Chapter 03: Dynamics

Dynamics descibes motion by translating the interactions that produce them in terms of forces.

Knowing the forces allows us to predict the nature of the motion that will occur

3-1 * Inertial frames: All reference with respect to which newtons first law is true.

* Newtons First Law: ~~of Newton~~ A free body remains at rest or in a state of uniform motion unless acted on by an external force.

* Free body : is not subjected to a force (no force acting to this body) or if the sum of the forces acting on the body is zero.

* Force = time rate of ~~momentum~~ : ie

~~where p = m~~ $F = \frac{dp}{dt}$

where $P = mv$ = momentum of body of mass m moving with the velocity v.

* The inertia of a body is the resistance that the body opposes to any variation caused by its speed.

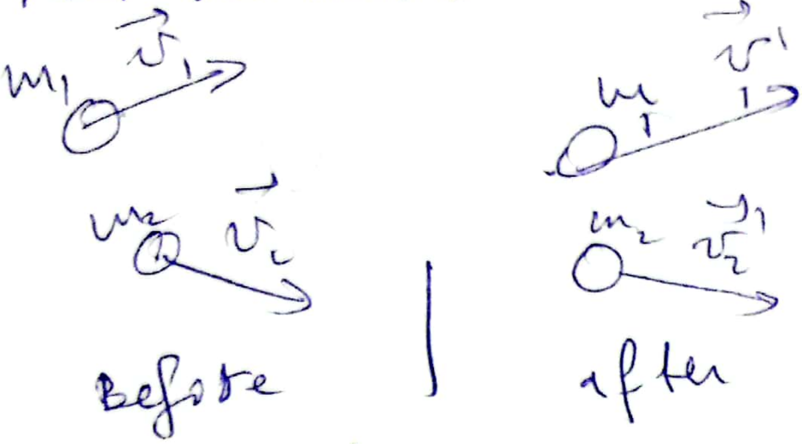
3-2 Conservation of Linear momentum:

Suppose we have ~~two~~ objects with no external ~~for~~ fo forces acting on them,

A the moment t : $\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2$

" " " t' $\vec{P}' = m_2 \vec{v}_2' + m_2 \vec{v}_2'$

We note that we have vectorial quantities.



$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_2 \vec{v}_1' + m_2 \vec{v}_2'$$

$$m_1 (\vec{v}_1' - \vec{v}_1) = -m_2 (\vec{v}_2' - \vec{v}_2)$$

$$\Delta \vec{P}_1 = -\Delta \vec{P}_2$$

A loss of the momentum of a particle ~~results~~ ~~in~~ results in the gain of the other's momentum.

3_3 Newton Laws: L1 Every body preserves in its state of resting, or uniformly in a straight line (ie with constant velocity) unless acted upon by a force.

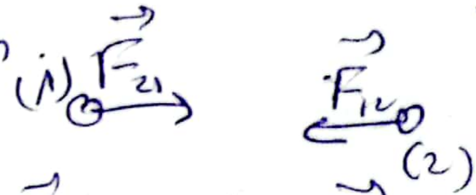
Second Law: If $\vec{F}$ is the (external) force acting on a particle of mass m which as a consequence is moving with velocity v, then,

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{P}}{dt}$$

where $\vec{p} = m\vec{v}$. If m is independent of time this becomes $\vec{F} = m\dfrac{d\vec{v}}{dt} = m\vec{a}$

where $\vec{a}$ is the acceleration of the particle

3rd Law: If 1 acts on particle 2 with a force $\vec{F}_{12}$ in a direction along the line joining the particles, while particle 2 acts on particle 1 with a force $\vec{F}_{21}$, then $\vec{F}_{21} = -\vec{F}_{12}$. In other words, to every action there is an equal and opposite reaction.



$$\vec{P} = \vec{P}_1 + \vec{P}_2 = \underline{cte}$$

$$\frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \frac{d\vec{P}_1}{dt} = -\frac{d\vec{P}_2}{dt} \Rightarrow \vec{F}_{21} = -\vec{F}_{12}$$

For the 2nd Law:

