Chapter 3: Viscous Incompressible Fluid Dynamics

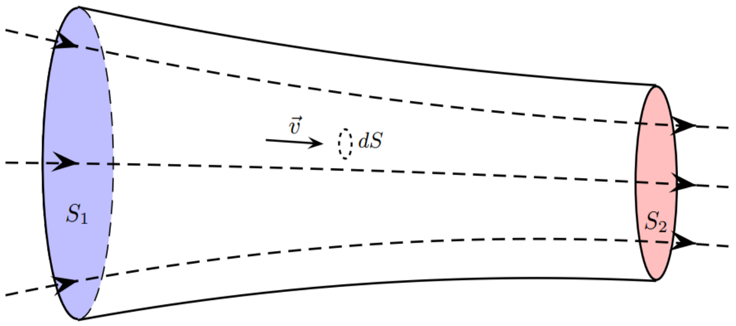
Introduction:

- ✓ In the previous chapter, we assumed that the fluid was ideal in order to apply the energy conservation equation.
- ✓ The flow of a real fluid is more complex than that of an ideal fluid. Indeed, friction forces due to the fluid’s viscosity arise both between the fluid particles and the walls, as well as among the particles themselves.
- ✓ A fluid is said to be real when, during its motion, the contact forces are not only perpendicular to the surface elements on which they act. They therefore have tangential components that oppose the sliding of fluid layers over one another. This resistance to shear is characterized by viscosity..

Conservation of mass and equation of continuity (General case)

Conservation of mass:

The conservation of mass stipulates that the amount of matter transported per unit of time in a system must remain constant (nothing is lost, nothing is created)



- in the Figure, the mass "entering" per unit time across the surface S_1, is equal to the mass "exiting" per unit time across the surface S_2
- For a small period of time dt, the fluid moves with a velocity $\vec{v}$ and travels a distance $dx = vdt$.
- The volume of fluid that passed through the section dS of the tube dt is therefore:

$$dV = vdsdt$$

- If the fluid has a density ρ, then the mass of fluid dm that has been displaced through the section dS during the time dt is:

$$dm = \rho dV = \rho vdsdt$$

- The mass of fluid $\boldsymbol{dM}$ that passes through the section S of the flow during dt would then be the sum of all the masses of fluid dm that each pass through a small portion dS of the section during that same interval dt:

$$dM = \iint_S dm \;= \iint_S \rho vdsdt \;= dt \iint_S \rho vds$$

The mass flow rate is therefore given by: $q_m = \frac{dM}{dt} = \iint_S \rho v ds = Constante$ *(conservation of mass)*

$= \iint_{S_1} \rho v ds$ = mass per unit of time crossing $S_1 = q_{m1}$

$= \iint_{S_2} \rho v ds$ = mass per unit of time crossing $S_2 = q_{m2}$

Remark:

The conservation of mass (i.e. constant mass flow rate) implies the conservation of the volume(i.e. a constant volume flow rate) if and only if the density is constant.

Indeed, the volume flow (the volume of fluid that passes through a section of the flow per unit of time), is defined by:

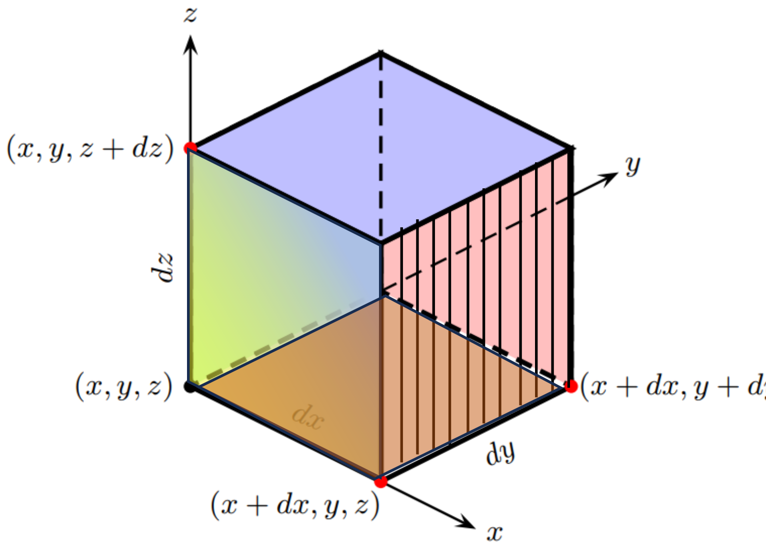
$$q_v = dV/dt.$$

Thus:

$$q_m = \frac{dM}{dt} = \frac{d(\rho V)}{dt} = \rho \frac{dV}{dt} + V \frac{d\rho}{dt} = \rho q_V + V \frac{d\rho}{dt} \iff q_v = \frac{q_m}{\rho} - \frac{V}{\rho}\frac{d\rho}{dt}$$

