Chapter 0

Mathematical reminder

1. What is physics :

Physics is the study of all physical phenomena - phenomeme exprienced. ~~throug the~~ either directly or with the aid of instruments.

2- Measurement and units

To find relationships that decrible physical phenomena we must be able to measure physical quantities. such as length, volume Velocity, mass, time. To do this we need units of measurement

2-1- Dimensions : there are three quantities of ~~Dimensions~~ fundamental dimensions

* Length : the basic unit is the meter L
* Mass : " " is the Kilogram M
* time : " " " the second T

* Remark : the is several units system.

1 the CGS system (Centimeter, gram, Second
2 " MSKA sytem (Meter, Kilogram, Second
3 " MTS (Meter, Ton, Second)

we note that there is three fundamental units: Meter, Kilogram, Second

2-2 Derived units:

The units of physical quantities other than the fundamental units are called derived units

Example: Area: the square meter m^2

Velocity: the meter per second.

3- Dimension and dimensional equation

Any physical quantity is characterized by its dimension which is a property associated with a units. the dimension of the quantity Q is noted [Q]. It informs us about the physical nature of the Quantity.

example: if Q has the dimension of mass we say that it is homogeneous to a mass the relation. $[Q] = M$ correponds to the dimensional equation of the quantity Q.

Physical quantity	Dimension	CGS system	MSKA system
Length	L	cm	m
Mass	M	g	Kg
Time	T	s	s
velocity	$L.T^{-1}$	cm/s	m/s
Acceleration	$L.T^{-1}$	g cm/s^2	kg m/s^2 = Newton
Energy work	ML^2T^{-2} = erg		N.m Joul
Density	ML^{-3}	g/cm^3	Kg/m^3

II Vector Calculation

Scalar quantity and vector quantity.

In physics, we use two types of quantities: scalar quantities and vector quantities.

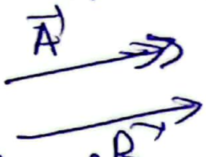
- Scalar quantities are defined by a number and appropriate unit.
  Example: the mass m of a body
  the length ℓ of an object
- Vector physical quantities: quantities specified by a ~~number and~~ magnitude on appropriate unit ~~and~~ plus direction, we can Bite the speed $\vec{V}$ and the weight $\vec{P}$ and displacement $\vec{XM}$
  these quantities require for their specification a direction as well as magnitude.
  denoted by $\|\vec{A}\|$ ~~or~~ A

Vector Algebra

The operations of addition, substraction and multiplication familiar in the algebra of real numbers are with suitable definition capable of extension to an algebra of vectors.

The following definitions are fundamental:

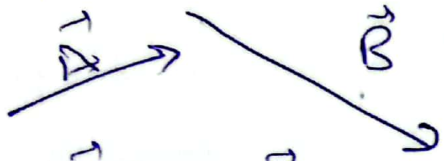
1 - Two vectors $\vec{A}$ and $\vec{B}$ are equal if they have the same magnitude and direction regardless of their initial points.
Thus $\vec{A} = \vec{B}$.

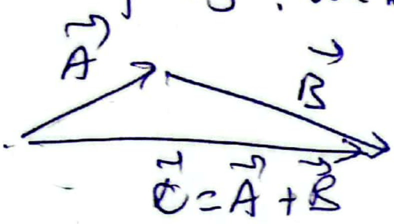


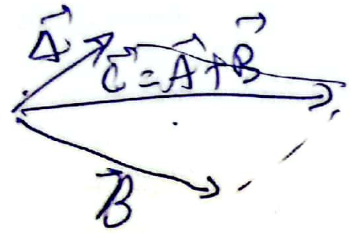
2 - $\vec{A} = -\vec{B}$

same length
opposite direction
$\vec{A} = -\vec{B}$

3 the sum or resultant of vectors $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ formed by placing the initial point of $\vec{B}$ on the terminal point of $\vec{A}$ and joining the initial point of $\vec{A}$ to the terminal point of $\vec{B}$: we write $\vec{C} = \vec{A} + \vec{B}$







parallelogram law

4/ The difference of vetors $\vec{A}$ and $\vec{B}$, represented by $\vec{A}-\vec{B}$, is that vector $\vec{C}$ which when added to $\vec{B}$ gives $\vec{A}$

if $\vec{A}=\vec{B}$ so $\vec{A}-\vec{B}=\vec{0}$ is the nulle or zero vector.

