Chapter 1: Kinematics

I Introduction:

Kinematics is the study of the physical quantities that describe the motion of an object. The most fundamental of these quantities are the displacement (where the object is) and the time (when it is there).

The motion and the rest are two relative motions, we have to choose a reference point.

I-2 Kinematic quantities:

I-2-1 Material point: for an object to be located at a definite point on the x axis at a given time, it would ~~have~~ to be infinitesimally small.

1-2 Trajectory: all positions occupied by the moving point.

1-3 Origin: We choose an origin to ~~describe~~ identify the different positions over time.

1-4 Analytical definition of motion:

Consider an oxyz reference. The position of a moving point is defined, If we know at every moment t its coordinates. (1) $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$

3_ Velocity and speed:

3-1 Average Velocity:

If a particle is located at position x_1 at time t_1 and moves so that is occupies a different position x_2 at a later time t_2, then the average velocity v_{av} of the particle over that time period is defined as the relative displacement, $d_2 = x_2 - x_1$ divided by the time elapsed, $t_2 - t_1$:

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} \qquad [v_{av}] = m/s$$

3-2 Average speed. is defined as the total distance in a given time interval devided by that time interval. Since distance traveled is always positive, the average speed is always positive

3-3 Instantaneous velocity:

If the average velocity is defined in an interval than the instantanous velocity is defined in a specific point

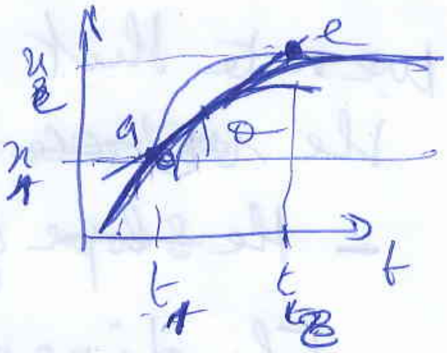
$$v(t) = \lim_{\Delta t \to 0} v_{av} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Example If $x(t) = 6t+3$ than $v(t) = 6$.

$$v(t) = 6 \ m/s$$

3-4 Geometrical interpretation of average velocity on an x vs. t Graph

$$v_{av}\Big|_{t_1}^{2} = \frac{\Delta x}{\Delta t}\Big|_{t_1}^{t_2}$$



According to the graph.

$$\frac{\Delta X}{\Delta T}\Big|_{t_1}^{t_2} = tg\,\alpha$$ = the slope of the chord ae
= ~~the angl~~ tangent of the angle θ

the slope is the ratio of the opposite side to the adjacent side of the triangle shown.

So the average velocity represent the slope of the chord (the tangent) of the graph $x = x(t)$.

4) Acceleration

4-1 Average acceleration

Just as average velocity is the "time rate of change" of ~~velocity~~ displacement

So average acceleration is the "time rate of change" of velocity:

$$a_{av} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$$

$$[a_{av}] = m/s^2$$

4-2 Instantaneous acceleration: a at a given time (say t_2), is defined as the limit of the average acceleration

$$a(t) = \lim_{\Delta t \to 0} a_{av} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

we note that there is two methods to determine the ~~acce~~ instantaneous acceleration:

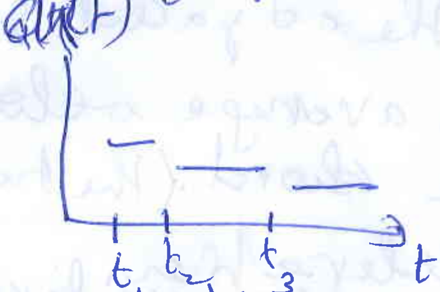
- the ~~slope~~ graphic method : the slope of the tagent.
- The direct method $a(t) = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

Example $x(t) = 15t^2 + 2$

$v(t) = 30t \Rightarrow a(t) = 30\ m/s^2$

5 - Acceleration diagram:

Acceleration diagram is the graph giving the acceleration in term of time.

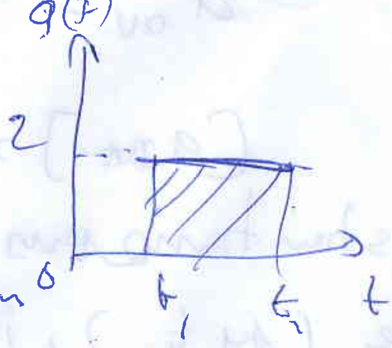


6 - finding Kinematic quantities.

6-1 Transition from acceleration to velocity

Taking for example the acceleration of a moving particle equal $a(t) = 2\ m/s^2$

$$\Rightarrow v(t) = \int_{t_1}^{t_2} a(t)\, dt$$



$\Rightarrow$ Variation of velocity between t_1 and t_2 equals to the area under a vs. t curve.

