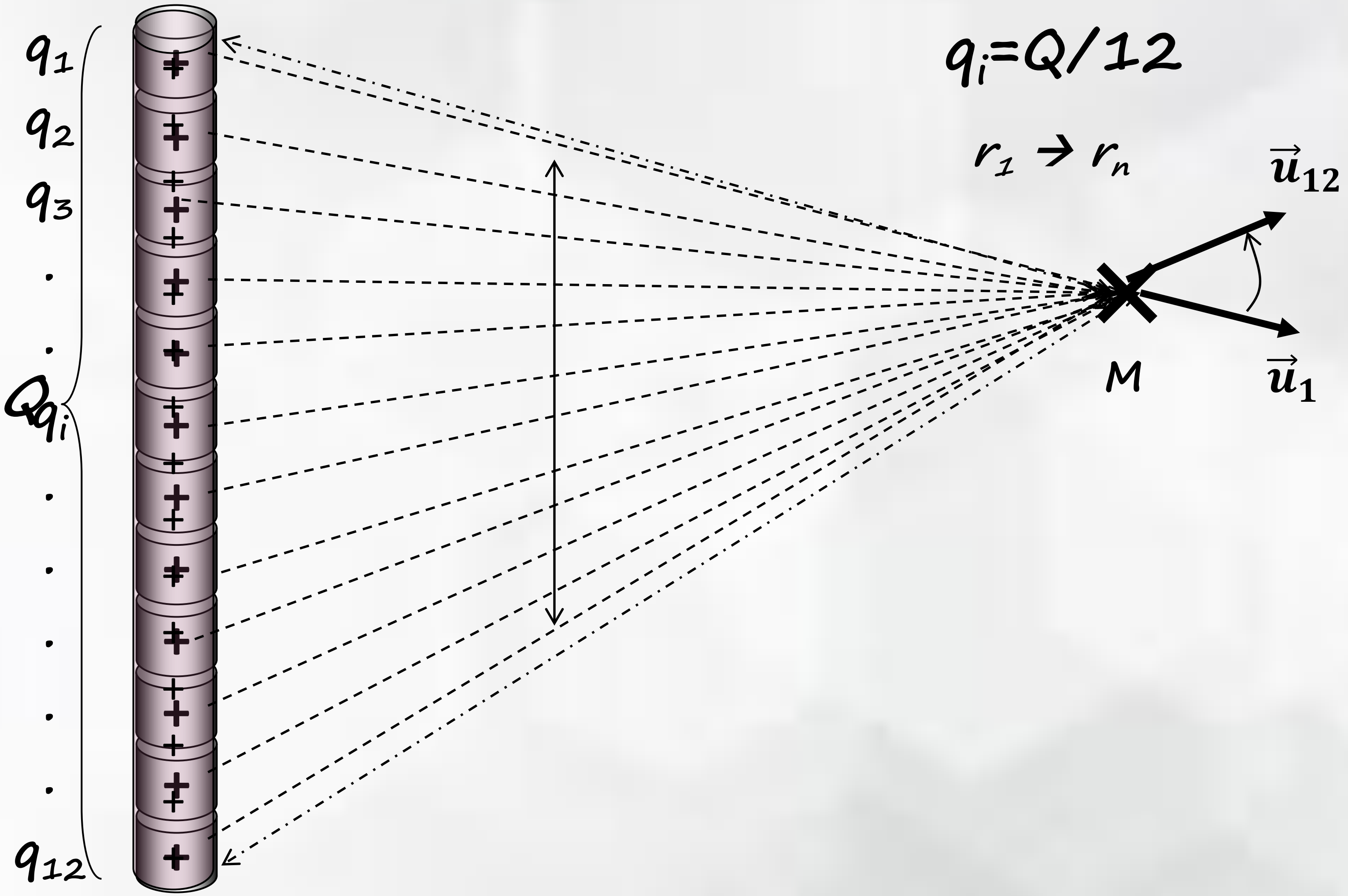
$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$\vec{E}_{TOT} = -9\vec{u}_x + -27\vec{u}_y + 18\vec{u}_x + 9\vec{u}_y$$

$$\vec{E}_{TOT} = 9\vec{u}_x - 18\vec{u}_y$$

II. Maxwell Equations

6. Electric Field of a charge distribution



$$\vec{e}_1 = K.\frac{q_1}{{r_1}^2}.\vec{u}_1$$

$$\vec{e}_2 = K.\frac{q_2}{{r_2}^2}.\vec{u}_2$$

$$\vec{e}_3 = K.\frac{q_3}{{r_3}^2}.\vec{u}_3$$

.

.

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.

.

$$\vec{e}_{12} = K.\frac{q_{12}}{{r_{12}}^2}.\vec{u}_{12}$$

II. Maxwell Equations

6. Electric Field of a charge distribution

$$\vec{E} = \vec{e}_1 + \vec{e}_2 + \ldots + \vec{e}_{12} = \sum_{i=1}^{12} \vec{e}_i$$

$$\vec{E} = k.q \sum_{i=1}^{12} \frac{1}{r_i^2}.\vec{u}_i$$

N : très grand

↓

q_i *: très petit* → $q_i = dQ$

↓

$$\vec{e}_i = d\vec{E} = K.\frac{dQ}{r_i^2}.\vec{u}_i$$

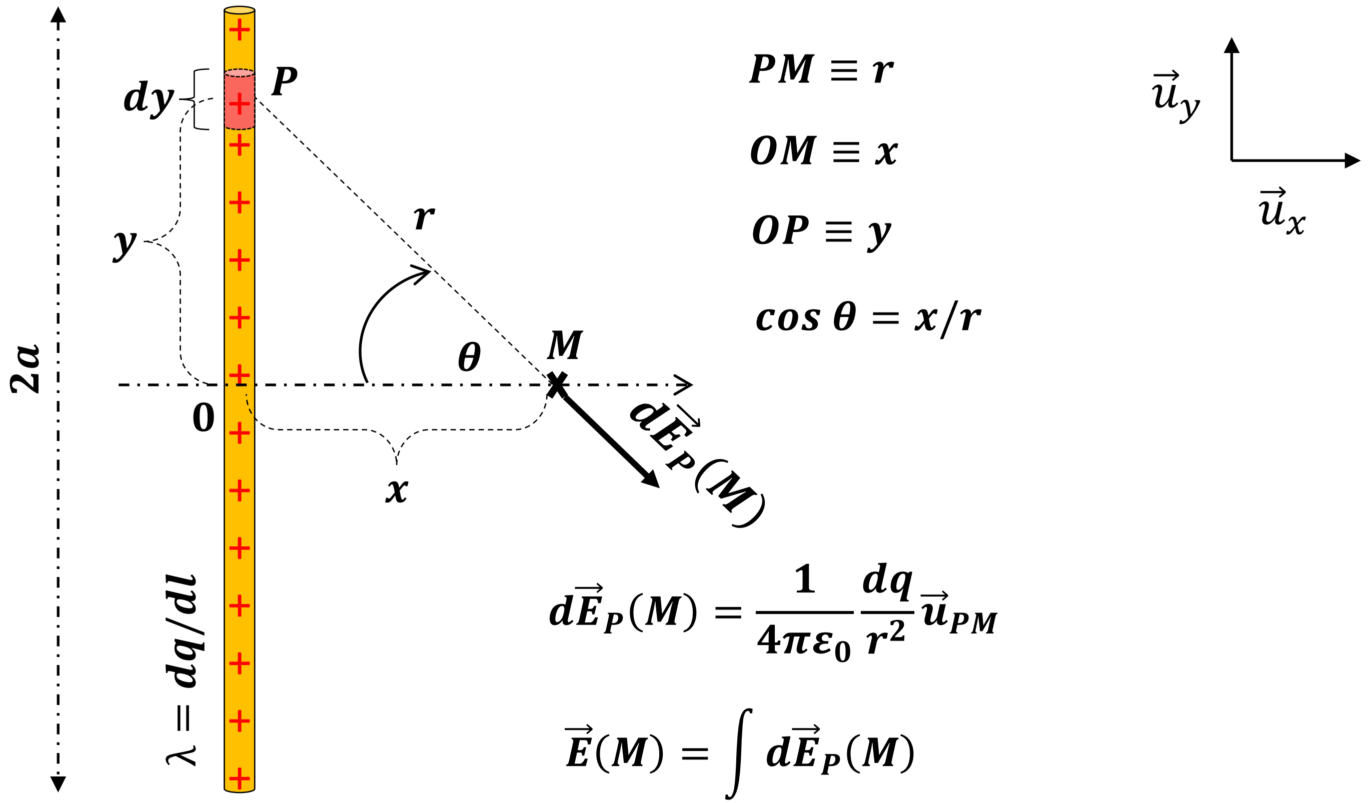
$$\vec{E} = \int_A^B d\vec{E} = K.\int_Q \frac{dQ}{r^2}.\vec{u}$$

we need to know:

- *The charge distribution: λ, σ or ρ;*
- *The geometry of the system;*
- *And exploit the symmetry if it exists (rectangular, cylindrical, spherical)*

- # Example 02:

Let's calculate the resultant field created in the point M by the linear distribution of charge on the wire of length 2A, as shown in the figure.



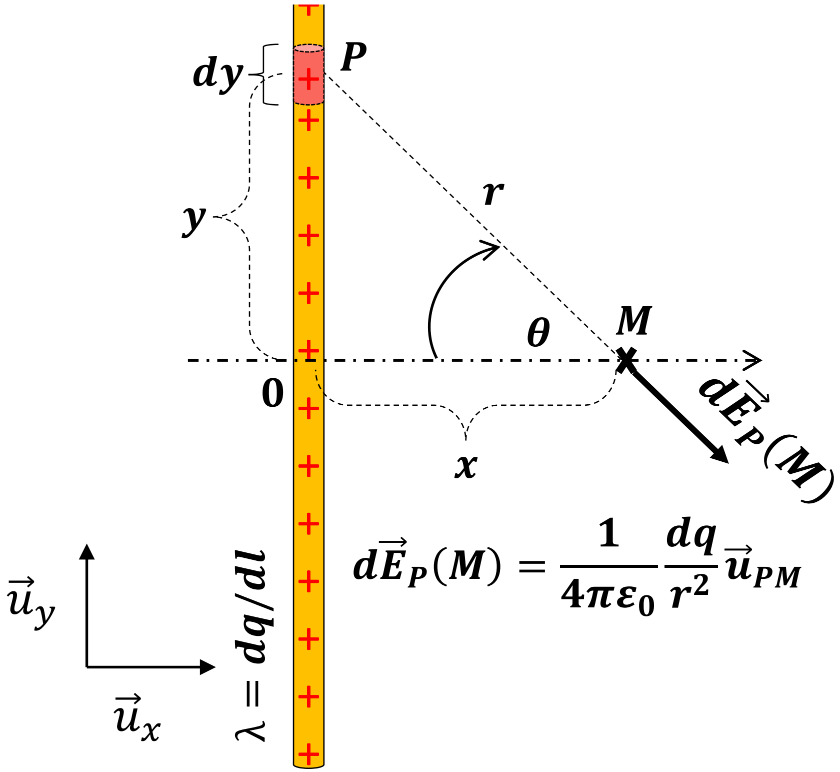
$$d\vec{E}_P(M) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u}_{PM}$$

$$\vec{E}(M) = \int d\vec{E}_P(M)$$

- # Example 02:

1. Write the expression of the unit vector $\vec{u}_{PM}$ as a function of $\vec{u}_x, \vec{u}_y$ et θ .

2. By using a convenient choice of symmetry, show that the total field will have only one component on the axis ox.



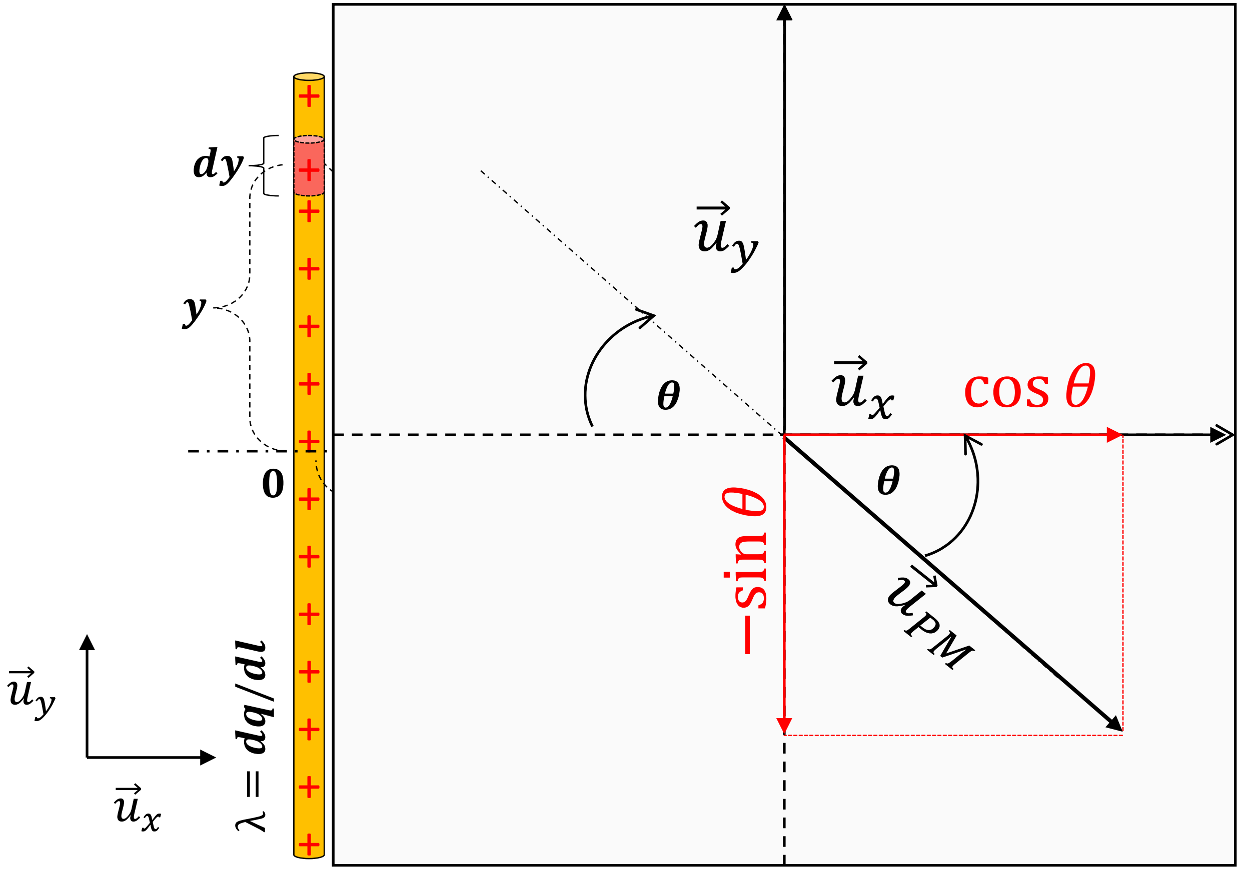
3. Find the expression of the non-null elementary field as a function of x and θ. Define the angle θ_{max} (***hint:*** *use the value* $\sin\theta_{max}$)

4. Find the total electrical field created by this wire in the point M. Deduce the result for $a \to \infty$

- **Solution of Example 02:**

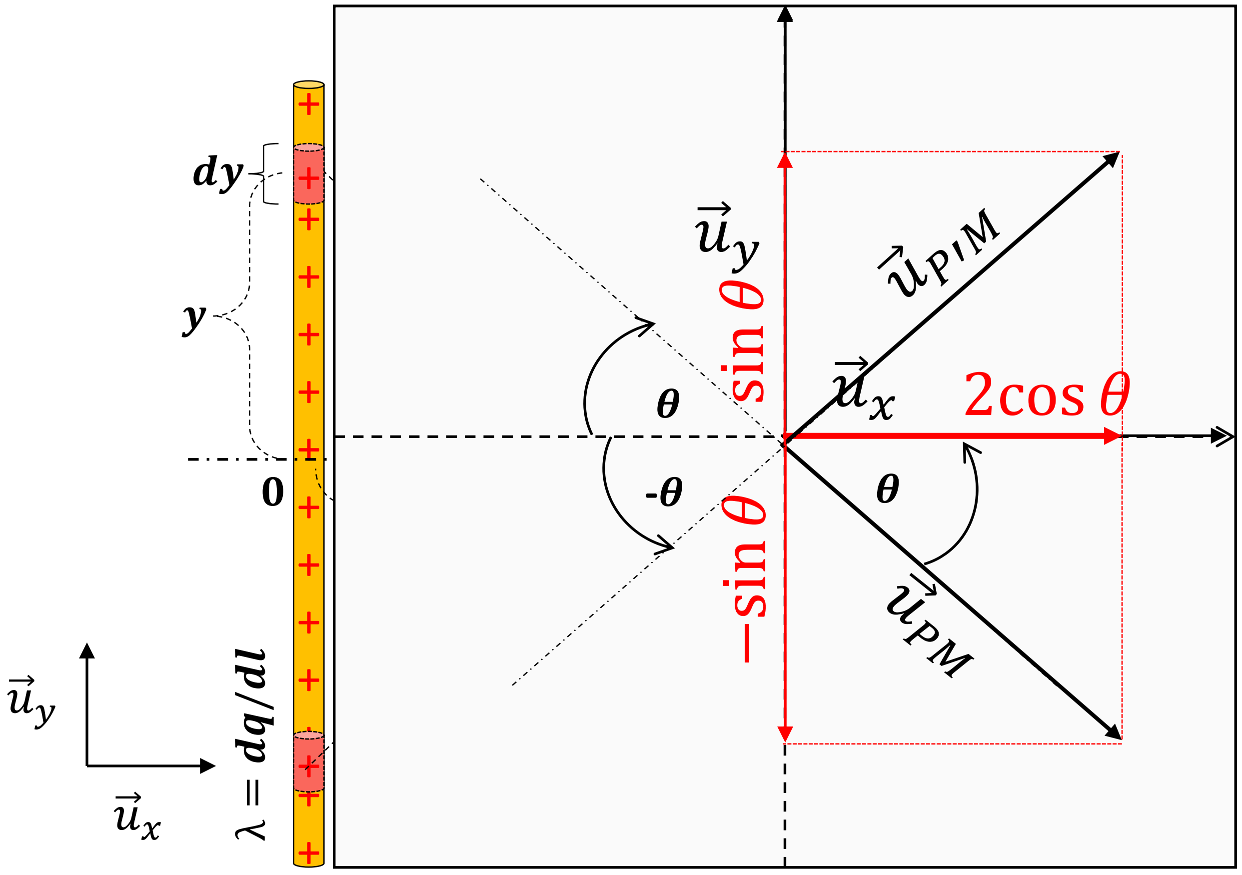
1. Write the expression of the unit vector $\vec{u}_{PM}$ as a function of $\vec{u}_x, \vec{u}_y$ et θ .

$$\vec{u}_{PM} = \cos\theta\, \vec{u}_x - \sin\theta\, \vec{u}_y$$



- **Solution of Example 02:**

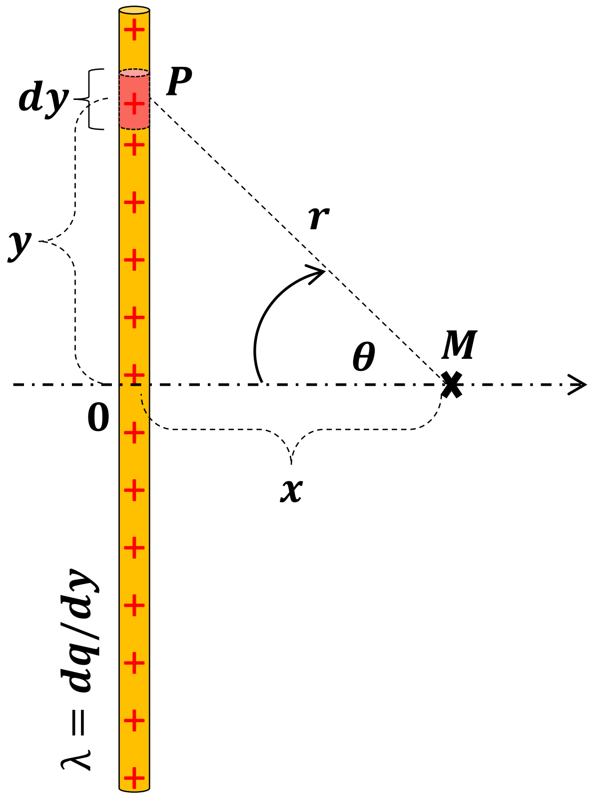
2. By using a convenient choice of symmetry, show that the total field will have only one component on the axis ox.



- **Solution of Example 02:**

3. Find the expression of the non-null elementary field as a function of x and θ.

Define the angle θ_{max} (***hint:*** *use the value* $\sin\theta_{max}$)



$$d\vec{E}_{Px}(M) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta.\vec{u}_x = \frac{\lambda}{4\pi\varepsilon_0}\frac{dy}{r^2}\cos\theta.\vec{u}_x$$

$$\tan\theta = \frac{y}{x} \rightarrow x = \frac{y}{\tan\theta} \rightarrow dy = \frac{x.d\theta}{\cos^2\theta}$$

$$\cos\theta = \frac{x}{r} \rightarrow r = \frac{x}{\cos\theta} \rightarrow \frac{1}{r^2} = \frac{\cos^2\theta}{x^2}$$

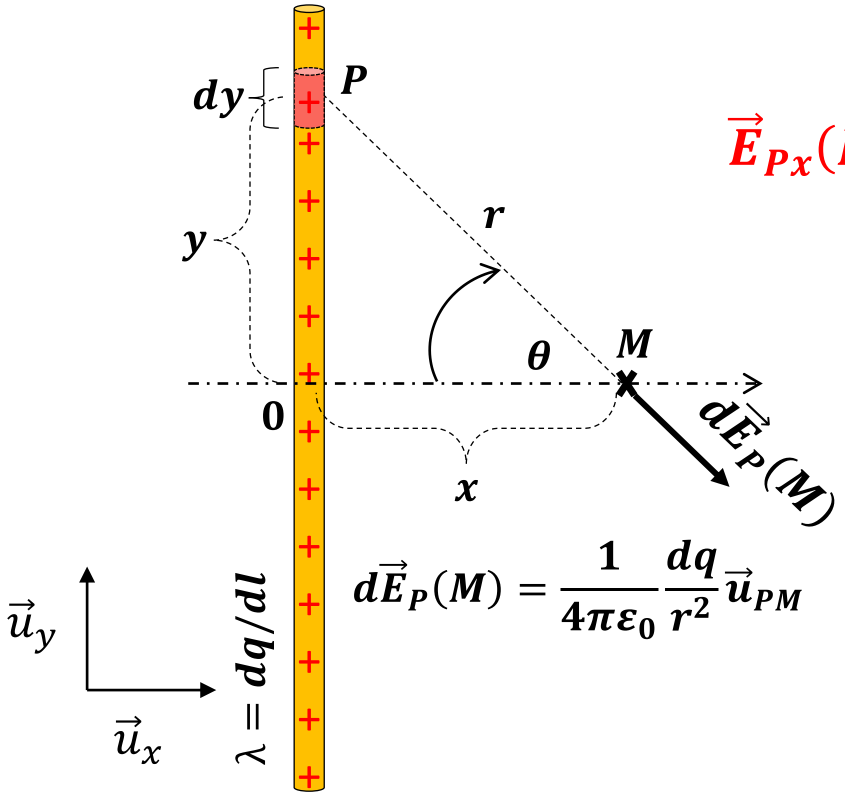
$$\frac{dy}{r^2}\cos\theta = \frac{x.d\theta}{\cos^2\theta}.\frac{\cos^2\theta}{x^2}.\cos\theta = \frac{\cos\theta.d\theta}{x}$$

$$d\vec{E}_{Px}(M) = \frac{\lambda}{4\pi\varepsilon_0}\frac{\cos\theta.d\theta}{x}.\vec{u}_x$$

$$0 \leq \theta \leq \theta_{max} \rightarrow \sin\theta_{max} = \frac{a}{\sqrt{a^2+x^2}}$$

