Propability Theory

Definition of propability: probability as a quantitative measure of the the likelihood of an event occurring.

Types	formula
Probabty calculation	P(A)=$\frac{Number\ of\ favorate\ outcomes}{Total\ number\ of\ possible\ outcomes}$ also you can write : P(A)=$\frac{\|A\|}{\|S\|} = \frac{cardinal\ of\ A}{cardinal\ of\ S}$
Probability Axioms	– $0 \leq P(A) \leq 1$ (probability is always between 0 and 1. – P(S)=1 . (The probability of the entire sample space is 1) – P(ø)=0 . (the probability of an impossible event is 0)
The fundamental Rules	
Addition Rule	**a–For Mutually Exclusive events** :(**Incompatible Events**) P(A ∪B)= P(A)+ P(B) **b–For Non–Mutually Exclusive events** : (**compatible Events**) P(A ∪B)= P(A)+ P(B)– P(A∩B) This formula is know as the general rule of addition. In general, if it has 3 or more events, you can generalized a formula about compatible events as follows: P(A∪B∪ C)=P(A) +P(B)+P(C) –P(A∩B)– P(A∩C)– P(B∩C)+ P(A∩B ∩C)
Subtraction Rule	P(A–B) = P(A∩ $\bar{B}) = P(A) - P(A \cap B)$

Complement Rule	P(A) + P($\bar{A}$) = 1 => $P(\bar{A}) = 1 - P(A)$ ($P(\bar{A})$ The probability of an event NOT happening)
Conditional probability and Independence This section explain how the occurrence of one event affects another.	a–Dependent Events : the occurrence of B depends on A. Multiplication rule : P(A∩B)=P(B)*P(A/B) Conditional Formula: If P(B)≠0, P(A/B)=$\frac{P(A\cap B)}{P(B)}$ b– Independent Events : A and B have no effect on each other: P(A∩B)=P(A)*P(B)
Advanced Theorems:These are used for complex systems where an event A can happen across several different scenarios (B_1,B_2,…..B_n)	
Total probability	We have a sample space is partitioned into several disjoint events Bi, if the following conditions are: - $\coprod_{i=1}^{n}$ Bi = S أي $B1 \cup B2 \cup \ldots.\cup Bn = S$ - $\forall i : 0 \leq P(Bi) \leq 1$ - $i \neq j : (Bi \cap Bj) = \emptyset$ The total probability of an event A is: P(A)=P(B_1) .P(A/B_1)+ P(B_2) .P(A/B_2)+ P(B_3) .P(A/B_3) Which is written in the following general form: P(A)=$\sum_{i=1}^{3} p(Bi).P(A/_{Bi})$
Bayes' Formula	This is used to "reverse" conditional probability – finding the probability of a specific cause Bi, given that result A has occurred: $P(Bi/_{A}) = \frac{P(Bi).P(A/_{Bi})}{\sum_{i=1}^{n} p(Bi).P(A/_{Bi})}$

Faculty of Economic, Commercial, and Management Sciences
Level: First Year (Branch 2)
Subject: Statistics 2

Exercise Series No. 3 (Probability)

Exercise 1
Three light bulbs are randomly selected from a lot of 15 bulbs, 5 of which are defective. Find the probability of the following events:
 * All selected bulbs are functional (non-defective).
 * Exactly one bulb is functional and the others are defective.
 * Exactly one bulb is defective.
 * At least one bulb is defective.

Exercise 2
A box contains 8 balls numbered from 0 to 7.
A. Suppose three balls are drawn sequentially without replacement. What is the probability of obtaining:
 * An odd number?
 * A number divisible by 5?
 * A number greater than or equal to 400?
 * A number greater than or equal to 200 and < 500?
B. Answer the same questions assuming the drawing is done with replacement.

Exercise 3
Three coins are tossed simultaneously. Determine:
 * The sample space (S).
 * The probability that the outcomes are heads (H) given that:
   * a. The first coin shows heads.
   * b. At least one of the three coins shows heads.

Exercise 4
Consider the random experiment of rolling two fair dice. Calculate the probability of the following events:
 * Obtaining a 2 on the first die.
 * The sum of the two dice is 11.
 * The absolute difference between the two dice is 6.
 * Obtaining the same number on both dice (a double).
 * The sum of the two dice is less than 6.

Exercise 5
Let A and B be two events such that $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$, and $P(\bar{A}) = 5/8$.
Required: Calculate P(A), P(B), and $P(A \cap \bar{B})$.

Exercise 6
Two hunters, A and B, fire at a target. Each fires exactly once:
 * The probability that hunter A hits the target is 0.7.
 * The probability that hunter B hits the target is 0.6.
 * The probability that both hunters hit the target is 0.5.
Required:
 * What is the probability that only hunter A hits the target?
 * What is the probability that at least one of them (A or B) hits the target?

Exercise 7
In a specific municipality, 50% of households own their homes, 75% own a car, and 30% own both. A household is selected at random.
Find the probability that:
 * The household owns a car, given that they own their home.
 * The household does not own their home, given that they own a car.
 * The household does not own a car, given that they do not own their home.

Exercise 8
Assume A and B are independent events where P(A) = 1/2 and P(A \cup B) = 2/3.
Required: Calculate P(B), P(A|B), and $P(\bar{B}|A)$.

Exercise 9
The total population of second-year Management Science students consists of 48% males and 52% females. Among them, 5% of male students and 25% of female students failed. A student is selected at random.
Required:
 * What is the probability that the selected student failed?
 * What is the probability that the selected student passed?
 * Given that the selected student failed, what is the probability they are male?

* Given that the selected student passed, what is the probability they are female?