If $\rho = cst$ and $v = cte$: $q_m = \rho v S$ and $q_v = Sv$

Equation of continuity :



- Let be a small cubic elementary volume of sidesdx, dy and dz in the flow
- The velocity of the fluid at this point in the flow est: $\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$.
- The flow enters the volume through the faces located in the x (green face)
  - In y (front left)
  - In z (underside)
  - and it emerges from it through the faces located in $(x + dx)$ (Front right)
  - In $(y + dy)$(red side) and in $z + dz$ (blue face).
- Conservation of mass requires that:

$$q_{m_{entrant}} = q_{m_{sortant}} \Longrightarrow q_{m_{entrant}} - q_{m_{sortant}} = 0$$

- Parallel to the Ox axis, the mass of fluid that enters the volume per unit of time through the face $S(x)$ is:

$$q_{mx}(x) = \rho(x)v_x(x)S(x) = \rho(x)v_x(x)dydz$$

- Similarly, the mass that exits per unit time from the face $S(x + dx)$ is:

$$q_{mx}(x + dx) = \rho(x + dx)v_x(x + dx)S(x + dx) = \rho(x + dx)v_x(x + dx)dydz$$

- The difference in mass per unit time between the quantity that goes out and the quantity that goes in parallel to Ox is:

$$q_{mx}(x+dx) - q_{mx}(x) = \big(\rho(x+dx)v_x(x+dx) - \rho(x)v_x(x)\big)dydz$$

$$= \frac{\big(\rho(x+dx)v_x(x+dx) - \rho(x)v_x(x)\big)}{dx}dxdydz$$

$$= \frac{\big((\rho v_x)(x+dx) - (\rho v_x)(x)\big)}{dx}\Delta V$$

$$= \frac{d(\rho v_x)}{dx}\Delta V$$

- Doing the same calculation parallel to Oy and Oz, we find:

$$q_{my}(y+dy) - q_{my}(y) = \frac{d\big(\rho v_y\big)}{dy}\Delta V \quad \text{and} \quad q_{mz}(z+dz) - q_{mz}(z) = \frac{d(\rho v_z)}{dz}\Delta V$$

- The difference in mass per unit of time that enters and leaves the entire elementary volume is therefore:

$$q_{m_{entrant}} - q_{m_{sortant}} = [q_{mx}(x+dx) - q_{mx}(x)] + \big[q_{my}(y+dy) - q_{my}(y)\big] + [q_{mz}(z+dz) - q_{mz}(z)]$$

$$= \left(\frac{d(\rho v_x)}{dx} + \frac{d\big(\rho v_y\big)}{dy} + \frac{d(\rho v_z)}{dz}\right)\Delta V \qquad = div(\rho\vec{v}\,)\Delta V \quad = \vec{\nabla}.(\rho\vec{v}\,)\Delta V$$

Principle of conservation of mass

$$q_{m_{entrant}} - q_{m_{sortant}} = \boxed{\Leftrightarrow \left(\frac{d(\rho v_x)}{dx} + \frac{d(\rho v_y)}{dy} + \frac{d(\rho v_z)}{dz}\right) = \vec{\nabla}.(\rho\vec{v}\,) = 0}$$

(continuity equation)

Special case: incompressible fluids (ρ=cst):