5 the product $\vec{D}=P\vec{A}$ is a vector with P is a scalar

4-3 Laws of vector Algebra

If $\vec{A}$, $\vec{B}$, $\vec{C}$ are vectors and p and q are scalars, then:

1. $\vec{A}+\vec{B}=\vec{B}+\vec{A}$: Commutative law for addition

2- $\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$ Associative law for addition

3 $P.(q\vec{A})=(pq)\vec{A}=q(P\vec{A})$ Associative law for multiplication

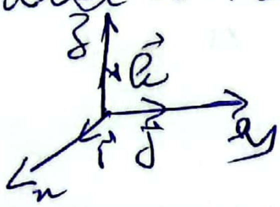
4-4 Unit Vectors:

Vectors having unit of length are called unit vectors. $\vec{A}=A\vec{u}$

A is the length of $\vec{A}$.

4-5 Rectangular unit vectors:

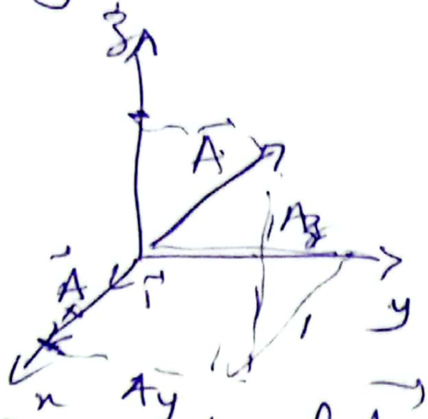
The rectangular unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ are mutually perpendicular unit vectors having directions of the positive x, y and z axes respectively of a rectangular coordinate system



4-6 Components of A vector

Any vector $\vec{A}$ in 3 dimensions can be represented with initial point at the origin O of a rectangular coordinate system

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$



$$\vec{A} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}, \quad \vec{A}(A_x, A_y, A_z)$$

A_x, A_y and A_z are called the components of $\vec{A}$

4-7

The magnitude of $\vec{A}$ is $\|\vec{A}\| = |\vec{A}| = A$

$$= \sqrt{A_x^2 + A_y^2 + A_z^2}$$

In particular, the position vector or radius vector $\vec{r}$ is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

and has magnitude $r = \|\vec{r}\| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

4-7 Dot or scalar product:

It is denoted by $\vec{A} \cdot \vec{B}$ (read $\vec{A}$ dot $\vec{B}$) and defined as the product magnitude of $\vec{A}$ and $\vec{B}$ and cosine of the angle between them. In symbols,

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos\theta$$

* $\vec{A} \cdot \vec{B}$ is a scalar and not a vector.

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the following laws are valid:

1. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ Commutative Law for dot Products
2. $\vec{A}\cdot(\vec{B}+\vec{C})$ Distributive Law

3- $p(\vec{A}\cdot\vec{B})$ $(p\vec{A})\vec{B} = \vec{A}\cdot(p\cdot\vec{B})$
$= (\vec{A}\cdot\vec{B})\,p$ p is a scalar

4- $\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$ $\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = \vec{0}$

5- If $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ and $\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$

then $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z)$

$\vec{A}\cdot\vec{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

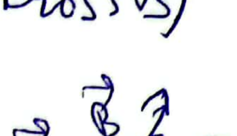
$\vec{B}\cdot\vec{B} = B^2 = B_x^2 + B_y^2 + B_z^2$

6- If $\vec{A}\cdot\vec{B} = \vec{0}$ and A and $\vec{B}$ are not null vectors, then A and B .