- **Solution of Example 02:**

4. Find the total electrical field created by this wire in the point M. Deduce the result for $a \to \infty$



$$d\vec{E}_{Px}(M) = \frac{\lambda}{4\pi\varepsilon_0}\frac{\cos\theta . d\theta}{x}.\vec{u}_x \to \vec{E}_{Px}(M) = 2\int_0^{\theta_{max}}\frac{\lambda}{4\pi\varepsilon_0}\frac{\cos\theta . d\theta}{x}.\vec{u}_x$$

$$\vec{E}_{Px}(M) = \frac{\lambda}{2\pi x\varepsilon_0}\int_0^{\theta_{max}}\cos\theta . d\theta . \vec{u}_x = \frac{\lambda}{2\pi x\varepsilon_0}\vec{u}_x[\sin\theta]_0^{\theta_{max}}$$

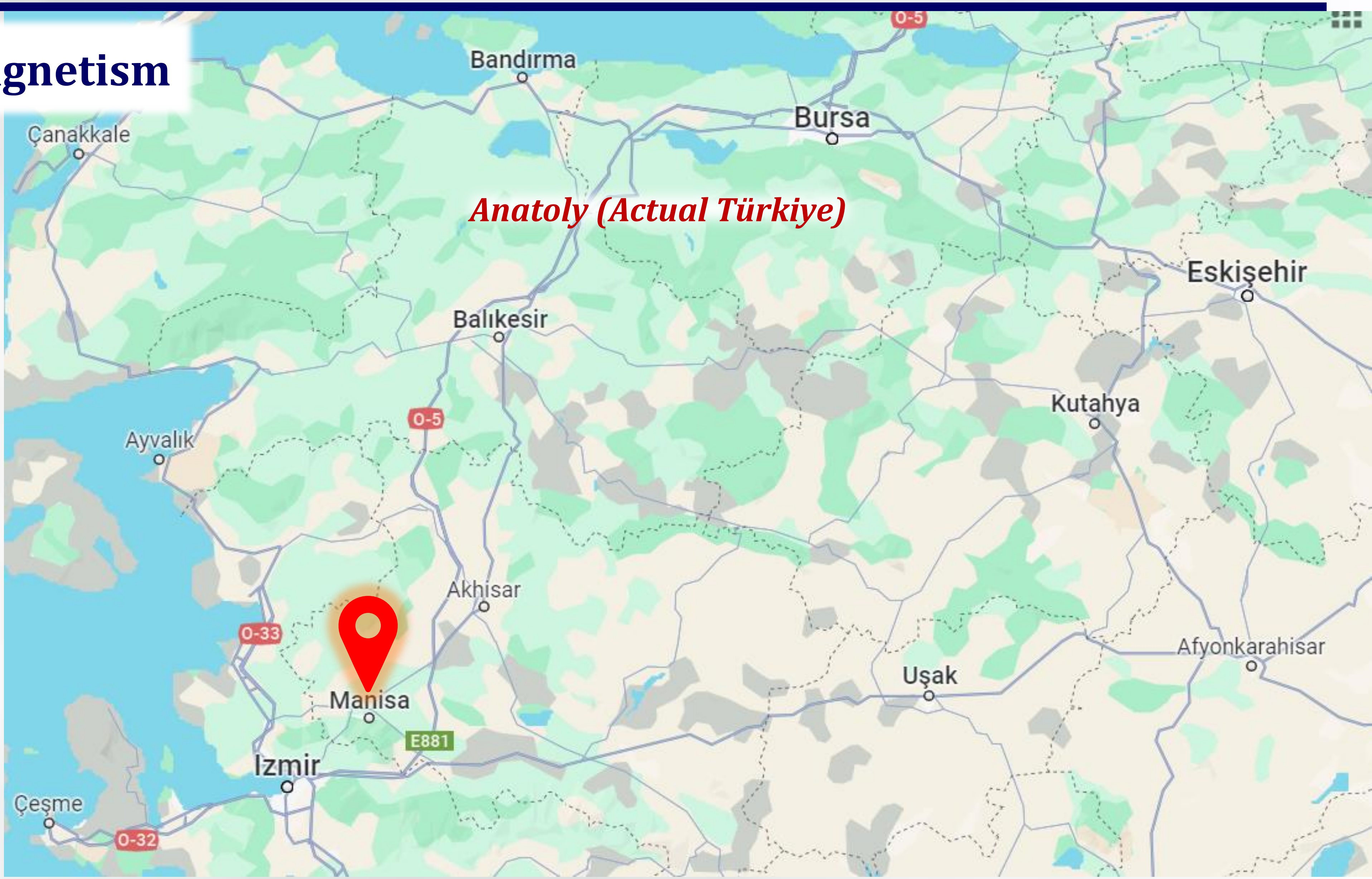
$$\vec{E}_{Px}(M) = \frac{\lambda}{2\pi x\varepsilon_0}\sin\theta_{max}\vec{u}_x = \frac{\lambda}{2\pi x\varepsilon_0}\frac{a}{\sqrt{a^2+x^2}}\vec{u}_x$$

pour $a \to \infty$

$$\lim_{a\to\infty}\vec{E}_{Px}(M) = \frac{\lambda}{2\pi x\varepsilon_0}\vec{u}_x \underbrace{\lim_{a\to\infty}\frac{a}{\sqrt{a^2+x^2}}}_{=1}$$

$$\lim_{a\to\infty}\vec{E}_{Px}(M) = \frac{\lambda}{2\pi x\varepsilon_0}\vec{u}_x$$

7. Magnets and magnetism

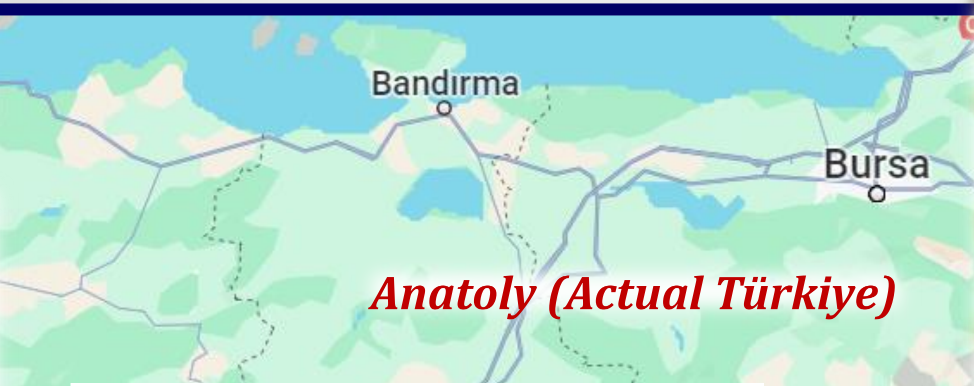


II. Maxwell Equations

Elec

7. Magnets and magnetism

Chinese Compass (~2500 years ealier)

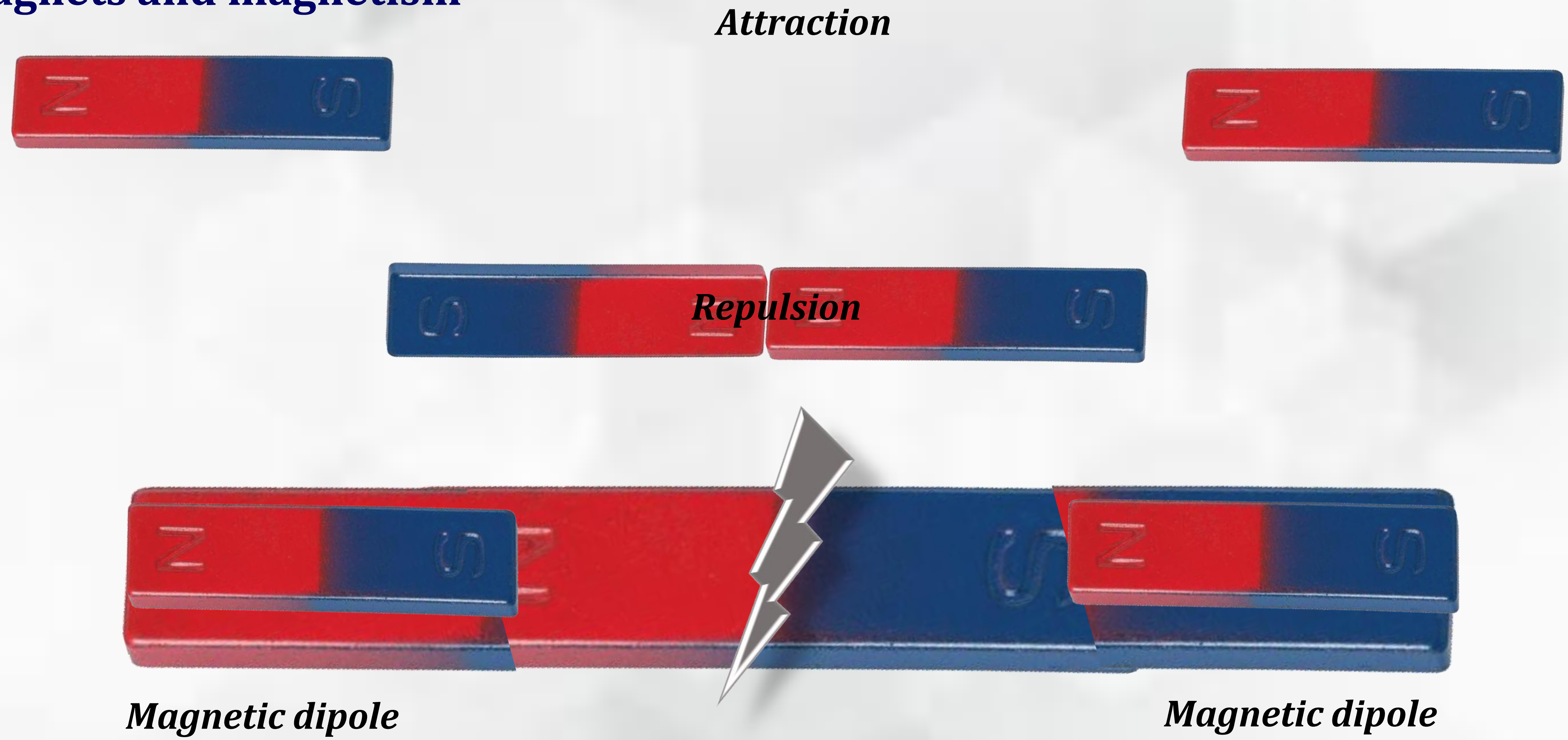


Anatoly (Actual Türkiye)

The magnetite (Fe_3O_4)