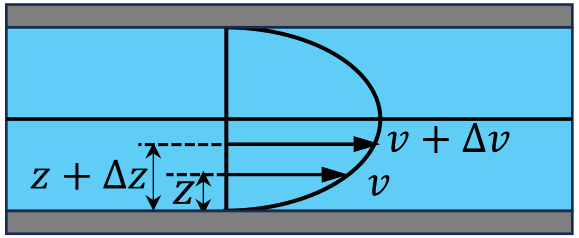
$$\vec{\nabla}.(\rho\vec{v}\,) = 0 \quad \Leftrightarrow \rho\vec{\nabla}.\vec{v} + \vec{v}\vec{\nabla}\rho = 0 \quad \Rightarrow \rho.\,div\vec{v} + \vec{v}.\overrightarrow{grad}\rho = 0$$

$$\rho = cst \Rightarrow \overrightarrow{grad}\rho = 0 \quad \Rightarrow div\vec{v} = \vec{\nabla}.\vec{v} = 0$$

$$\Rightarrow \frac{dv_x}{dx} + \frac{dv_y}{dy} + \frac{dv_z}{dz} = 0$$

Viscosity phenomenon:

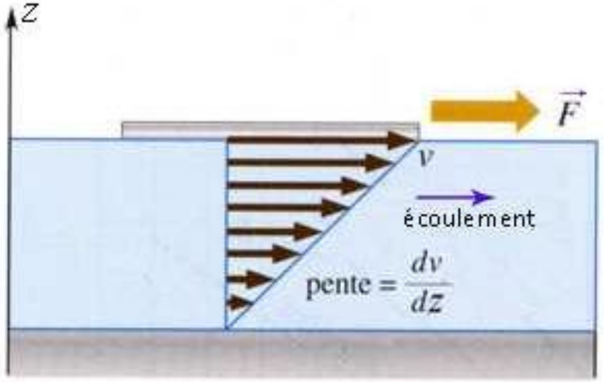
- Water, oil, honey flow differently: water flows quickly, but with turbulent, honey flows slowly, but in a very regular way
- In a real fluid, the contact forces are not perpendicular to the surface elements on which they are exerted.
- Viscosity is due to these friction forces that oppose the sliding of fluid layers over one another.
- Phenomena due to the viscosity of fluids occur only when these fluids are in motion.
- Under the effect of the interaction forces between the fluid molecules and those of the wall, each fluid molecule does not flow at the same velocity. It is said that there is a velocity profile.
- The curve of the ends of these vectors represents the velocity profile.



- The velocity is a function of the distance z from this curve to the fixed plane $(v = v(z))$

Dynamic Viscosity - Kinematic Viscosity

1. Dynamic viscosity



- We consider two adjacent layers of fluid that are distant from z (Figure).
- The friction force exerted at the interface between these two layers opposes the sliding of one layer over the other..
- It is proportional to the difference in velocity of the layers, i.e. $\mathrm{d}v$, to their surface S and inversely proportional to $\mathrm{d}z$.
- The proportionality factorμ is the dynamic viscosity coefficient of the fluid and the relationship between the sliding force F and μ is given by:

$$F = \mu S \frac{dv}{dz} \quad \Longrightarrow \mu = \frac{F/S}{dv/dz}$$

In this case, the fluid is said to be Newtonian

- The force per unit area applied by the fluid to the wall is called tangential stress, given by:

$$\boxed{\tau = \mu \frac{dv}{dz}}$$

2. Kinematic viscosity

In many formulas, the ratio of dynamic viscosity and density appears. This ratio is called kinematic viscosity

$$\boxed{\nu = \frac{\mu}{\rho}}$$

The different flow regimes: Reynolds number

Studying the flow of a viscous fluid means solving the fundamental equation of fluid dynamics (known as the Navier-Stokes equation)

Equation of Navier-Stokes

The Navier-Stokes equation reflects Newton's second law, applied to fluids, which states that the sum of the forces exerted on a body is equal to the mass of this body multiplied by its acceleration:

$$\sum \vec{F} = m\vec{a}$$

It is therefore necessary to determine how the three terms of this equation are written for a fluid in motion.

