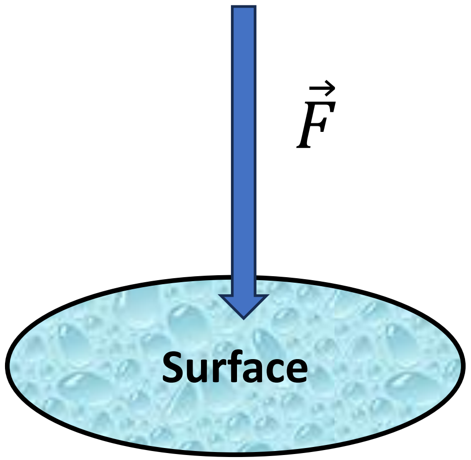
Chapter 2: Fluid Statics



Introduction:

- Fluid statics is the branch of fluid mechanics that primarily studies fluids at rest.
- The study of the properties of fluids at rest constitutes fluid statics.

Concept of pressure: The pressure exerted by a force F acting perpendicularly on a surface S is given by:

$$P = \frac{F}{S}$$

In general, if we consider a surface element dS and the force dF that is normally exerted at the surface, given the isotropy of the force, the pressure is given by:

$$P = \frac{dF}{dS}$$

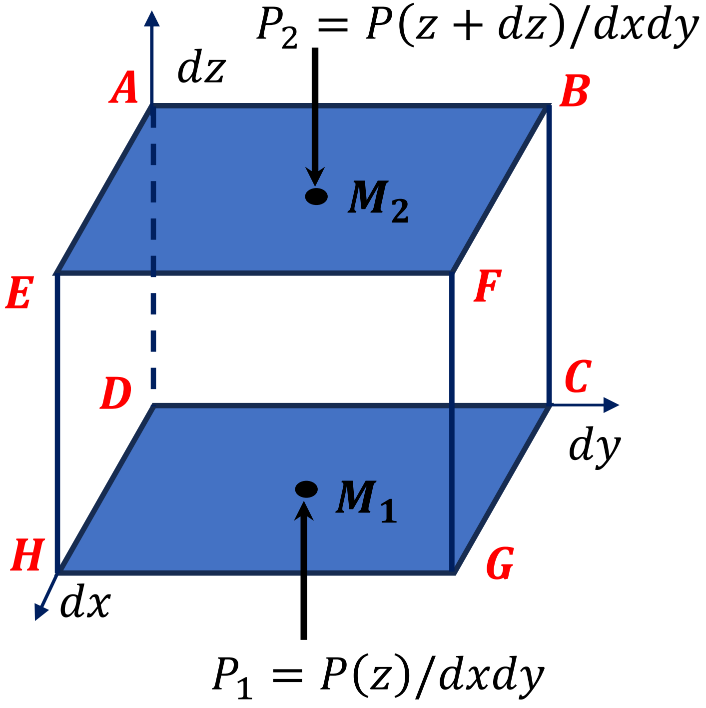
The unit of pressure in (SI) is the Pascal (Pas): $1\ Pas = 1\ N/m^2$

We also use:

- **The Hectopascal (hPa):** $1\ hPas = 100\ Pas$
- **The bar:** $1\ bar = 10^5\ Pas$
- **The atmosphere (**Atmospheric pressure) **:** $1\ atm = 101325\ Pas$
- **Cm of water:** $1\ Cm\ H_2O = 98.04\ Pas$
- **Cm of mercury:** $1\ Cm\ Hg = 133.3\ Pas$

The basic principle of fluid statics:

- Let be an elementary volume $dV = dx.dy.dz$ within a fluid whose faces are defined parallel to the axes (Ox, Oy, Oz).
- Consider the two faces **ABFE** and **CGHD** perpendicular to Oz.
- The pressing forces exerted on the two sides considered are:



$$\textbf{Pressure } \boldsymbol{P_1} : d\vec{F_1} = P_1\, dxdy\vec{k} \quad ; \quad \textbf{Pressure } \boldsymbol{P_2} : dF_2 = -P_2\, dxdy\vec{k}$$

The resulting force is: $d\vec{F_z} = d\vec{F_1} + d\vec{F_2} \quad = (P_1 - P_2)dxdy\vec{k}$