At $t = t_1 \Rightarrow \vec{p}_1 = m\vec{v}_1$

$t = t_2$ $\vec{p}_2 = m\vec{v}_2$

the variation is in the time interval $\Delta t = t_2 - t_1$

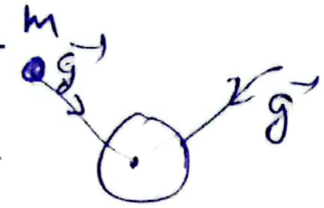
$$\Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = m(\vec{v}_2 - \vec{v}_1) \Rightarrow \Delta\vec{P} = m\,\Delta\vec{v}$$

$\dfrac{\Delta\vec{P}}{\Delta t}$ represent the interaction between the body and the $\quad \vec{F}_{AV} = \dfrac{\Delta\vec{P}}{\Delta t} = m\dfrac{\Delta\vec{v}}{dt} = m\vec{a}_m$

8-4 Applications of Newton's Laws:

8-4-1 Weight and acceleration due to gravity:

It is found experimentally that near the earth's surface objects fall with a vertical acceleration which is constant provided that air resistance is negligible. It is denoted by g and is called the acceleration due to gravity or the gravitational acceleration. $g \simeq 9.80\ m/s^2$: This value varies at different parts of the earth.



The force acting on particle of mass m is given by $\vec{W} = m\vec{g}$

This force is called weight of particle.

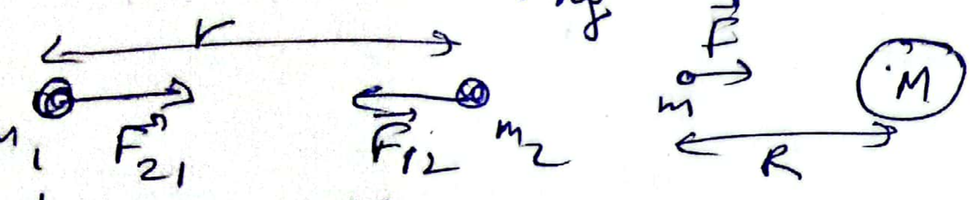
8.4-2 The Law of Gravity

Newton's law of universal gravitation states, that every particle of matter in the universe attracts every other particle of matter in the universe with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them

$$\vec{F} = \frac{-G\, m_1 m_2}{r^2}$$

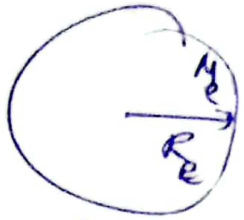
The constant G is called the universal gravitational constant and has the value

$$G = 6.676 \times 10^{-11}\ N.m^2/kg^2$$



This leads to $F = \frac{G m M}{R^2}$

Appling this to an object of mass m just at the earth's surface, we have $F = \frac{G m M_e}{R_e^2}$ where M_e and R_e are the earth's mass and Radius, respectively. Since $\vec{F}$ is just the weight of the object, we also have $\vec{F} = m\vec{g}$

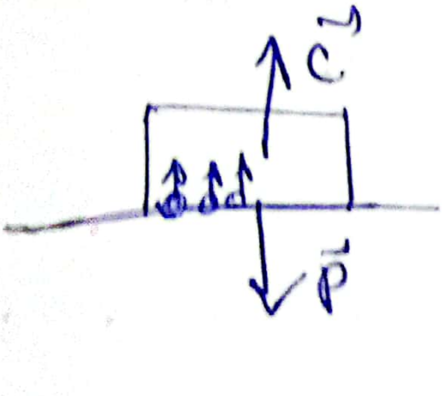


Thus $mg = \frac{G m M_e}{R_e^2}$ $\Rightarrow$ $\boxed{g = \frac{G M_e}{R_e^2}}$

The gravitational force is concretized in the phenomenon of tides at the coasts.