1. ***Mass of the fluid:*** the mass of a small elementary volume of sides dx, dy and dz:

$$\Delta M = \rho \Delta V = \rho dxdydz$$

2. ***Acceleration of fluid***

Since the velocity $\vec{v}$ of a fluid particle can vary in time and space, it can be written:

$$\vec{v} = \vec{v}(x,y,z,t) = (v_x, v_y, v_z) = \begin{pmatrix} v_x(x,y,z,t) \\ v_y(x,y,z,t) \\ v_z(x,y,z,t) \end{pmatrix}$$

$$\vec{a}(x,y,z,t) = \begin{pmatrix} \dfrac{dv_x(x,y,z,t)}{dt} \\ \dfrac{dv_y(x,y,z,t)}{dt} \\ \dfrac{dv_z(x,y,z,t)}{dt} \end{pmatrix}$$

Total differential dv_x is written: $$dv_x = \frac{\partial v_x}{\partial t}dt + \frac{\partial v_x}{\partial x}dx + \frac{\partial v_x}{\partial y}dy + \frac{\partial v_x}{\partial z}dz$$

$$\Longrightarrow \frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x}\frac{dx}{dt} + \frac{\partial v_x}{\partial y}\frac{dy}{dt} + \frac{\partial v_x}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}$$

$$= \frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_x \quad = \frac{\partial v_x}{\partial t} + (\vec{v}.\vec{\nabla})v_x$$

Similarly, we find:

$$\frac{dv_y}{dt} = \frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_y = \frac{\partial v_y}{\partial t} + (\vec{v}.\vec{\nabla})v_y$$

$$\frac{dv_z}{dt} = \frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_z = \frac{\partial v_z}{\partial t} + (\vec{v}.\vec{\nabla})v_z$$

So, we find:

$$\vec{a}(x,y,z,t) = \left(\frac{\partial v_x}{\partial t} + (\vec{v}.\vec{\nabla})v_x\right)\vec{\imath} + \left(\frac{\partial v_y}{\partial t} + (\vec{v}.\vec{\nabla})v_y\right)\vec{\jmath} + \left(\frac{\partial v_z}{\partial t} + (\vec{v}.\vec{\nabla})v_z\right)\vec{k}$$

$$= \left(\frac{\partial v_x}{\partial t}\vec{\imath} + \frac{\partial v_y}{\partial t}\vec{\jmath} + \frac{\partial v_z}{\partial t}\vec{k}\right) + (\vec{v}.\vec{\nabla})\left(v_x\vec{\imath} + v_y\vec{\jmath} + v_z\vec{k}\right) = \frac{d\vec{v}}{dt} + (\vec{v}.\vec{\nabla})\vec{v}$$

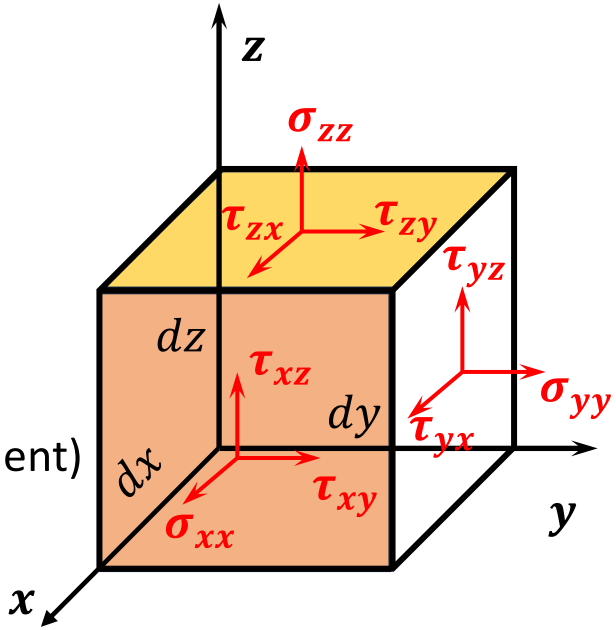
3. *Sum of forces:* The sum of the forces acting on an elementary volume is the sum of the forces $\vec{F}_V$ acting on its entire volume and the forces $\vec{F}_S$ acting on its surfaces.

