



Series 02: (Functions with two variables)

First exercise:

Let the following function be:

$$f(x, y) = 2x^2 + y^2 + 7$$

1. Find the function definition set.
2. Calculate:

$$f(0, 0), f(-1, 2), f(3, 5), f(0, -1)$$

Second exercise:

Find the definition field of the following functions:

$$f(x, y) = \frac{x + y}{x^2 + y^2}$$

$$f(x, y) = \frac{5x + y^2 - 3}{x - y}$$

$$f(x, y) = \sqrt{y + x}$$

$$f(x, y) = \frac{\sin(xy)}{\sqrt{4 + x^2 + y^2}}$$

$$f(x, y) = \frac{3xy}{x^2 + y^2 + 3}$$

$$f(x, y) = \sqrt{y^2 - 5x + 6y}$$

$$f(x, y) = e^{xy} + \ln(xy)$$

$$f(x, y) = e^{5x - y^2 + 1}$$

$$f(x, y) = \frac{x - y}{x^2 - y^2}$$

Third exercise:

Find the first-order partial derivatives of the following functions:

$$f(x, y) = x^3 + y^2$$

$$f(x, y) = e^{2x} \cos(3y)$$

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$f(x, y) = x^2 + y^2 + xy - 5$$

$$f(x, y) = (x^2 + 4y^3)^5$$

$$f(x, y) = \sin(x^2 + y^2)$$

Fourth exercise:

Calculate:

$$f'_x(1, -1), f'_y(1, -1)$$

For function:

$$f(x, y) = \ln(1 + xy^2)$$

Fifth exercise:

Find the second-order partial derivatives of the following functions:

$$1) f(x, y) = x^2 + xy^2, \quad 2) f(x, y) = \ln(3x - 5y), \quad 3) f(x, y) = x^2 + 2y^2 - \frac{x^3}{y}$$

$$4) f(x, y) = e^{2x^2+xy+7x+y^2}, \quad 5) f(x, y) = \sin(xy)$$

Sixth exercise:

We say that a given function satisfies Laplace's equation if it satisfies the following condition:

$$f(x, y) = e^{-3y} \cos 3x \quad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$