8.4-3 Contact forces :

If an object placed on a table, this last exerts an upward action force on the object. The reaction of the table ($\vec{C}$) on the object m is distributed over the entire table-object contact surface and represents the resultant of all actions exerted on the contact surface

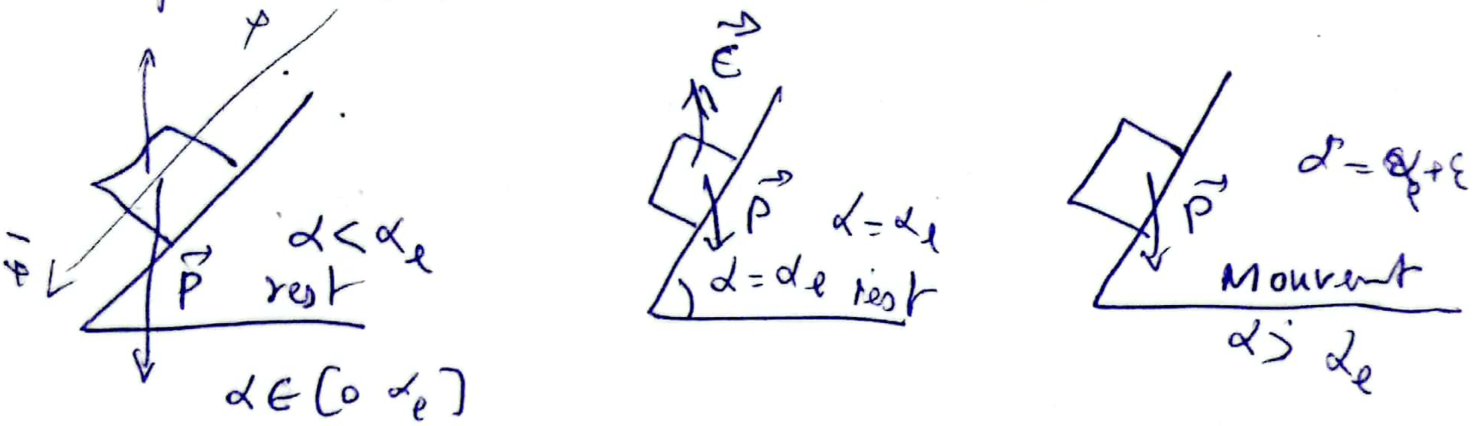


8-4-4 Friction forces

Frichon is the rubbing force between two objects whose surfaces are in contact. The force of friction always acts parallel to the touching surfaces. There is two types of friction

- Solid - Solid friction (solid friction)
- Solid - fluid friction (viscos friction)

We are interessing of the experimental study of this forces. Consider the system below:



We gradually increase the angle α for the inclined plane to a value for which the body will start moving.

For $\alpha = \alpha_e$ The body is always in rest

$$\vec{W} + \vec{C}_0 = \vec{0}$$

$\|\vec{C}_{0x}\| = mg \sin\alpha_e$ and $\|\vec{C}_{0y}\| = mg\cos\alpha_e$

The ratio $\frac{\|\vec{C}_{0x}\|}{\|\vec{C}_{0y}\|}$ is called the coefficient of static friction, that depends on the nature of the two surface: $\mu_s = \tan\alpha_e$

α_e is the friction angle.

Remarks:

- μ_s is independent to the mass of the studied body
- μ_s depend only on the nature of the contacted bodies.

b/ ~~Dynamic~~ Coefficient of kinetic friction

Beyond the limit value α_ℓ, The body of the mass m slide on the inclined plane with $\alpha = \alpha_\ell + \varepsilon$

$$\vec{C'} + \vec{W} = m\vec{a} \text{ with } \vec{a} \neq 0$$

After projection we get:

$$-\|\vec{C'}_x\| + mg \sin(\alpha_\ell + \varepsilon) = ma$$

$$\|\vec{C'}_y\| - mg\cos(\alpha_\ell + \varepsilon) = 0$$

$$mg\sin(\alpha_\ell + \varepsilon) \simeq \|\vec{C}_{0x}\| = W\sin\alpha_\ell$$

This gives

$$-\|\vec{C'}_x\| + \|\vec{C}_{0x}\| > 0 \Rightarrow \|\vec{C}_{0x}\| > \|\vec{C'}_x\|$$

with projection on the oy axes, we get:

$$\|\vec{C'}_y\| = \|\vec{C}_{0y}\| \simeq mg\cos\alpha_\ell$$

The coefficient of kinetic friction is given by:

$$\mu_k = \mu_d = \frac{\|\vec{C'}_x\|}{\|\vec{C'}_y\|} < \mu_s$$

Experimentally, we have:

- $\mu_k < \mu_s$.
- μ_k depends only on the nature of the two surfaces.