3.1. Volume Forces$\vec{F}_V$: Volume forces are the forces induced by gravity, which acts on all fluid particles in the elementary volume

$$\vec{F}_V = \Delta m\vec{g} = \rho\vec{g}\Delta V \Longrightarrow \begin{pmatrix}F_{v_x}\\ F_{v_y}\\ F_{v_z}\end{pmatrix} = \rho\begin{pmatrix}g_x\\ g_y\\ g_z\end{pmatrix}\Delta V$$

3.2. Surface Forces $\vec{F}_S$:

- The forces exerted on the elementary volume surfaces can act either:
  - ✓ *Perpendicular to these surfaces*: forces are induced by the pressure of the fluid
  - ✓ *Tangential to these surfaces:* The forces are induced by shear stresses (contraintes cisaillement)
- These stresses are shown in Figure

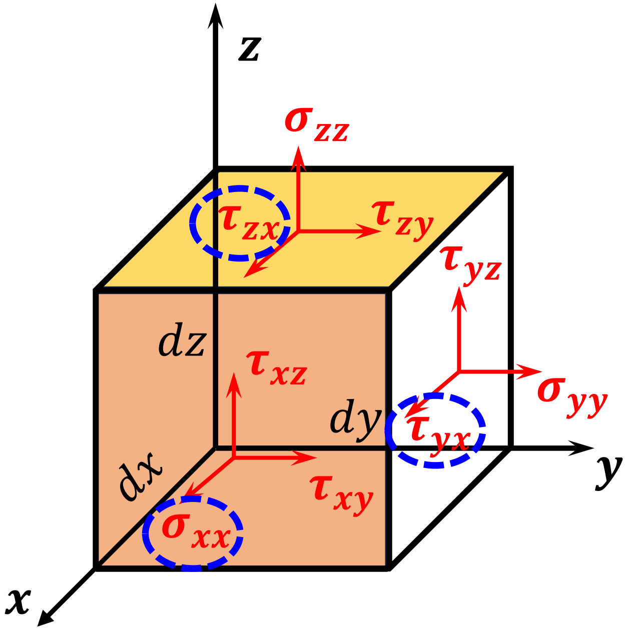


- A stress acts on a surface identified by position i and is oriented in the direction j
- Stresses with different indices i and j represent shear stresses, which are then denoted τ_{ij}
- Stresses with identical indices represent the stresses normal to the surfaces, which are then denoted σ_{ii}

- Each of these surface forces is the product of a stress by the surface on which that stress acts
- the sum of the surface forces parallel to the Ox axis is:

$$\begin{aligned}\sum \vec{F}_{S_x} &= (\sigma_{xx}(x+dx)dydz - \sigma_{xx}(x)dydz) & &= \left(\frac{\sigma_{xx}(x+dx) - \sigma_{xx}(x)}{dx}\right) dxdydz \\ &+ (\tau_{zx}(z+dz)dxdy - \tau_{zx}(z)dxdy) & &+ \left(\frac{\tau_{zx}(z+dz) - \tau_{zx}(z)}{dz}\right) dxdydz \\ &+\left(\tau_{yx}(y+dy)dxdz - \tau_{zx}(z)dxdz\right) & &+ \left(\frac{\tau_{yx}(y+dy) - \tau_{yx}(z)}{dy}\right) dxdydz\end{aligned}$$

$$\Rightarrow \sum \vec{F}_{S_x} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) \Delta V$$



Similarly, we find:

$$\sum \vec{F}_{S_y} = \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right) \Delta V$$

$$\sum \vec{F}_{S_z} = \left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) \Delta V$$

For a flow (1D), the viscous stress (viscous force per unit of surface) is given by:

$$\tau = \mu \frac{dv}{dz}$$

For a flow (3d), the viscous stress becomes a tensor (3×3 matrix), related to the strain rate:

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial j} + \frac{\partial v_j}{\partial i} \right)$$

So there will be:

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad ; \quad \tau_{xz} = \tau_{zx} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \quad ; \quad \tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)$$

We also accept that for a viscous fluid (Experimental Law):

$$\sigma_{xx} = -P + 2\mu \frac{\partial v_x}{\partial x} \quad ; \quad \sigma_{yy} = -P + 2\mu \frac{\partial v_y}{\partial y} \quad ; \quad \sigma_{zz} = -P + 2\mu \frac{\partial v_z}{\partial z}$$