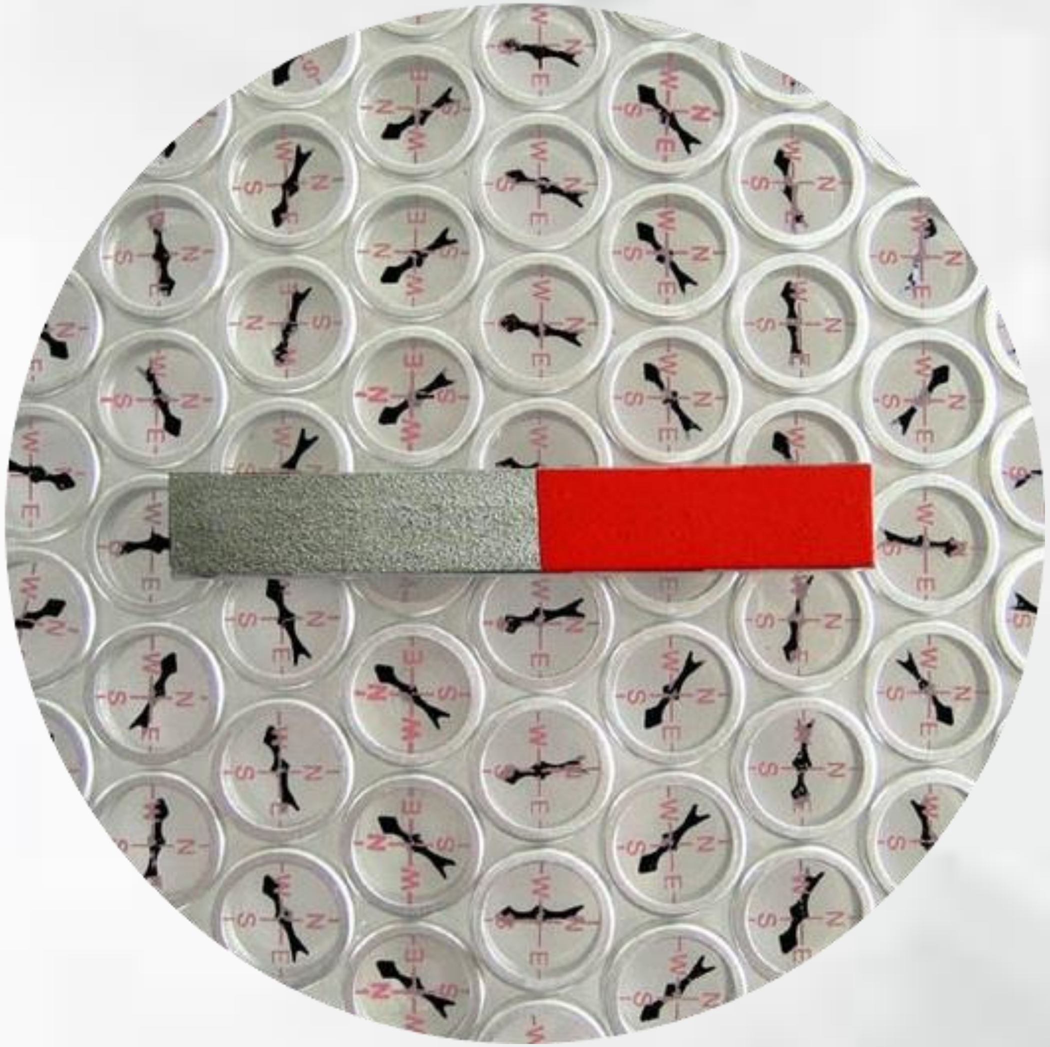
II. Maxwell Equations

7. Magnets and magnetism

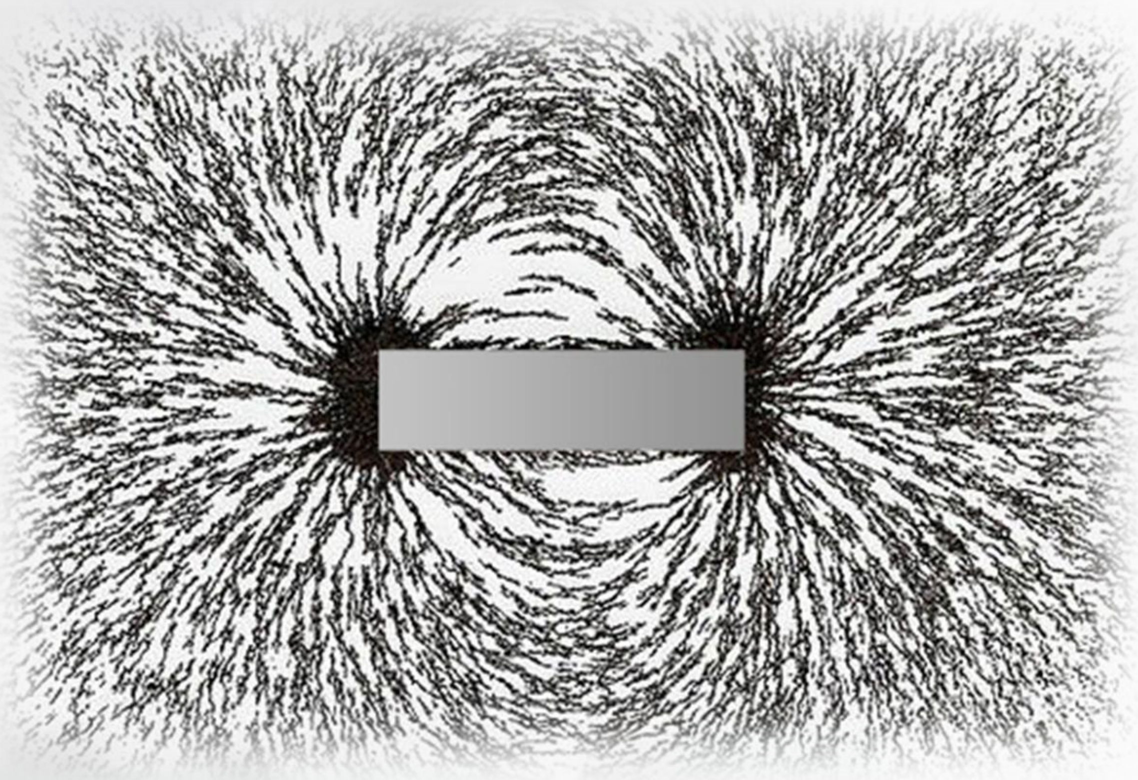


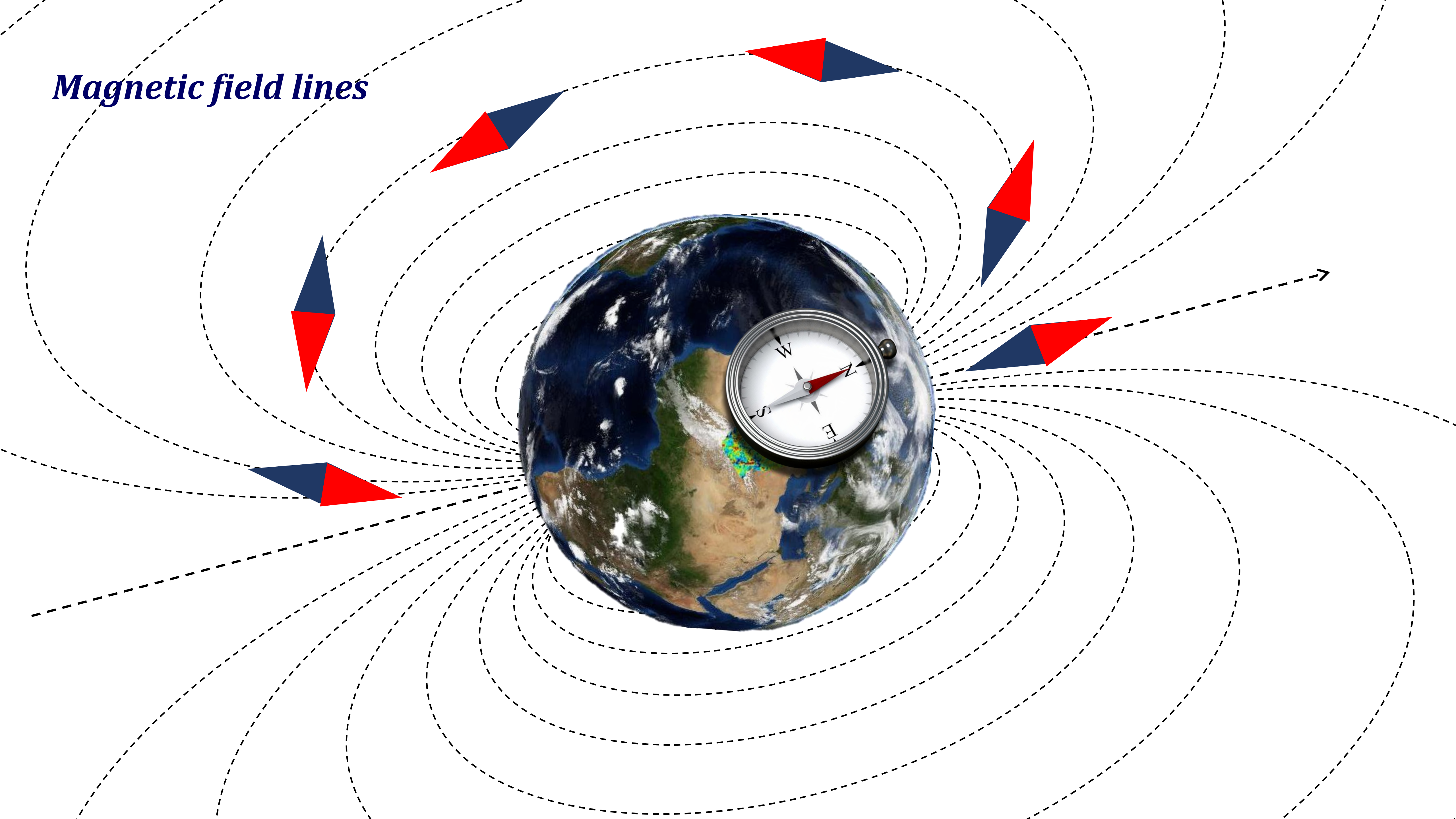
II. Maxwell Equations

7. Magnets and magnetism



Magnetic field lines



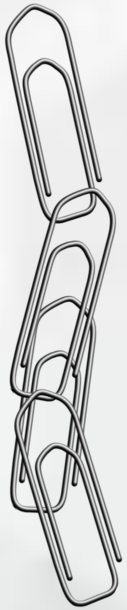

Magnetic field lines
W
N
S
E

II. Maxwell Equations

8. Induced magnet

- Once the paper clip is pulled by the permanent magnet, it becames a scondary non-permanent magnet.