Finding the displacement from the v-vs.t Graph

If $v(t) = \frac{dx}{dt}$; We can write:

$$x(t) = \int_{t_1}^{t_2} v(t)\, dt$$

The variation of position of a moving particle between two moments t_1 and t_2 is given by the area under v vs. t curve.

7. Study of some particular motions.

7.1 Uniform linear Motion: (ULM)

In this type of motions, there is no change in the velocity ($V = cte$) and direction of the object at any point throughout the motion and hence, the acceleration is zero ($a = 0$). The motion is in a straight line.

To find $v(t)$ and $x(t)$, we consider the initial conditions: at $t = 0\,s \Rightarrow v(0) = v_0$ and $x(0) = x_0$

While $a = 0$ and $v = \frac{dx}{dt} = cte$

$$\begin{cases} dx = v\, dt \Rightarrow \int x(t) = \int v\, dt \\ \frac{dv}{dt} = 0 \Rightarrow dv = 0 \Rightarrow v(t) = v_0 \end{cases}$$

$$v(t) = v_0$$

So $\int_{x_0}^{x} dx = \int_0^t v_0 \, dt \Rightarrow x - x_0 = v_0 t$

We have : $\boxed{x(t) = v_0 t + x_0}$

I-2 Non ~~linear Motion~~ Uniform linear motion

- Trajectory is straight line.
- $a = \underline{Cte} \neq 0$

The initial conditions are;

at $t = 0$ s $v(0) = v_0$ and $x(0) = x_0$

Calcul method :

If $a = \frac{dv}{dt}$ then $dv = a\,dt$

$$\int_{v_0}^{v} dv = \int_0^t a\,dt \Rightarrow v(t) = at + v_0$$

Similary $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

$$\int_{x_0}^{x} dx = \int_0^t v\,dt \Rightarrow x(t) - x_0 = \frac{a}{2}t^2 + v_0 t$$

$$\boxed{x(t) = \frac{a}{2}t^2 + v_0 t + x_0}$$

Also we can write $v = f(x, a)$

$a = \frac{dv}{dt} \Rightarrow v = \frac{dx}{dt} \Rightarrow v\,dt = dx$

$dv = a\,dt$

$v\,dv = a v\,dt$

$\Rightarrow v\,dv = a\,dx \Rightarrow \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx$

8- Vector quantities in the case of linear motion

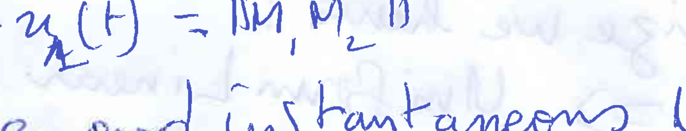
To describe motion for a particle when changing direction we use vector quantities.

8-1 Position vector:

The position of a moving particle is represented by a position vector defined as $\overrightarrow{OM} = x\vec{i}$

8-2 Displacement vector:

If a particle moving along the x-axis is in position M_1 at t_1 and in M_2 at t_2, then the displacement in the time interval $[t_1\ t_2]$ is presented by the vector $\overrightarrow{M_1M_2}$ such as $\overrightarrow{M_1M_2} = \overrightarrow{OM_2} - \overrightarrow{OM_1}$,



we get $\overrightarrow{M_1M_2} = [x(t_2) - x(t_1)]\,\vec{i}$

So $x_2(t) - x_1(t) = \|\overrightarrow{M_1M_2}\|$

8-3 Average and instantaneous velocities vectors

The average velocity vector of the particle over the time interval $\Delta t = t_2 - t_1$ is defined as

$$\vec{v}_{av}\Big|_{t_1}^{t_2} = \frac{\overrightarrow{M_1M_2}}{t_2 - t_1} = \frac{\Delta x}{\Delta t}\,\vec{i}$$

This Vector is parallel and have the same sens of the motion.