By injecting the expressions of σ_{xx}, τ_{yx} et τ_{zx} in $\sum \vec{F}_{S_x}$ we obtain:

$$\Rightarrow \sum \vec{F}_{S_x} = \left(\frac{\partial}{\partial x} \left(-P + 2\mu \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \right) + \frac{\partial}{\partial z} \left(\mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \right) \right) \Delta V$$

$$\Rightarrow \sum \vec{F}_{S_x} = \left(-\frac{\partial P}{\partial x} + 2\mu\frac{\partial^2 v_x}{\partial x^2} + \mu\frac{\partial^2 v_x}{\partial y^2} + \mu\frac{\partial^2 v_y}{\partial y \partial x} + \mu\frac{\partial^2 v_x}{\partial z^2} + \mu\frac{\partial^2 v_z}{\partial z \partial x}\right)\Delta V$$

$$\Rightarrow \sum \vec{F}_{S_x} = \left(-\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v_x}{\partial x^2} + \mu\frac{\partial^2 v_x}{\partial y^2} + \mu\frac{\partial^2 v_x}{\partial z^2} + \mu\frac{\partial^2 v_x}{\partial x \partial x} + \mu\frac{\partial^2 v_y}{\partial y \partial x} + \mu\frac{\partial^2 v_z}{\partial z \partial x}\right)\Delta V$$

$$\Rightarrow \sum \vec{F}_{S_x} = \left(-\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \mu\frac{\partial}{\partial x}\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\right)\Delta V$$

We have: $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$ (Continuity equation)

$$\Rightarrow \sum \vec{F}_{S_x} = \left(-\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)\right)\Delta V \qquad = \left(-\frac{\partial P}{\partial x} + \mu\nabla^2 v_x\right)\Delta V$$

$$\Rightarrow \sum \vec{F}_{S_y} = \left(-\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right)\right)\Delta V \qquad = \left(-\frac{\partial P}{\partial y} + \mu\nabla^2 v_y\right)\Delta V$$

$$\Rightarrow \sum \vec{F}_{S_z} = \left(-\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right)\right)\Delta V \qquad = \left(-\frac{\partial P}{\partial z} + \mu\nabla^2 v_z\right)\Delta V$$

$$\sum \vec{F} = m\vec{a} \quad \Longrightarrow \vec{F}_v + \vec{F}_S = \Delta M \frac{dv}{dt}$$

In Cartesian coordinates:

$$\Longrightarrow \rho\cancel{\Delta V}\begin{pmatrix}\frac{dv_x}{dt}\\ \frac{dv_y}{dt}\\ \frac{dv_z}{dt}\end{pmatrix} = \rho\cancel{\Delta V}\begin{pmatrix}\frac{\partial v_x}{\partial t}+(\vec{v}.\vec{\nabla})v_x\\ \frac{\partial v_y}{\partial t}+(\vec{v}.\vec{\nabla})v_y\\ \frac{\partial v_z}{\partial t}+(\vec{v}.\vec{\nabla})v_z\end{pmatrix} = \rho\begin{pmatrix}g_x\\ g_y\\ g_z\end{pmatrix}\cancel{\Delta V} + \begin{pmatrix}-\frac{\partial P}{\partial x}+\mu\nabla^2 v_x\\ -\frac{\partial P}{\partial y}+\mu\nabla^2 v_y\\ -\frac{\partial P}{\partial z}+\mu\nabla^2 v_z\end{pmatrix}\cancel{\Delta V}$$

In vector form, we find:

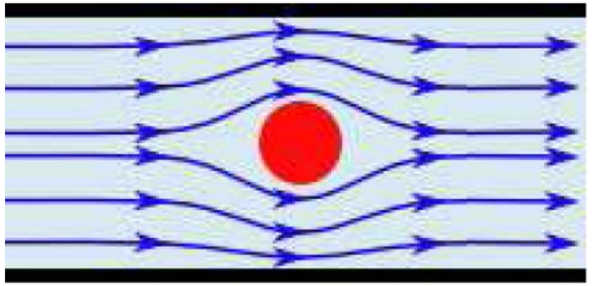
$$\boxed{\rho\frac{d\vec{v}}{dt} = \rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \mu\nabla^2\vec{v} + \rho\vec{g}}$$

(Equation of Navier Stokes)

Laminar and turbulent flows: the Reynolds number

In practice, the Navier Stokes equation can only be solved analytically by introducing assumptions that will allow this equation to be solved within the framework of particular flow regimes: laminar regime and turbulent regime

