Operations Research (OR)

course08- Modeling 5- Routing Problems
Traveling Salesman Problem (TSP)

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Outline

- Routing problem definition
- Traveling Salesman Problem (TSP)
  - Overview
  - Formulation
  - Example
- Summary

What is a Routing Problem?

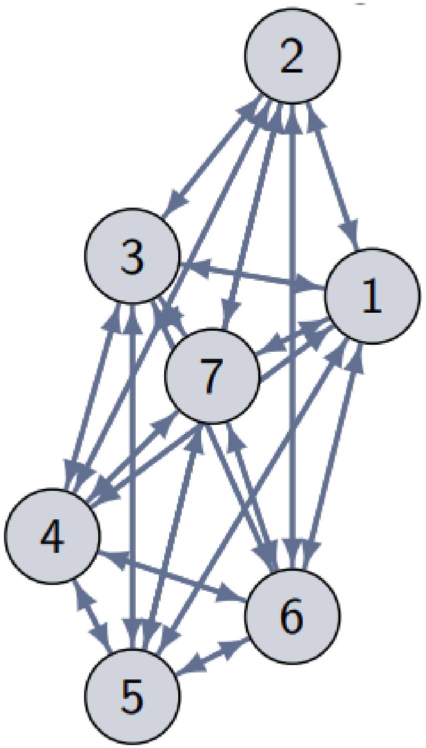
A routing problem refers to a class of optimization problems that involve finding the most efficient way to allocate and schedule resources, usually vehicles, to perform a set of tasks or visit a set of locations. The primary objective is to minimize or optimize a certain criterion, such as the total distance traveled, time taken, cost incurred, or a combination of these factors.

There are various types of routing problems, each with its own set of constraints and characteristics:

- Travel salesman problem (TSP)
- Vehicle routing problem (VRP)
- Capacitated vehicle routing problem (CVRP)
- Traveling Salesman Problem with Time Windows (TSPTW)
- Vehicle Routing Problem with Time Windows (VRPTW)

Traveling Salesman Problem (TSP) : Overview

- **Given:** Graph $G = (N ; E)$, distance matrix d
  Goal: Find the minimal distance **tour** that visits every node exactly once and returns
  to the starting node of the tour.



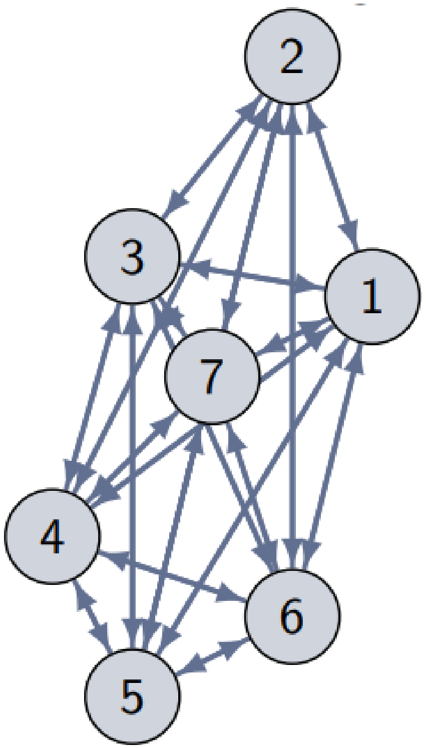
Number of potential routes?

- All permutations of n nodes: $n!$
- Many symmetries: Position of a city in the route is irrelevant
- If the distances are symmetric, we need to check only one direction

$$|S| = \frac{n!}{2n} = \frac{(n-1)!}{2}, n \geq 3$$

Traveling Salesman Problem (TSP) : Overview

- **Combinatorics**



$$|S| = \frac{n!}{2n} = \frac{(n-1)!}{2}, n \geq 3$$

Growth of S in terms of the input size n.

n	$\|S\|$
7	360
10	181,000
20	$\sim$ 10,000,000,000,000,000
50	$\sim 1x10^{62}$

TSP: Mathematical Formulation

Parameters:

- ***N*** Set of nodes
- ***E*** Set of edges ($N \times N$)
- d_{ij} Distance from node ***i*** to ***j***

Decision Variables:

- x_{ij} : 1 iff $(i, j) \in E$ is part of the solution

Objective Function: Min total distance

$$Min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$$

Constraints:

S.t.

(exactly one incoming edge)

$$\sum_{\{i \mid (i,j) \in E\}} x_{ij} = 1, \qquad \forall j \in N$$

(exactly one outgoing edge)

$$\sum_{\{k \mid (j,k) \in E\}}^{n} x_{jk} = 1, \qquad \forall j \in N$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in N$$

TSP: Example

Problem descritption

- Joe owns business in cities 1 through 5
- At the end of each year he must take a tour , i.e. visit each of these cities exactly once and come back to the city he starts
- What order of visiting will minimize the total distance traveled?

TSP: Example

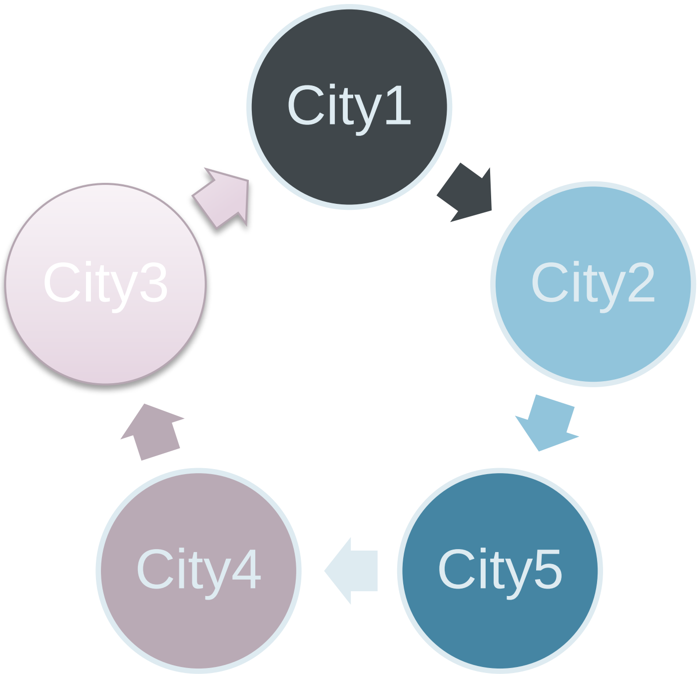
Distance (or cost) matrix

	City 1	City 2	City 3	City 4	City 5
City 1	∞	132	217	164	58
City 2	132	∞	290	201	79
City 3	217	290	∞	113	303
City 4	164	201	113	∞	196
City 5	58	79	303	196	∞

- c_{ij}: distance from city i to city j
- $c_{ii} = \infty$, a large positive number
- Symmetric: $c_{ij} = c_{ji}$, for all $i,j=\{1,\ldots,5\}$
- Otherwise, the problem is asymmetric

TSP: Example

- **Solution: 1-2-5-4-3-1**
- Minimal Total distance = 1134



Check video: Branch and bound Method to solve the problem

Summary

Today we learned:

- What is an routing problem
- What is the travel salesman problem(TSP)
- How to model TSP problem into a linear programming problem (LPP)

Questions?