

## The first axis : Set theory

- 1- Set definition : sets are defined as « a well-defined collection of objects ».

For Example :

A set of natural numbers is given by  $A = \{1, 2, 3, \dots\}$

A set of Even numbers :  $B = \{2, 4, 6, \dots\}$

A set of prime numbers :  $C = \{2, 3, 5, 7, \dots\}$

- 2- Types of sets :

**Empty set** : A set that has no elements inside it is called an Empty set.

It is represented by

$$A = \{\} \quad / \quad A = \emptyset$$

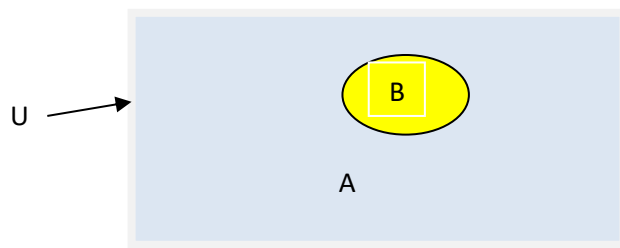
**Subset** : When every element in Set A is also found in Set B.. it is represented as :

**$B \subset A / \in A$  : read as ‘B is a subset of A’**

**Universal Set (U)**: The "big picture" set that contains everything currently under consideration.

**Disjoint sets** : Two sets A and B are disjoint if their **intersection** is the empty set. In mathematical terms:

$$A \cap B = \emptyset \quad \text{but} \quad A \cup B = U$$



For Example : set  $A = \{1,2,3\}$  and set  $B = \{4,5,6\}$  then we observe that there is no common element between set A and set B. Hence, set A and B are disjoint sets.

### 3- Set operation :

- **Union of sets** : union of sets basically refers to uniting two set and writing their elements in a single set without repeating element if common elements are present. The union of sets is given by  $A \cup B$ .

$$A \cup B = \{w, w \in A \text{ or } w \in B\}$$

- **Intersection of sets** : intersection of sets refers to finding the common elements between two sets. It given by  $A \cap B$ .

$$A \cap B = \{w, w \in A \text{ and } w \in B\}$$

- **Difference of sets (Substraction)** : The difference ( $A - B$ ) consists of elements that are in A but **not** in B. It's like taking Set A and "cutting out" any piece of B that was inside it.

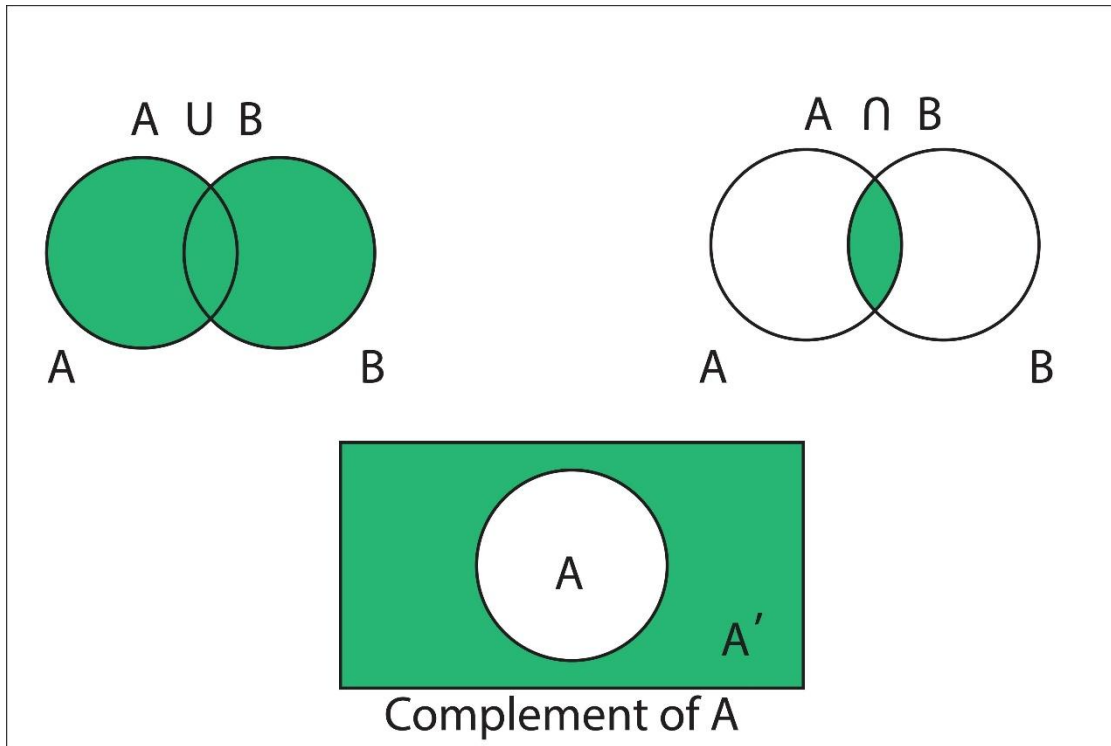
$$A - B = \{w, w \in A \text{ and } w \notin B\}$$

For Example : if  $A = \{2,4\}$  and  $B = \{4,6\}$  then  $A - B = \{2\}$

- **Complement of sets** : If A is a subset of a Universal Set U, the complement of A consists of all elements in U that are not in A.

$$A \cap A^c = \emptyset \text{ and } A \cup A^c = U$$

Operation	Symbol	What it means
Union	$A \cup B$	Everything in A <b>OR</b> B (or both).
Intersection	$A \cap B$	Only the things in <b>BOTH</b> A and B.
Difference	$A - B$	Only the thing in A not in B
Complement	$A^c$ or $A'$	Everything <b>NOT</b> in A.



#### 4- Essential Laws of Set Theory:

Law	Definition
Commutative	$A \cup B = B \cup A$ (Order doesn't matter for union/intersection)
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Difference	$(A - B) = A \cap \bar{B}$
1 <sup>st</sup> law of Morgan	$\overline{A \cup B} = \bar{A} \cap \bar{B}$
2 <sup>nd</sup> law of Morgan	$\overline{A \cap B} = \bar{A} \cup \bar{B}$

**Exercise:** Given the following sets:  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Let the subsets be as follows:

A: obtain a number less than 3

B: obtain an even number

C: obtain an odd number

1. Find the following the elements sets:  $A$ ;  $B$ ;  $C$
2. Identify:  $A \cap B$ ;  $A \cup B$ ;  $A \cap C$ ;  $B \cup C$ ;  $A - B$ ;  $B - A$   
 $\bar{A}$ ;  $\bar{B}$ .....