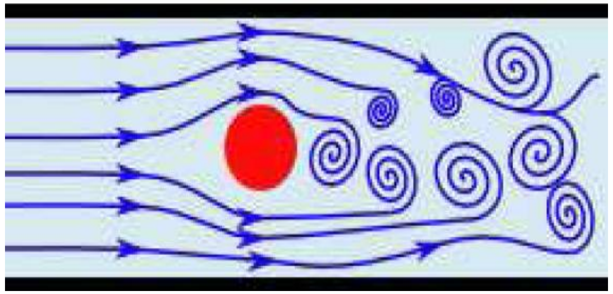
- ❑ A flow is said to be **laminar** when the fluid particle is made in a regular and orderly manner



The main physical characteristics of laminar flow are as follows:

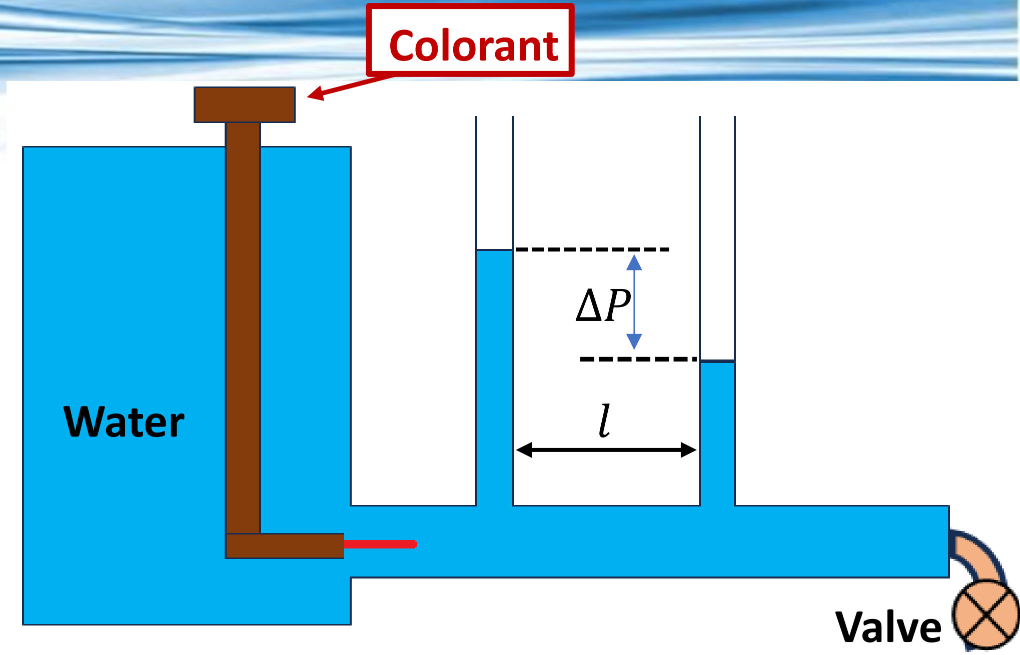
- ➢ Constant velocity over time at any point of the flow;
- ➢ Each fluid element moves along a well-defined (streamline) and "smooth" trajectory;
- ➢ All fluid particles starting from the same given point of the flow (at different times) follow the same trajectory, or streamline.

- ❑ Flow is **turbulent** when the displacement is irregular and random fluctuations in velocity are superimposed on the average movement of the fluid



- ➢ The flow is irregular with rapid and sudden variations in velocity both in time and space
- ➢ The path followed by a fluid particle is very irregular and changes continuously
- ➢ different fluid particles starting from the same point of the flow (at different times) follow different trajectories;

Reynolds' Experience (1883):



The historical experience of O. Reynolds consists in making a coloured fluid of the same liquid flow through a transparent tube as that which circulates in the tube and at the same velocity.

He varied the velocity of the flow, the diameter of the tube and the viscosity of the fluid (use of different fluids).

Observations:

1. For low velocities, large diameters and large viscosities, the flow is smooth, it is said to be laminar.
2. For large velocities, small diameters and low viscosity, the flow is chaotic ,it is said to be turbulent.

The parameter that determines the nature of a flow is a combination of the three parameters: average flow velocity, tube diameter and viscosity.

It is a dimensionless number, called the Reynolds number: $\boxed{R_e = \frac{\rho v d}{\mu}}$ $\boxed{= \frac{vd}{\nu}}$

v : Average flow velocity through the sectionconsidérée in (m/s)

d : Diameter of the pipe or width of the fluid vein in (m).