4-8 Cross or vector product:

The vector product $\vec{C} = \vec{A}\times\vec{B}$ (read A cross B) is a vector:

* $\|\vec{C}\| = \|\vec{A}\|\,\|\vec{B}\|\,\sin(\vec{A},\vec{B})$



* The direction of the vector $\vec{C} = \vec{A}\times\vec{B}$ is perdiculan to the plane of $\vec{A}$ and $\vec{B}$ and such that $\vec{A}$, $\vec{B}$ and $\vec{C}$ form a right-handed system.

The following laws are valid

1. $\vec{A} \times \vec{B} = -\vec{B} \wedge \vec{A}$ Commutative Law for Cross products Fails

2. $\vec{A} \wedge (\vec{B} + \vec{C}) = \vec{A} \wedge \vec{B} + \vec{A} \wedge \vec{C}$ Distributive Law

3. $\vec{i} \wedge \vec{i} = \vec{j} \wedge \vec{j} = \vec{k} \wedge \vec{k} = \vec{0}$

$\vec{i} \wedge \vec{j} = \vec{k}$, $\vec{j} \wedge \vec{k} = \vec{i}$, $\vec{k} \wedge \vec{j} = \vec{j}$

4. If $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$ and $\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$

then:

$$\vec{C} = \vec{A} \wedge \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

5. $\|\vec{A} \wedge \vec{B}\|$ is the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$

6. If $\vec{A} \wedge \vec{B} = \vec{0}$; $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are parallel.

7. $\vec{A} \wedge (\vec{B} \wedge \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

V- Dérivatives of vectors:

If to each value assumed by a scalar variable t there corresponds a vector $\vec{A}(t)$ then $\vec{A}(t)$ is called a function of t; the derivative of $\vec{A}(t)$ is given by:

$$\frac{d\vec{A}}{dt} = \lim_{\Delta t \to 0} \frac{(\vec{A}(t+\Delta t) - \vec{A}(t))}{\Delta t}$$

If $\vec{A}(t) = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$, then

$$\frac{d\vec{A}}{dt} = \frac{dA_x}{dt}\vec{i} + \frac{dA_y}{dt}\vec{j} + \frac{dA_z}{dt}\vec{k}$$

Example: $\vec{A} = (2t^2 - 3t)\vec{i} + 5\cos t\,\vec{j} - 3\sin t\,\vec{k}$

$$\frac{d\vec{A}}{dt} = (4t-3)\vec{i} - 5\sin t\,\vec{j} - 3\vec{k}$$

5:- Gradient divergence and Curl

If to each point (x, y, z) of a rectangular coordinate system there corresponds a vector $\vec{A}$ we say that $\vec{A} = \vec{A}(x, y, z)$ is a vector function of x, y, z. Similarly we call the scalar function $\phi(x, y, z)$.

It is convinient to consider a vector differential operator called del given by

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

Using this, we define the following important quantities.

1. Gradient $\nabla\phi = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\phi$

$$= \vec{i}\frac{\partial \phi}{\partial x} + \vec{j}\frac{\partial \phi}{\partial y} + \vec{k}\frac{\partial \phi}{\partial z}$$

This is a vector called the gradient of ϕ and is also written $\overrightarrow{grad}\,\phi$

2 Divergence $\vec{\nabla}\cdot\vec{A} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\cdot\left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right)$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

This is a Scalar called divergence of $\vec{A}$ and is also written $div\,\vec{A}$

3 Curl $\vec{\nabla}\wedge\vec{A} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\times\left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{j}$$

$$+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}$$

This Vector called the Curl of A and is also written $Curl\,\vec{A}$

$$div\,Curl\,\vec{A} = \vec{\nabla}(\vec{\nabla}\wedge\vec{A}) = 0$$

$$Curl\,grad\,\phi = \vec{\nabla}(\vec{\nabla}\phi) = \vec{0}$$