- This magnetized clip could pull another paper clip, and so on … each touched clip will become a new magnet by magnetic induction but with weaker intensity

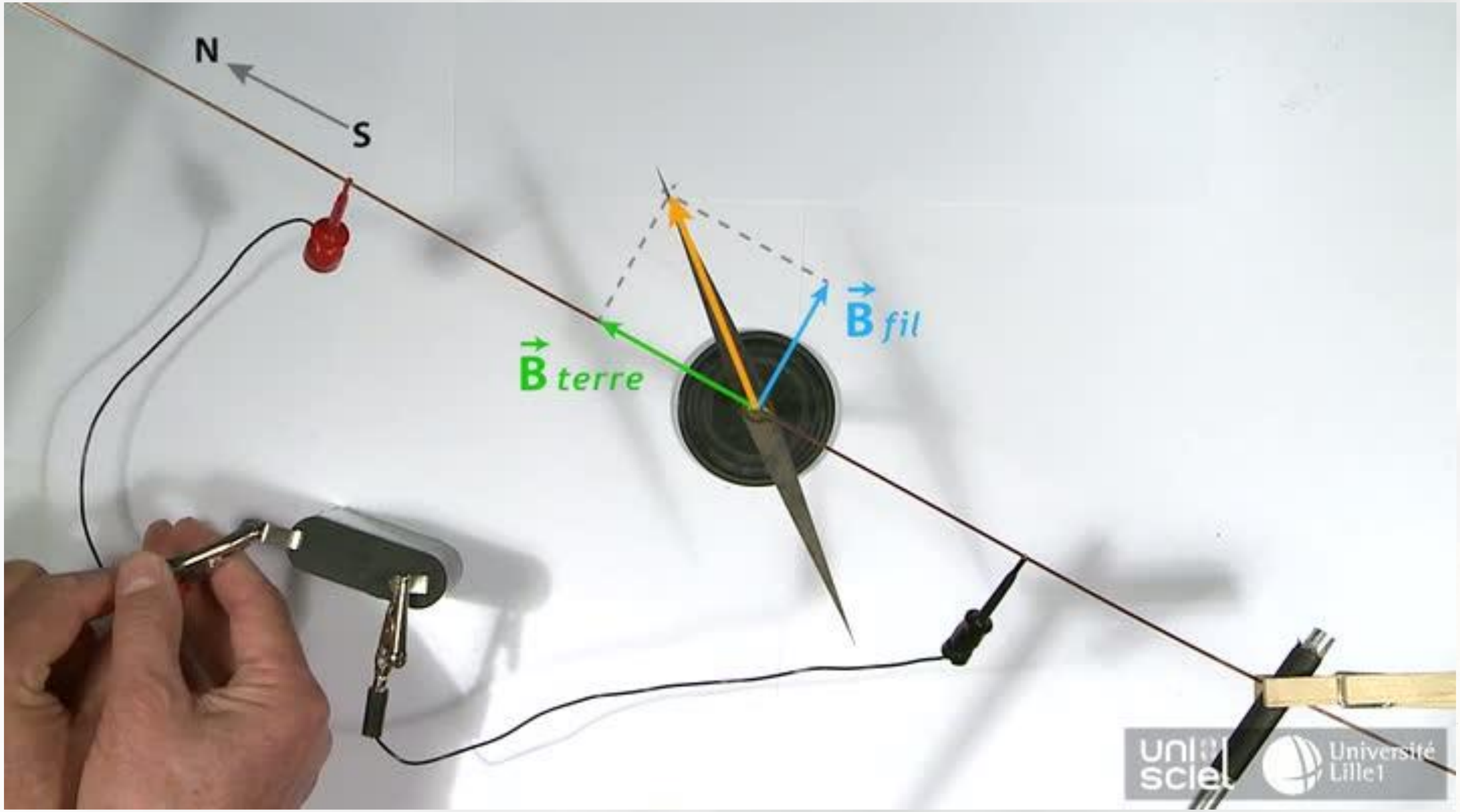


A permanent magnetized piece (compass needle) could be processed by using high quality hot steel, cooled near a strong magnet.

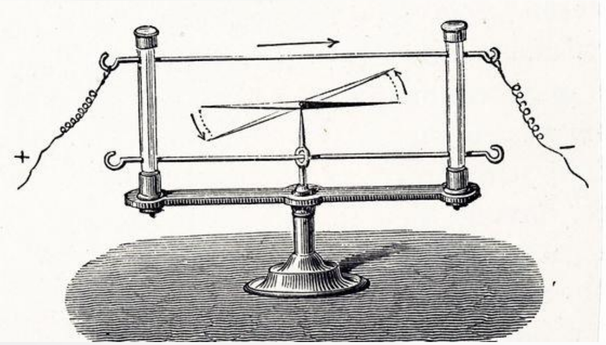
Nowadays, strong permanent magnets are obtained by using alloy of Iron, Rare Earth Element (Neodymium) and Boron, in similar way but magnetized using a powerful electromagnet.

II. Maxwell Equations

8. ØRSTED EXPERIMENT (1819)



Hans Christian Ørsted (1777-1852), Danemark



II. Maxwell Equations

9. The Ørsted's law

Ørsted found that, for a straight wire traversed by a steady direct current (DC):

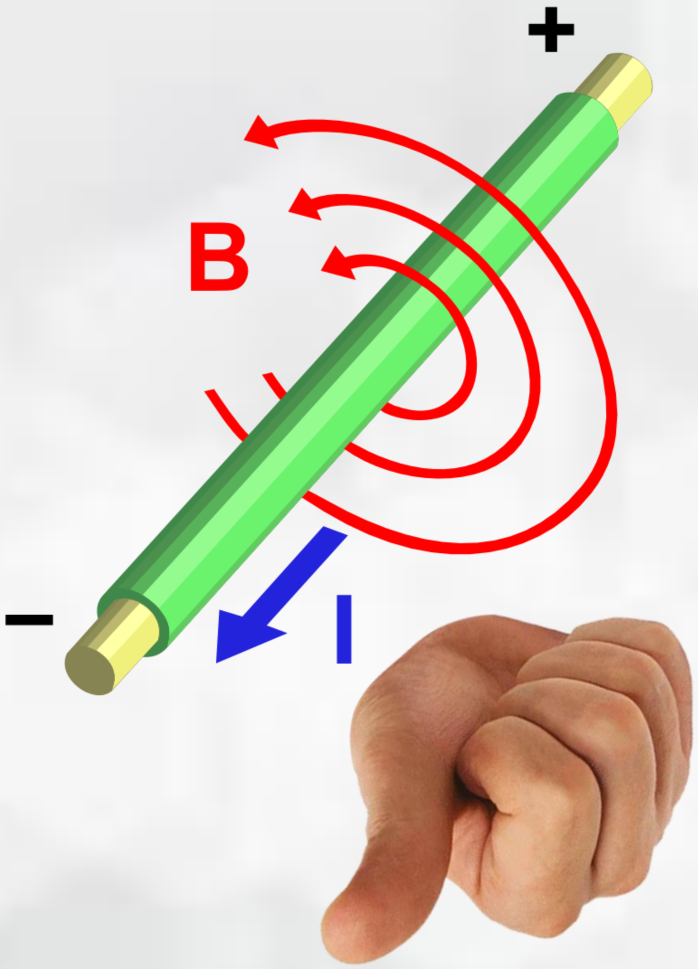
- The magnetic field lines encircle the current-carrying wire and they lie in a plane perpendicular to the wire;

- If the direction of the current is reversed, the direction of the magnetic field reverses;

- The strength of the field: $B \propto I$

- The strength of the field : $B \propto {}^{1}/_{r^2}$

- the direction of the field lines: thumb rule

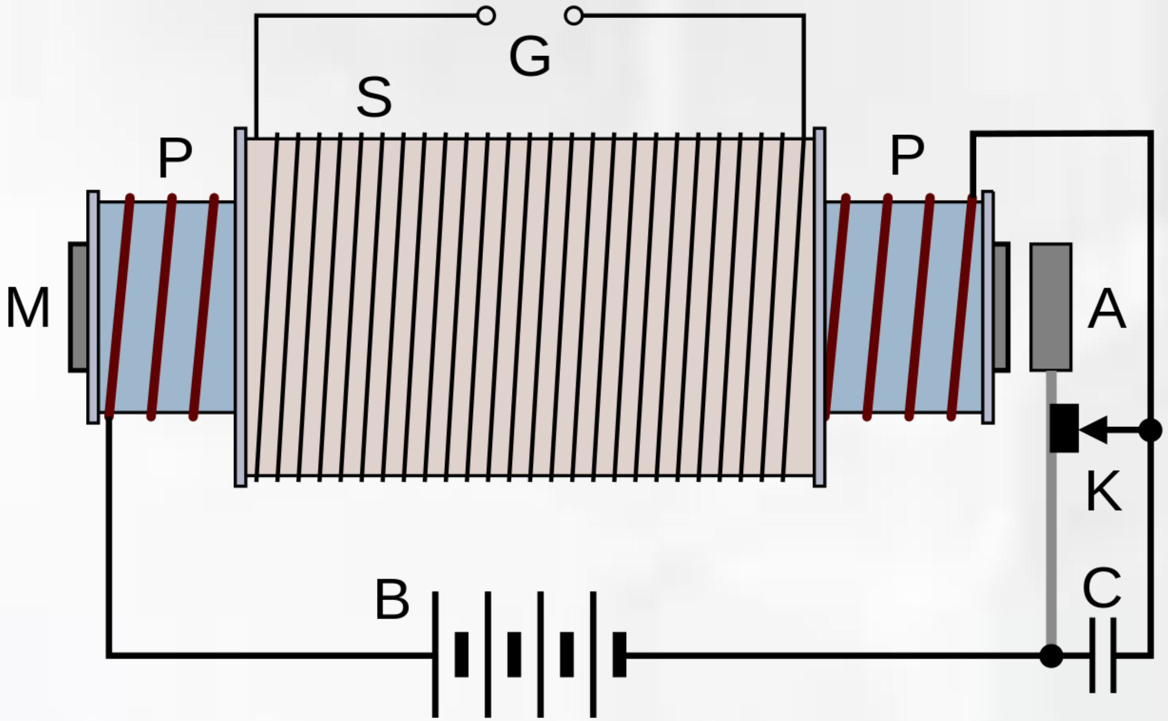


Hans Christian Ørsted (1777-1852), Danemark

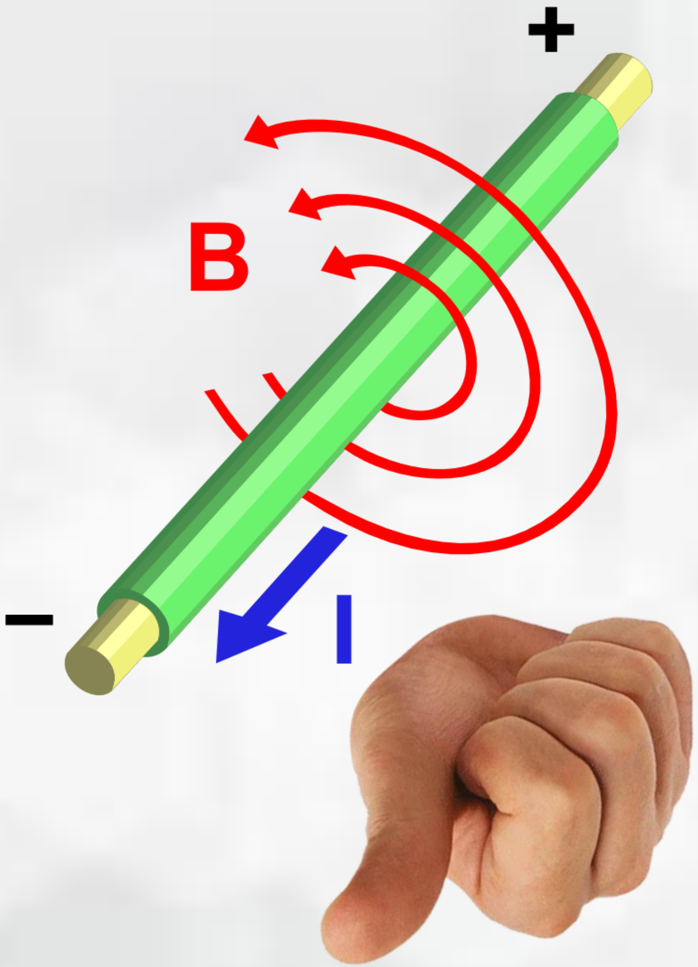
Phenomenology: Electricity ↔ Magnetism

II. Maxwell Equations

9. The Ørsted's law



Electromagnet for a ring doorbell



Hans Christian Ørsted
(1777-1852), Danemark

Phenomenology: Electricity ↔ Magnetism

II. Maxwell Equations

9. Biot-Savart law:

All the cited observations allowed Both two French scientists to deduce the mathematical formulation of an elementary magnetic field induced in a point P by an element $d\vec{l}$ (located at O) of the wire crossed by the electrical current intensity I:

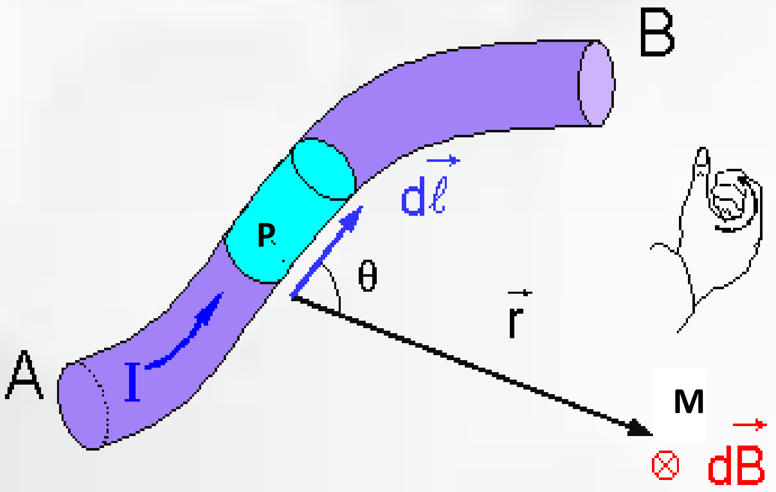
$$d\vec{B}(M) = \frac{\mu_0}{4\pi} I d\vec{l} \wedge \frac{\overrightarrow{PM}}{\left\|\overrightarrow{PM}\right\|^3} = \frac{\mu_0}{4\pi}\frac{I}{r^2} d\vec{l} \wedge \vec{u}$$

The magnetic field, could be then obtained via the integral form:

$$\vec{B}(M) = \int_{M\in(C)} d\vec{B}_P(M) = \int_{M\in(C)} \frac{\mu_0}{4\pi} \cdot \frac{I.d\vec{l} \wedge \vec{u}_{PM}}{r^2}$$

"which is not an easy calculation to do!!!"

Magnetic permeability: $\mu_0 = 4\pi \times 10^{-7}[H.m^{-1}]$



II. Maxwell Equations

9. Biot-Savart law:

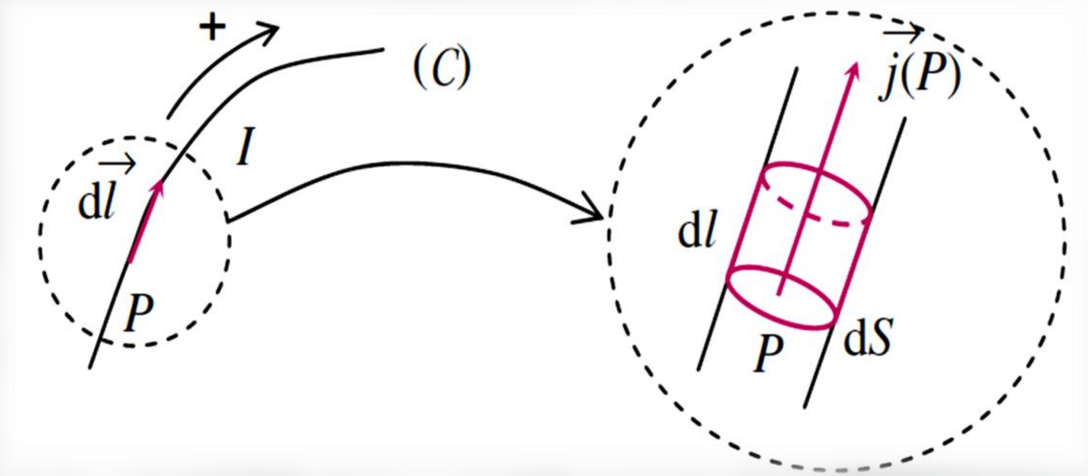
If the current density is known, it will be more convenient to calculate the magnetic field using the density instead of the current intensity:

$$I = \vec{J}.\overrightarrow{dS} \rightarrow I.d\vec{l} = J(P).dS.d\vec{l} = \overrightarrow{J(P)}.dV$$

Thus, the Biot-Savart law becomes in the case of volume density:

$$\vec{B}(M) = \int_{P\in(C)} d\vec{B}_P(M) = \iiint_V \frac{\mu_0}{4\pi}.\frac{\overrightarrow{J(P)}.dV \wedge \vec{u}_{PM}}{PM^2}$$

$$= \iiint_V \frac{\mu_0}{4\pi}.\frac{\overrightarrow{J(P)} \wedge \vec{u}_{PM}}{PM^2}.dV$$



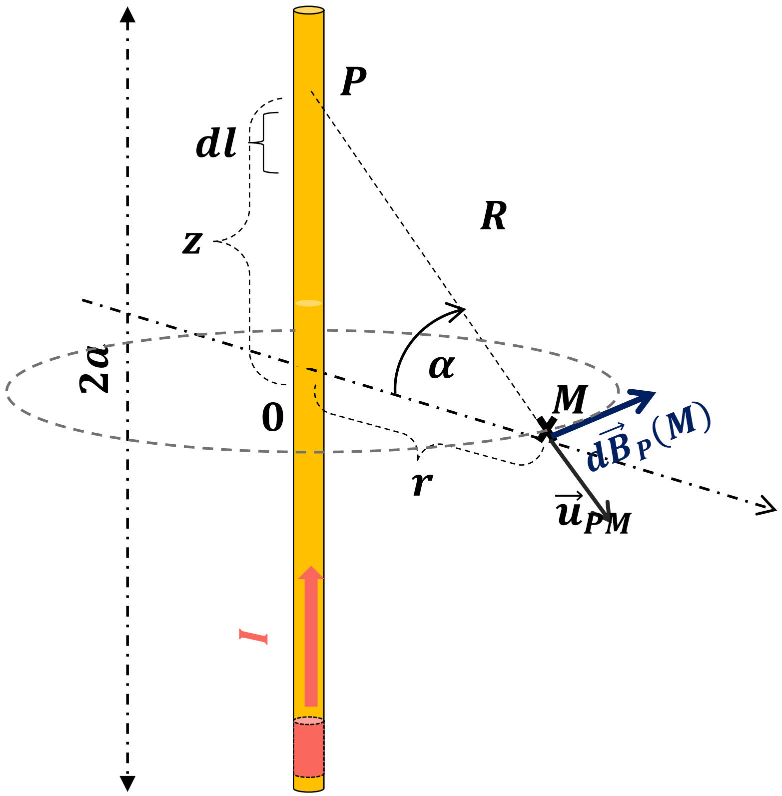
Besides that, the Biot-Savart law becomes in the case of surface density:

$$\vec{B}(M) = \int_{P\in(C)} d\vec{B}_P(M) = \iint_S \frac{\mu_0}{4\pi}.\frac{\overrightarrow{J_S(P)}.dS \wedge \vec{u}_{PM}}{PM^2}$$

$$= \iint_S \frac{\mu_0}{4\pi}.\frac{\overrightarrow{J_S(P)} \wedge \vec{u}_{PM}}{PM^2}.dS$$

Example 03:

In this exercise, we will calculate the magnetic field $\vec{B}(M)$ induced by a straight wire with a length $l = 2a$, crossed by a steady direct current I. We will examine the case $l \to \infty$



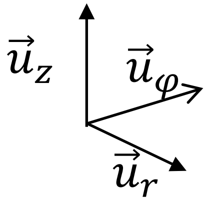
$$PM \equiv R$$

$$OM \equiv r$$

$$OP \equiv z$$

$$\cos\alpha = r/R$$

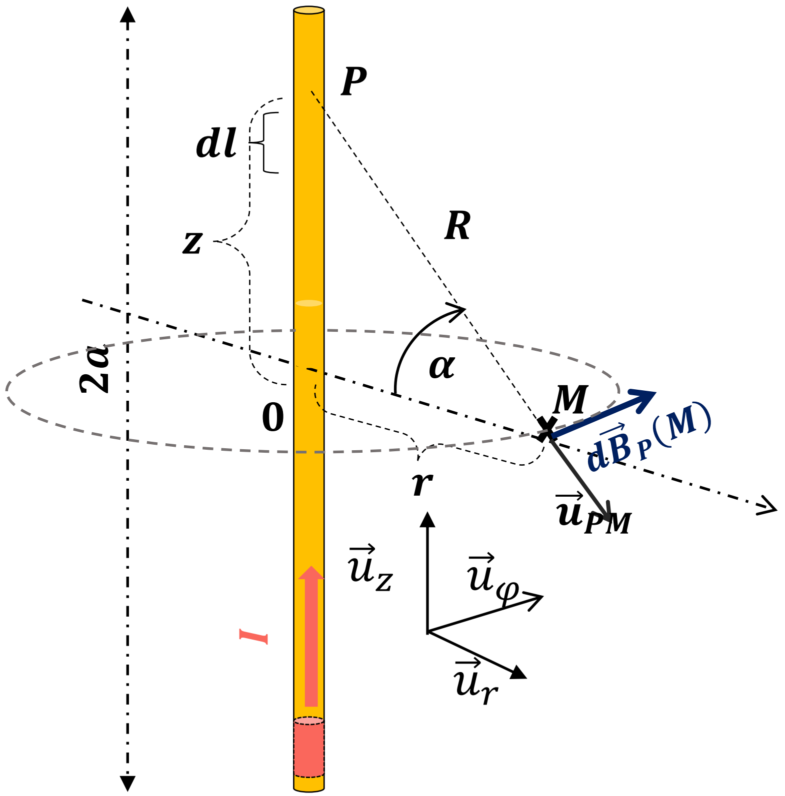
$$dl \equiv dz$$



$$d\vec{B}_P(M) = \frac{\mu_0}{4\pi}\frac{I.\,d\vec{l}\wedge\vec{u}_{PM}}{PM^2}$$

Solution of example 03:

Due to the cylindrical symmetry of the problem, the only non-zero component of $\vec{B}(M)$ is the azimuthal one $\overrightarrow{B_\varphi}(M)$:



$$d\vec{B}_P(M) = \frac{\mu_0}{4\pi}\frac{I.d\vec{l}\wedge\vec{u}_{PM}}{PM^2} = \frac{\mu_0}{4\pi}\frac{I.dl.\vec{u}_z\wedge\vec{u}_{PM}}{R^2}$$

With: $\vec{u}_{PM} = \cos\alpha.\vec{u}_r - \sin\alpha.\vec{u}_z$

$$\cos\alpha = \frac{r}{R} = \frac{r}{\sqrt{r^2+z^2}}$$

$$\frac{z}{r} = tan\alpha \rightarrow dl \equiv dz = \frac{r d\alpha}{\cos^2\alpha}$$

$$d\vec{B}_P(M) = \frac{\mu_0}{4\pi}\frac{I.\vec{u}_z\wedge\vec{u}_r}{R^2}dz.\cos\alpha = \frac{\mu_0 I}{4\pi}\frac{\cos\alpha.dz}{R^2}\vec{u}_\varphi$$