From our discussion in the previous section we can write the instantaneous velocity vector as: $\vec{v} = \lim_{\Delta t \to 0} \vec{v}_m$; $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\vec{i} = \frac{dx}{dt}\vec{i}$

8-4 Average and instantaneous acceleration vectors

Again as in 8.3, the average acceleration of the particle in $\Delta t = t_2 - t_1$ is defined as:

$$\vec{a}_{av}\Big|_{t_1}^{t_2} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{\Delta t} \quad \text{or} \quad \vec{a}_{av}\Big|_{t_1}^{t_2} = \frac{(v_2 - v_1)}{\Delta t}\vec{i}$$

This gives $a_{av}\Big|_{t_1}^{t_2} = \frac{\Delta v}{\Delta t}\vec{i}$

when $\Delta t \to 0$ the average acceleration vector became the instantaneous acceleration vector

we have: $\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{av}$; $\vec{a} = \frac{dv}{dt}\vec{i}$

8-5 Nature of a Linear motion:

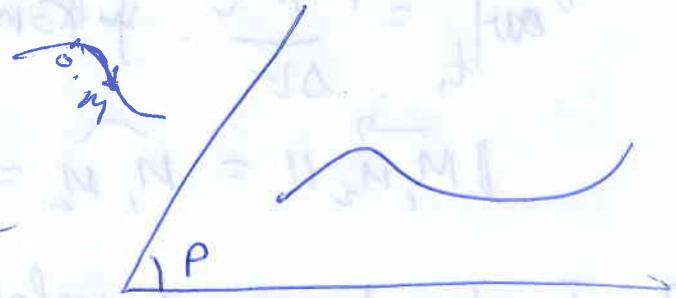
to summarize we have:

$\|\vec{a}\| = 0 \Rightarrow$ Uniform Linear motion ULM

$\|a\| = cte \Rightarrow \begin{cases} \vec{a}\cdot\vec{v} > 0 & \text{accelerated non Uniform Linear motion ANULM} \\ \vec{a}\cdot\vec{v} < 0 & \text{decelerated non Uniform Linear motion DNULM} \end{cases}$

9 Motion in a plane = Curvilinear motion

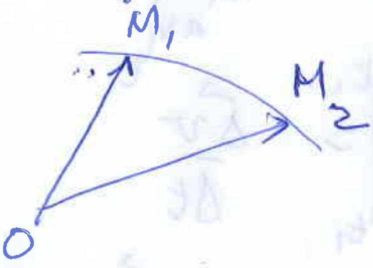
To study the motion in a plane we present the position of particle by the curvilinear abscissa $s(t) = \overset{\frown}{OM}$ → the arch



the velocity by $v(t) = \frac{ds}{dt}$ and $a = \frac{d^2s}{dt^2}$

These quantities are similar than those of the ~~stra~~ Linear motion but this is contrary to what a driver feels when he ~~is conducting~~ However, is driving on a winding road, so we must have to component of the acceleration (Vector quantity). centripetal acceleration
normal
tangential.

9-1 position vector



$\vec{OM_1}$ is the position vector at t_1

OM_2 " " " at t_2

the displacement vector $\vec{M_1M_2} = \vec{OM_2} - \vec{OM_1}$

$\vec{M_1M_2} = \Delta \vec{OM}$

We work in sufficiently small time intervals

$\overset{\frown}{M_1M_2} = \|\vec{M_1M_2}\|$

9 ~~..~~

9-2 Average velocity vector:

$$\vec{v}_{av}\Big|_{t_1}^{t_2} = \frac{\overrightarrow{M_1M_2}}{\Delta t} \quad \text{from where } \|\vec{v}_{M_1M_2}\| = \frac{\|\overrightarrow{M_1M_2}\|}{\Delta t}$$

$$\|\overrightarrow{M_1M_2}\| = \overset{\frown}{M_1M_2} = \Delta s$$

9-3 - Instantaneous velocity vector:

$$v(t) = \lim_{\Delta t} \frac{\Delta \overrightarrow{OM}}{\Delta t} \Rightarrow \vec{v}(t) = \frac{d\overrightarrow{OM}}{dt}$$

* The velocity vector is tangent to the trajectory.

$$\vec{v} = \frac{ds}{dt}\vec{u}_t$$

$\vec{u}_T$ = is the unit vector tangent to the trajectory

$$\|\vec{v}\| = \frac{ds}{dt}$$

Acceleration Vector:

We have $\vec{a}_{av}\Big|_{t_1}^{t_2} = \dfrac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$

$$\vec{a}_{av}\Big|_{t_1}^{t_2} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \vec{a}_m \text{ and } a(t) = \frac{d\vec{v}}{dt}$$

$$\vec{a}(t) = \frac{d}{dt}(v\vec{u}_T) = \frac{dv}{dt}\vec{u}_T + \|\vec{v}\|\frac{d\vec{u}_T}{dt}$$

With $\dfrac{d\vec{u}_T}{dt} = \dfrac{v}{R}\vec{u}_N \Rightarrow \vec{a} = \underbrace{\dfrac{dv}{dt}}_{a_T}\vec{u}_T + \underbrace{\dfrac{v^2}{R}}_{a_N}\vec{u}_N$

R: curvate radius.