Operations Research (OR)

course07- Assignment Problem and Hungarian Method

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Outline

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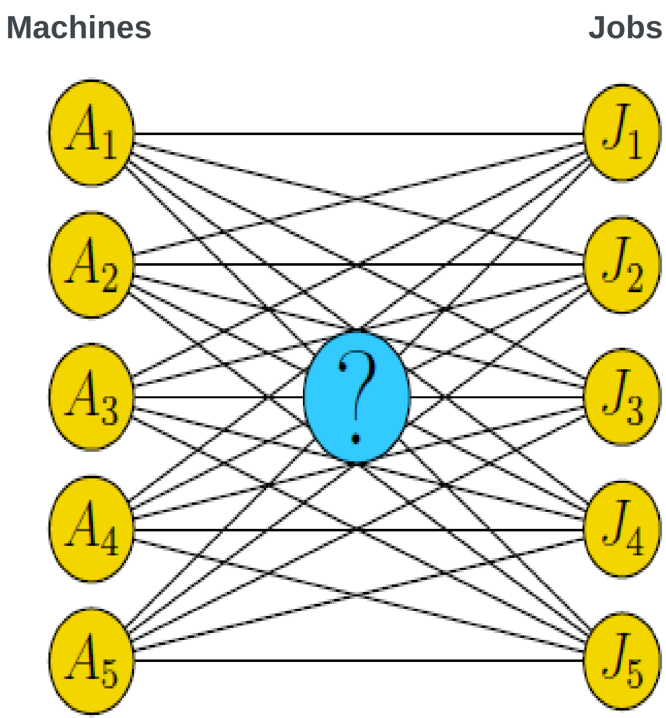
Assignment Problem: Overview

- An assignment problem may be considered as a special type of transportation problem in which the number of sources and destinations are equal.
- The capacity of each source as well as the requirement of each destination is taken as 1.
- In the case of an assignment problem, the given matrix must necessarily be a square matrix which is not the condition for a transportation problem.

Assignment Problem: Overview

- Consider the problem of assigning *n* jobs to *n* machines (one job to one machine). Let C_{ij} be the cost of assigning i^{th} job to the j^{th} machine and x_{ij} represents the assignment of i^{th} job to the j^{th} machine.

- x_{ij} = 1 if i^{th} *job is assigned to* j^{th} *machine*.
- x_{ij} = 0 if i^{th} *job is not assigned to* j^{th} *machine.*



Assignment Problem: Mathematical Formulation

Parameters:

- Job j_i
- Machine m_j
- Cost c_{ij}

Decision Variables:

- x_{ij} : Assignment of job *i* to machine *j*

Objective Function:

Minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Constraints: S.t.

$$\sum_{j=1}^{n} x_{ji} = 1, \qquad \forall i = 1,2,\cdots,n$$

$$\sum_{i=1}^{n} x_{ji} = 1, \qquad \forall j = 1,2,\cdots,n$$

$$x_{ij} \in \{0,1\}\ for\ all\ i,j$$

Hungarian Method: Steps

1. **Step 1:** Select the smallest value in each row. Then, subtract this value from each value in that row,

2. **Step 2:** in the new matrix, select the smallest value in each column. Then, subtract this value from each value in that column,

3. **Step 3**: Draw the min number of (raw or col) lines to cover all 0s. If the number of lines >= n, an optimal solution is available among the covered zeros. If number of lines is < n, go to step 4

4. **Step 4:** Find the smallest nonzero elements (k) that is not covered by the lines drawn in step 3. subtract k from each element that is not covered and add k to each element that is covered by two lines. Return to step 3.

Hungarian Method: Example

1. A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	II	III	IV
A	8	13	25	28
B	15	12	26	25
C	0	6	16	17
D	6	0	14	16

2. How should the tasks be allocated, one to a man, so as to minimize the total man hours?

Hungarian Method: Example

1. **Step 1 :** Subtracting the smallest element in each row from every element in that row, we get the first reduced matrix.

	I	II	III	IV
A	8	13	25	28
B	15	12	26	25
C	0	6	16	17
D	6	0	14	16

	I	II	III	IV
A	0	5	17	20
B	3	0	14	13
C	0	6	16	17
D	6	0	14	16

Hungarian Method: Example

1. **Step 2:** Next, we subtract the smallest element in each column from every element in that column; we get the second reduced matrix.

	I	II	III	IV
A	0	5	17	20
B	3	0	14	13
C	0	6	16	17
D	6	0	14	16

→

	I	II	III	IV
A	0	5	3	7
B	3	0	0	0
C	0	6	2	4
D	6	0	0	3

Hungarian Method: Example

1. **Step 3:** Now we test whether it is possible to make an assignment using only zero distances.

	I	II	III	IV
A	0	5	3	7
B	3	0	0	0
C	0	6	2	4
D	6	0	0	3

	I	II	III	IV
A	0	5	3	7
B	3	0	0	0
C	0	6	2	4
D	6	0	0	3

Number of lines < n ➔ this is not optimal solution **we go to step 4**

Hungarian Method: Example

1. **Step 4:** Find the smallest nonzero elements (k) that is not covered by the lines drawn in step 3. subtract k from each element that is not covered and add k to each element that is covered by two lines

	I	II	III	IV
A	0	5-k	3-k	7-k
B	3+k	0	0	0
C	0	6-k	2=k	4-k
D	6+k	0	0	3

	I	II	III	IV
A	0	3	1	5
B	5	0	0	0
C	0	4	0	2
D	8	0	0	3

Hungarian Method: Example

1. **Step 3:** Find the smallest nonzero elements (k) that is not covered by the lines drawn in step 3. subtract k from each element that is not covered and add k to each element that is covered by two lines

	I	II	III	IV
A	0	3	1	5
B	5	0	0	0
C	0	4	0	2
D	8	0	0	3

	I	II	III	IV
A	0	3	1	5
B	5	0	0	0
C	0	4	0	2
D	8	0	0	3

Number of lines = n ➔ this is the **optimal solution**

Hungarian Method: Example

1. The minimum total man hours are computed as :

	I	II	III	IV
A	0	3	1	5
B	5	0	0	0
C	0	4	0	2
D	8	0	0	3

	I	II	III	IV
A	1			
B				1
C			1	
D		1		

Z = 8+ 25 + 16 + 0= 49

Hungarian Method: Example

1. The minimum total man hours are computed as :

Optimal assignment	Man hours
A → I	8
B → IV	25
C → III	16
D → II	0
Total	49 hours

Summary

Today we learned:

- What is an assignment problem
- How to model this problem into a linear programming problem (LPP)
- Hungarian method for assignment problem

Questions?