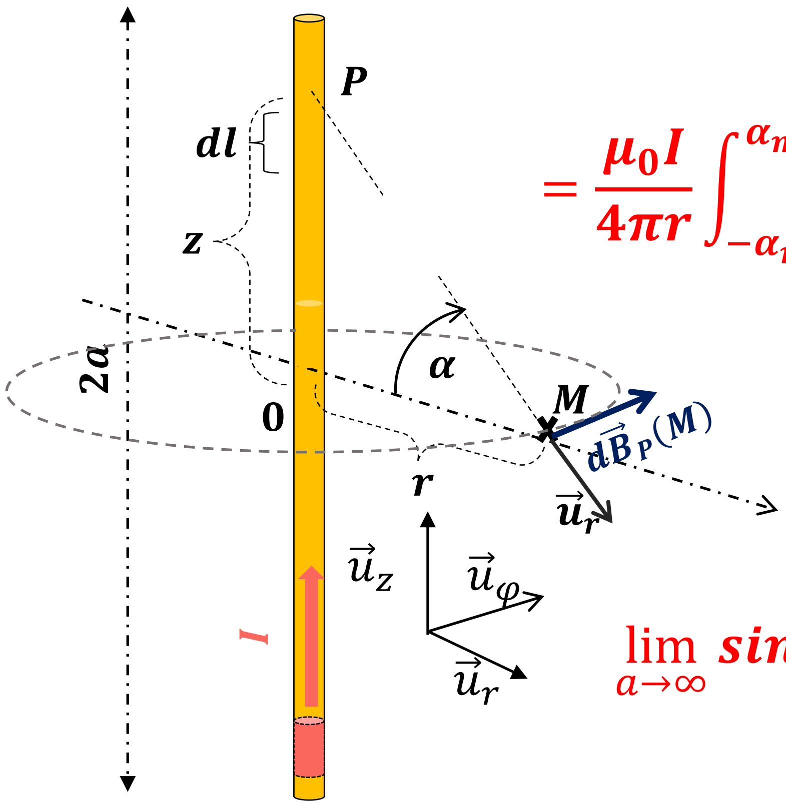
$$= \frac{\mu_0 I}{4\pi}\cos\alpha\frac{\cos^2\alpha}{r}\frac{d\alpha}{\cos^2\alpha}\vec{u}_\varphi$$

Solution of example 03:

$$d\vec{B}_P(M) = \frac{\mu_0}{4\pi}\frac{I.\vec{u}_z \wedge \vec{u}_r}{R^2}cos\alpha = \frac{\mu_0 I}{4\pi}\frac{cos\alpha.r.dz}{R^2}\vec{u}_\varphi = \frac{\mu_0 I}{4\pi}cos\alpha\frac{cos^2\alpha}{r}\frac{d\alpha}{cos^2\alpha}\vec{u}_\varphi$$

$$\vec{B}_P(M) = \int d\vec{B}_P(M) = \frac{\mu_0 I}{4\pi}\int\frac{cos\alpha.d\alpha}{r}\vec{u}_\varphi$$

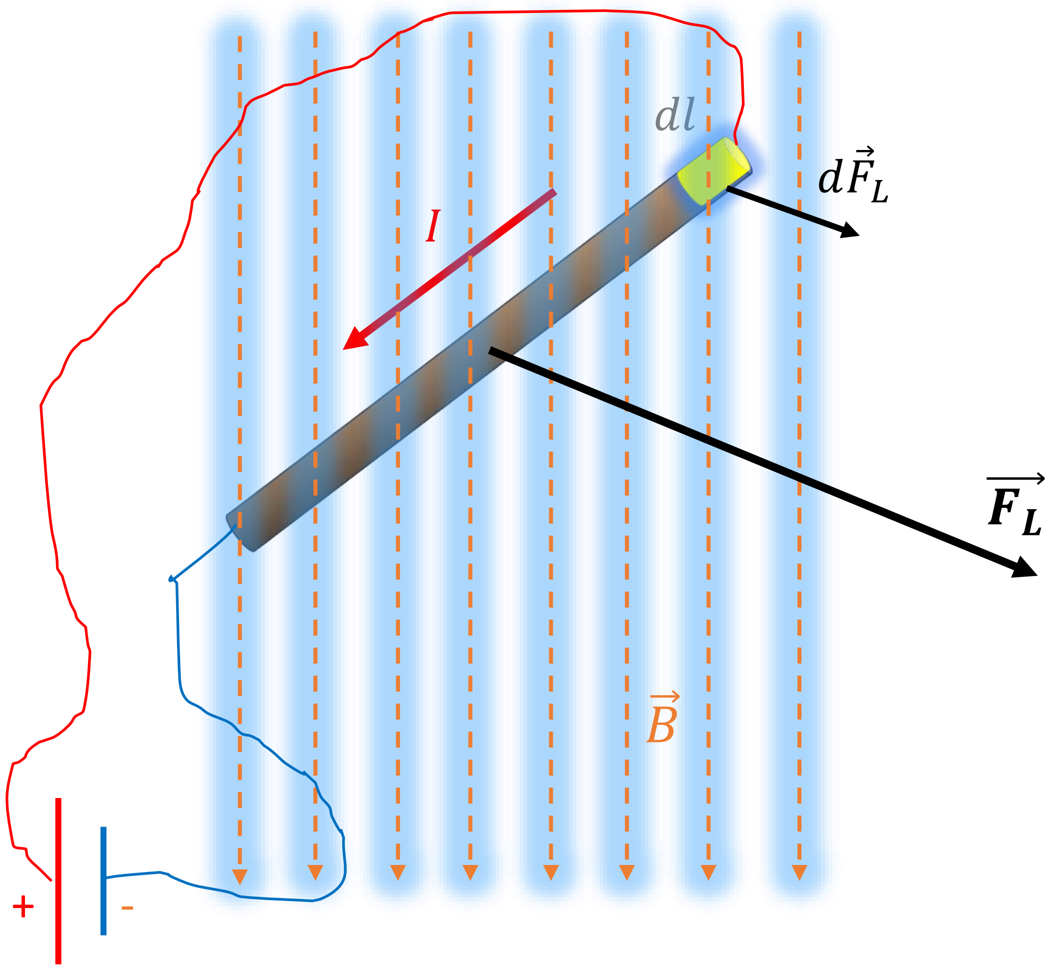
$$= \frac{\mu_0 I}{4\pi r}\int_{-\alpha_{max}}^{\alpha_{max}} cos\alpha.d\alpha\,\vec{u}_\varphi = \frac{\mu_0 I}{4\pi r}[2sin\,\alpha_{max}] = \frac{\mu_0 I}{2\pi r}[sin\,\alpha_{max}]$$

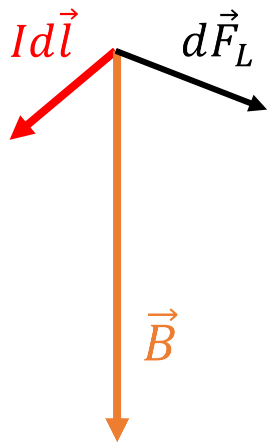


$$sin\,\alpha_{max} = \frac{a}{\sqrt{a^2+r^2}}$$

$$\lim_{a\to\infty} sin\,\alpha_{max} = 1 \rightarrow \lim_{a\to\infty}\frac{\mu_0 I}{2\pi r}[sin\,\alpha_{max}] \rightarrow \vec{B}_P(M) = \frac{\mu_0 I}{2\pi r}\vec{u}_\varphi$$

10. Laplace force:

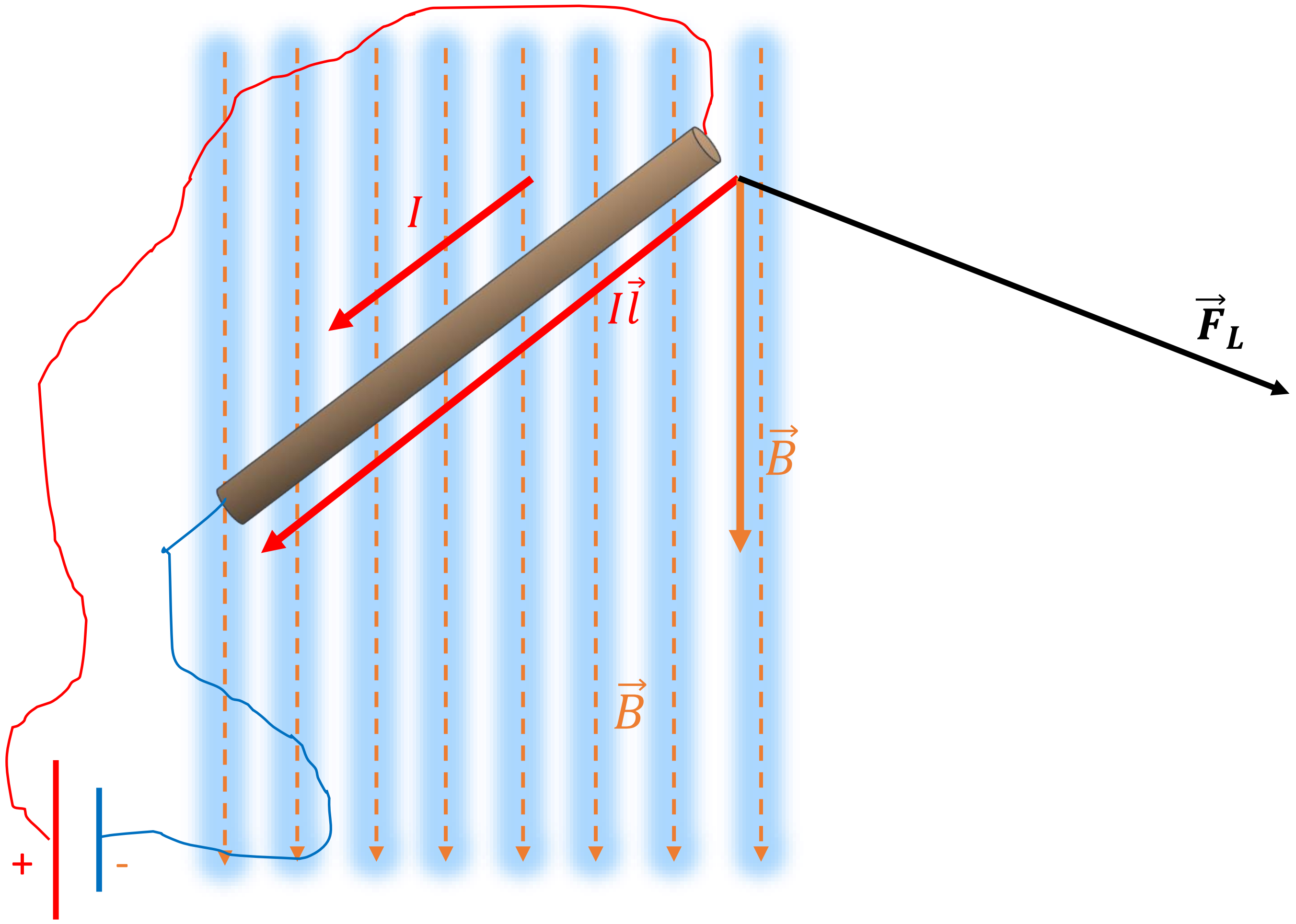




Pierre-Simon de Laplace

(1749-1827) France

10. Laplace force:



Pierre-Simon de Laplace

(1749-1827) France

II. Maxwell Equations

10. Laplace force:

When a conductor carrying a direct current intensity *I*, is put near a magnetic field, a mechanical force is applied on the wire and it tends to displace him in a perpendicular direction on both magnetic field and current flow. This force known as Laplace force is given by:

$$d\vec{F}_L = I.\overrightarrow{dl}\wedge\vec{B} \rightarrow d\vec{F}_L = \int I.\overrightarrow{dl}\wedge\vec{B}$$

