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Series and differential equations

- ❖ Simple and multiple integrals
 - ❖ Improper integrals
 - ❖ Differential equations
 - ❖ Series: Numerical, integer and Fourier series
 - ❖ Fourier transform
 - ❖ Laplace transform

2nd year Licence Fundamental Physics

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Introduction

This document is the culmination of a decade's worth of dedicated teaching experience, spanning the years from 2013 to 2023, during which I had the privilege of instructing second-year undergraduate physics students. The content within these pages is a testament to the evolution and refinement of my teaching methods and materials over this substantial period.

Our primary focus lies in providing a comprehensive and meticulously crafted course curriculum that seamlessly integrates with the "**Series and Differential Equations**" course, an essential component of the Fundamental Teaching Unit (UEF-F121) tailored specifically for second-year licence's students pursuing degrees in fundamental physics. It is our intention that this material not only complements the established syllabus but enriches the learning experience, offering students a robust foundation in the subject matter.

However, the potential reach of this resource extends far beyond the confines of fundamental physics. Its value is equally significant for students pursuing undergraduate studies in various disciplines, including but not limited to electronics, electromechanics, mechanical engineering, civil engineering, and automation. As the boundaries of science and technology continue to expand, this document aims to serve as a versatile tool for any learner seeking to grasp the intricate concepts of series and differential equations.

The document itself is organized into six fundamental chapters, each representing a cornerstone of mathematical understanding and practical application:

Simple and Multiple Integrals: We delve into the world of integrals, exploring both their basic and more complex forms, equipping students with the mathematical tools needed for a wide range of physics and engineering problems.

Improper Integrals: This chapter guides students through the subtleties of improper integrals, demonstrating their significance in various real-world scenarios.

Differential Equations: We demystify differential equations, a crucial language of science, and engineering, breaking them down into understandable components and solving techniques.

Series; Numerical, Integer, and Fourier Series: The study of series, both numerical and integer, introduces students to the realm of infinite summations, paving the way for a deeper understanding of Fourier series and their applications in diverse fields.

Fourier Transform: This chapter explores the Fourier transform, a powerful mathematical tool that finds applications in signal processing, image analysis, and quantum mechanics, among others.

Laplace Transform: We conclude with a thorough examination of the Laplace transform, a transformative technique essential for solving differential equations, making it indispensable in engineering and physics.

Within each chapter, students will find carefully crafted subchapters, each containing a selection of application exercises, bolstered by detailed solutions that illuminate the problem-solving process. Furthermore, at the end of each chapter, we present additional exercises, intended to challenge students, reinforce their comprehension, and stimulate further exploration of these mathematical concepts.

In essence, this document represents not only the culmination of a decade of teaching but also a bridge to deeper understanding and success in the diverse and ever-evolving fields of science and technology. It is our sincere hope that it serves as an invaluable resource on the educational journey of every student who engages with its content.

The second part of this document will be dedicated to solving the exercises presented in the preceding sections. Additionally, it will include solutions to past examination questions from the last ten years, which were administered to 2nd-year students pursuing a Fundamental Physics License. These exercises and past exams serve as valuable practice and provide a comprehensive understanding of the topics covered in this document. By working through these problems, students can reinforce their knowledge of simple and multiple integrals, Series, differential equations, Fourier and Laplace transforms, and their applications in physics.

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