ρ : Density of the fluid

μ : Dynamic viscosity

ν: Kinematic viscosity

It is generally observed that the transition from a laminar to a turbulent regime takes place when:

$$R_e \approx 2000 = R_{e_C} \text{ : nombre de Reynolds critique}$$

- ✓ $R_e < 2000$: the flow remains laminar
- ✓ $R_e > 3000$: the flow is turbulent
- ✓ $2000 < R_e < 3000$: Transition phase between laminar and turbulent flow

Exercise 01:

We consider a vertical volcanic conduit whose size (section) narrows upwards and with a simplified geometry:

A vent is circular in shape with a radius $R_2 = 10m$ while at depth, the conduit is connected to a magma reservoir with a horizontal section of radius $R_1 = 1km$. In addition, the velocity of ascent of magma v is considered to be uniform at each depth z and the density of magma ρ to be constant.

1. If the magma exit velocity v_2 during a Plinian eruption is of the order of $100\ m/\ s$, at what vertical speedv_1 moves magma into the reservoir?
2. How does the ratio of velocities v_1/v_2 vary with the Rays $R_1\ and\ R_2$?

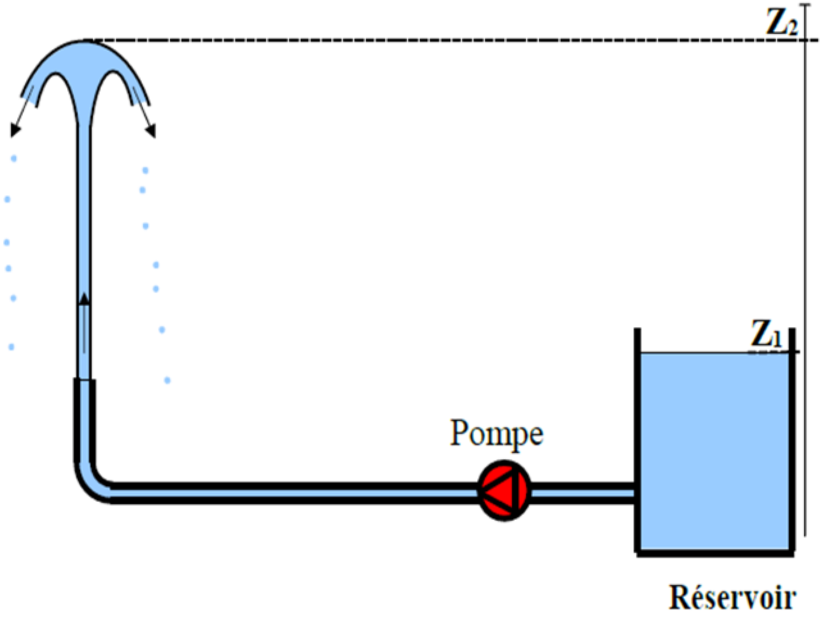
Exercise 02 :

Determine the critical velocity v_C :

a) for a fuel oil at 15°C circulating in a pipe with 15 cm of diameter;

b) for water at 15°C circulating in the same pipe. The kinematic viscosity at 15°C is: $\nu = 4{,}47.10^{-6}\ m^2/s$ for fuel and $\nu = 1{,}142.10^{-6}\ m^2/s$ for water.

Exercise 04:



A water jet is fed from a reservoir by means of a pump with a volume flow rate $q_v = 2\ L/s$ and a pipe of length $L = 15\ m$ and diameter $d = 30\ mm$. The level of the free surface of the reservoir, assumed to be slowly variable, is at an altitude $Z_1 = 3m$ above the ground. The jet rises to a height $Z_2 = 10\ m$.

It is assumed that:

- Pressures: $P_1 = P_2 = P_{atm}$.
- The dynamic viscosity of water: $\mu = 10^{-3}\ Pa.s$.
- The density of mass of water: $\rho = 1000\ kg/m^3$.
- The acceleration of gravity: $g = 9{,}81\ m/s^2$.

1. Calculate the velocity V of water flow in the pipe in m/s.
2. Calculate the Reynolds number Re.
3. Specify the nature of the flow.