Pierre-Simon de Laplace (1749-1827) France

In the case of uniform magnetic field, it is possible de perform the integration to obtain the force expression:

$$\vec{F}_L = I.\vec{l}\wedge\vec{B} = I.l.B.\sin\theta\,\vec{u}$$

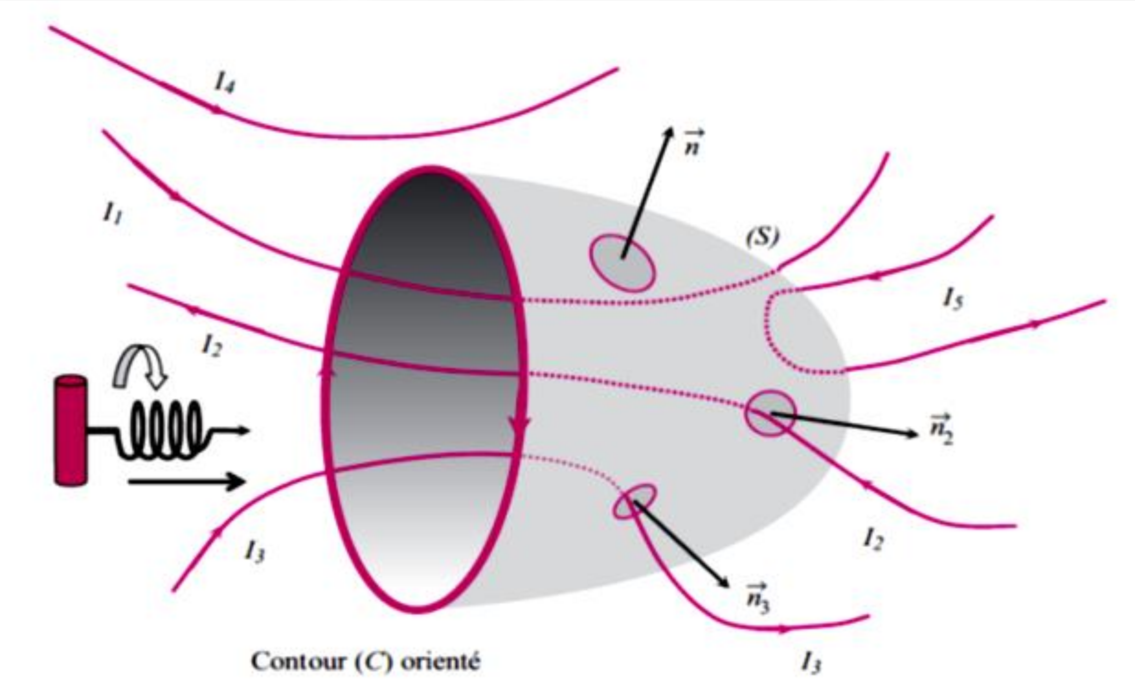
$$\vec{F}_L\Big|_{max} = I.\vec{l}\wedge\vec{B} = I.l.B.\sin\frac{\pi}{2}\,\vec{u} = I.l.B$$

II. Maxwell Equations

11. Ampere Theorem:

The Ampere theorem states that the magnetic field circulation through a closed path enclosing several currents I_k is directly proportional to the sum of these currents $\sum_k I_k$:

$$\oint dC = \oint \vec{B}.d\vec{l} = \mu_0 \sum_k I_k$$



André-Marie Ampère
1775-1836 (France)

II. Maxwell Equations

11. Ampere Theorem:

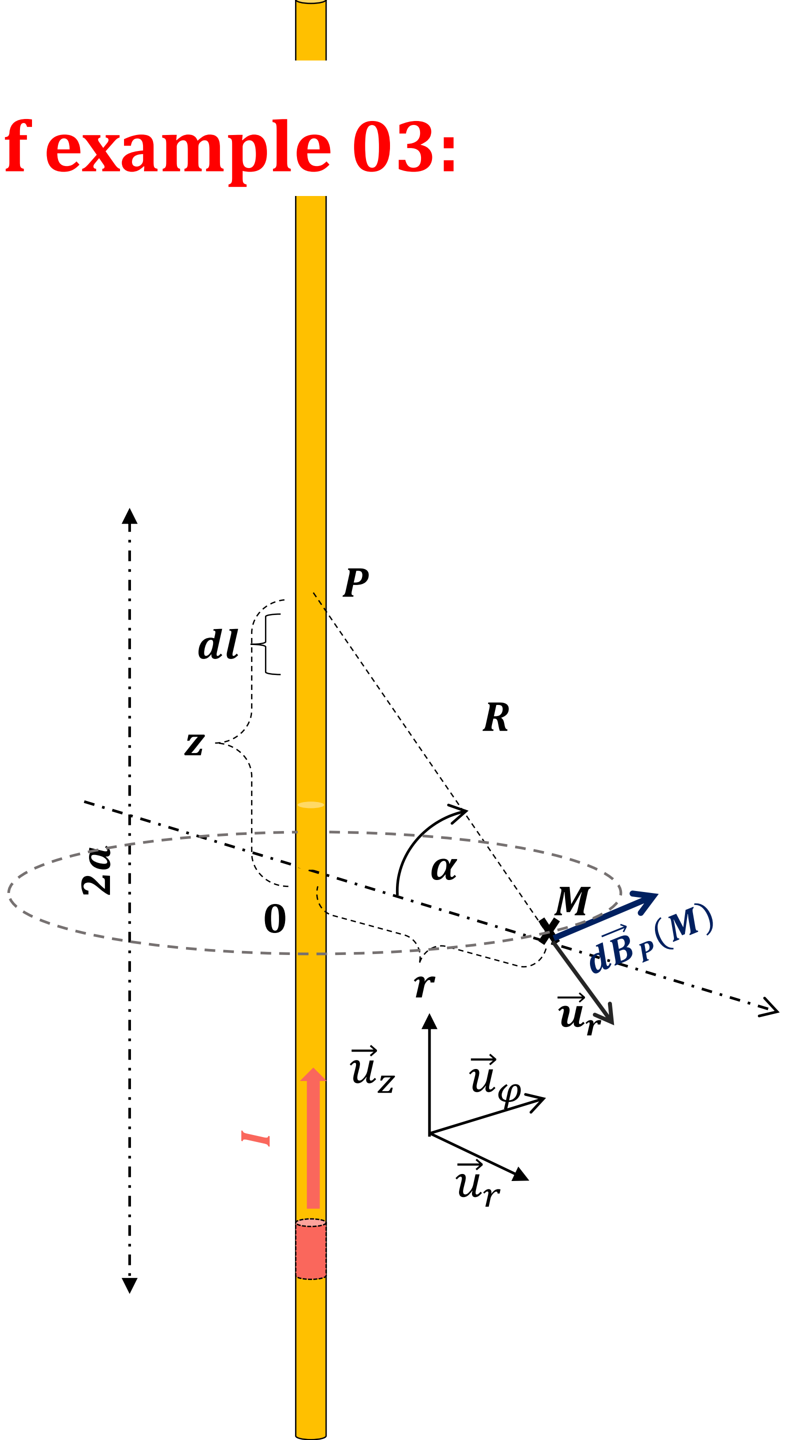
In the case of colinear straight currents, we obtain a uniform magnetic field parallel to the contour given by the Ampere law:

$$\oint dC = \oint \vec{B}.d\vec{l} = \vec{B}\oint d\vec{l} = B.L = \mu_0 \sum_k I_k \rightarrow B = \frac{\mu_0 \sum_k I_k}{L}$$

Application: ***by using this law to calculate again the magnetic field induced by the straight wire traversed by steady direct current I, in a given point M located at the radial distance r from the wire.***

André-Marie Ampère
1775-1836 (France)

Solution of example 03:



$$\int \vec{B}_P \, . \, d\vec{l} = \mu_0 \sum_k I_k$$

$$\rightarrow B_P \int_0^{2\pi} r d\,\varphi \vec{u}_\varphi . \vec{u}_\varphi = B_P 2\pi r = \mu_0 I$$

$$\rightarrow B_P = \frac{\mu_0 I}{2\pi r}$$

II. Maxwell Equations

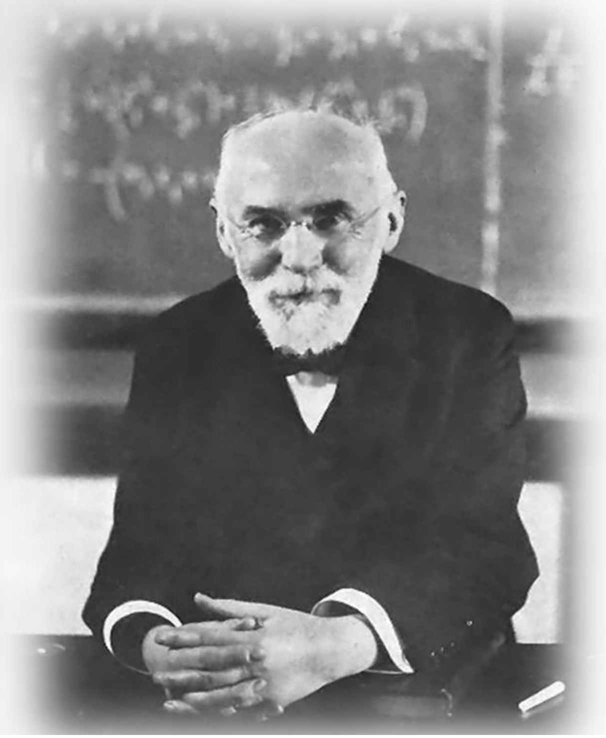
12. Lorentz force:

In the presence of electric field, any charged particle will feel an applied electrical force given by: $\vec{f}_e = q\vec{E}$

Similarly, if the same charged particle is animated with a celerity $\vec{v}$ in presence of a magnetic field, it will feel a magnetic force known as Lorentz force: $\vec{f}_m = q.\vec{v}\wedge\vec{B}$

In the case, where both fields are present, we get the general electromagnetic Lorentz force:

$$\vec{f}_L = \vec{f}_e + \vec{f}_m = q.\vec{E} + q.\vec{v}\wedge\vec{B} = q.\left(\vec{E} + \vec{v}\wedge\vec{B}\right)$$



Hendrik LORENTZ

(1853-1928) Netherland

12. Deriving Laplace force from magnetic Lorentz force:

If we consider a density n of charged particles animated with an average celerity $\vec{v}$ crossing a wire section S in presence of a magnetic field $\vec{B}$, where each individual particle will feel the force: $\vec{f}_m = q.\vec{v}\wedge\vec{B}$

Over an elementary distance dl, an elementary volume $dV = S.dl$ will represent a number of charges: $N = n.S.dl$

This will constitute an element of macroscopic force:

$$d\vec{F}_m = n.S.dl.q.\vec{v}\wedge\vec{B} = (q.n.S.v)d\vec{l}\wedge\vec{B} = Id\vec{l}\wedge\vec{B}$$

With by definition, we have: $I = q.n.S.v$

P-S. Laplace

H. LORENTZ

II. Maxwell Equations

13. Magnetic induction and excitation:

In physics the term magnetic field points usually to the physical value measured in Tesla: $\vec{B}[T]$, While the physical value : $\vec{H}[A/m] = \mu\vec{B}[T]$ is defined as *"magnetic excitation".*

Where $\mu = \mu_r\mu_0$ points to the magnetic permeability of the given media where $\vec{B}$ is present.

In engineering, $\vec{B}[T]$ is called the magnetic induction While : $\vec{H}[A/m] = \mu\vec{B}[T]$ is defined as "magnetic field".

Relative and absolute magnetic permeability for some media

Medium	μ_r	$\mu[H.m^{-1}]$
Vaccum	1.00000000	$1.25663062 \times 10^{-6}$
Air	1.00000037	$1.25663753 \times 10^{-6}$
Water	0.999992	1.256627×10^{-6}
Wood	1.00000043	$1.25663760 \times 10^{-6}$
Concrete	1.00000000	$1.25663062 \times 10^{-6}$
Iron	2×10^5	2.5×10^{-1}