The variation of the pressure along the Oz axis can be written: $\dfrac{\partial P}{\partial z} = \dfrac{P_2 - P_1}{dz}$

$$\Rightarrow P_1 - P_2 = -\frac{\partial P}{\partial z}dz \Rightarrow d\vec{F_z} = -\frac{\partial P}{\partial z}dxdydz\vec{k} \quad \boxed{\Rightarrow d\vec{F_z} = -\frac{\partial P}{\partial z}dV\vec{k}}$$

In the same way, we find: $\boxed{d\vec{F_x} = -\dfrac{\partial P}{\partial x}dV\vec{\imath}}$ and $\boxed{d\vec{F_y} = -\dfrac{\partial P}{\partial y}dV\vec{\jmath}}$

Therefore: $d\vec{F} = d\vec{F_x} + d\vec{F_y} + d\vec{F_z} = -\left(\dfrac{\partial P}{\partial x}\vec{\imath} + \dfrac{\partial P}{\partial y}\vec{\jmath} + \dfrac{\partial P}{\partial z}\vec{k}\right)dV$ $\boxed{\Rightarrow \boldsymbol{d\vec{F} = -\overrightarrow{grad}P.dV}}$

Equilibrium within a fluid

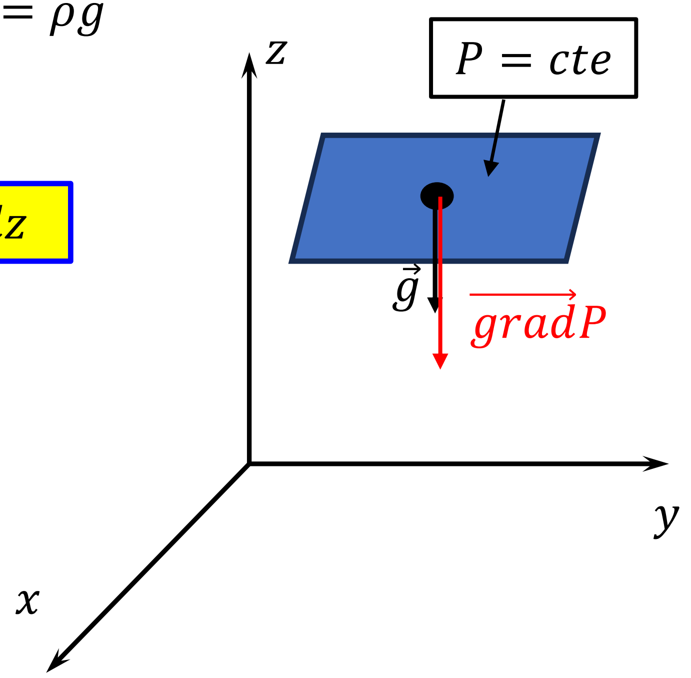
In addition to the pressure forces acting on the fluid element, there is also the force of the weight of the fluid element itself:

$$d\vec{w} = dm\vec{g} \quad = \rho dV\vec{g}$$

For the fluid in equilibrium, we have:

$$\sum \vec{F} = \vec{0} \quad \Rightarrow d\vec{F} + d\vec{w} = \vec{0} \quad \Rightarrow -\overrightarrow{grad}P.dV + \rho dV\vec{g} = \vec{0} \quad \Rightarrow \overrightarrow{grad}P = \rho\vec{g}$$

$$\Rightarrow \frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} = -\rho g\vec{k} \qquad \Rightarrow \frac{\partial P}{\partial z}\vec{k} = -\rho g\vec{k} \qquad \boxed{\Rightarrow dP = -\rho g dz}$$



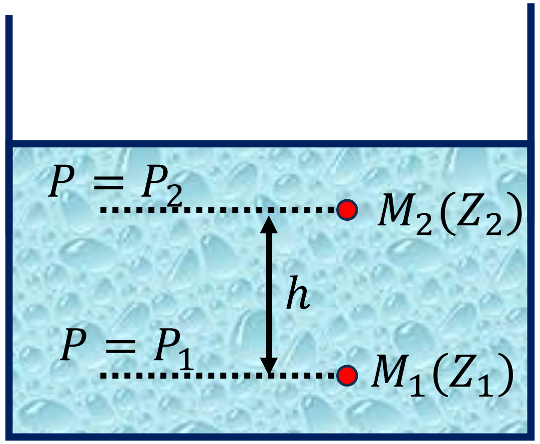
The pressure increases with height and depends only on z : $P = P(z)$

$$\Rightarrow \frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0$$

Case of incompressible fluids:

Fundamental principle of hydrostatics:

To obtain the pressure difference between two points M_1 at height Z_1 and M_2 at height Z_2 , just integer the expression: $dP = -\rho g dz$



$$\Rightarrow \int_{P_1}^{P_2} dP = -\rho g \int_{Z_1}^{Z_2} dz \quad \Rightarrow P_2 - P_1 = -\rho g (Z_2 - Z_1)$$

$$\Rightarrow \Delta P = -\rho g h$$

With:

- ρ is the mass density of the fluid in (kg/m^3)
- $h = Z_2 - Z_1$ is the difference in level between the two points M_1 and M_2 in (m)
- g is the acceleration of gravity $(9,81\ N/kg)$
- $\Delta P = P_2 - P_1$ is the pressure difference in (Pa)

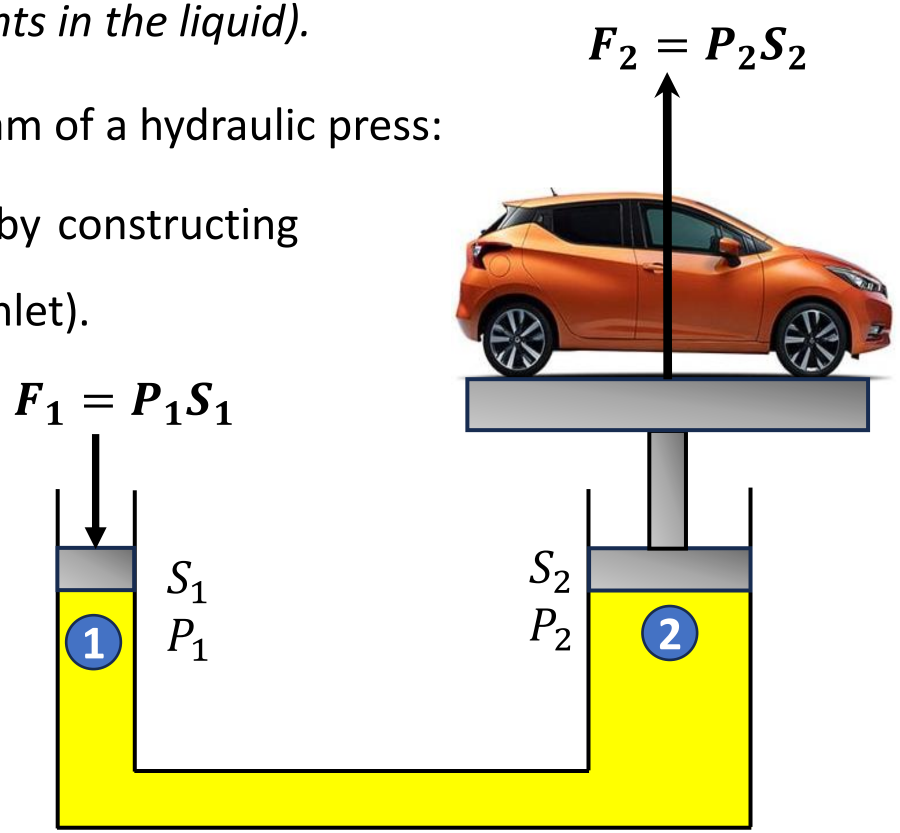
Pressure transmission in liquids:

Pascal's theorem: The theorem of Pascal is stated as follows:

In an incompressible fluid in equilibrium, any change in pressure at one point causes the same change in pressure at any other point (the change in pressure is transmitted in full to all other points in the liquid).

Application: Principle of hydraulic press: Let us consider the schematic diagram of a hydraulic press:

In this case, a considerable force is produced from a relatively small force by constructing the surface of one piston (at the outlet) wider than that of the other (at the inlet).

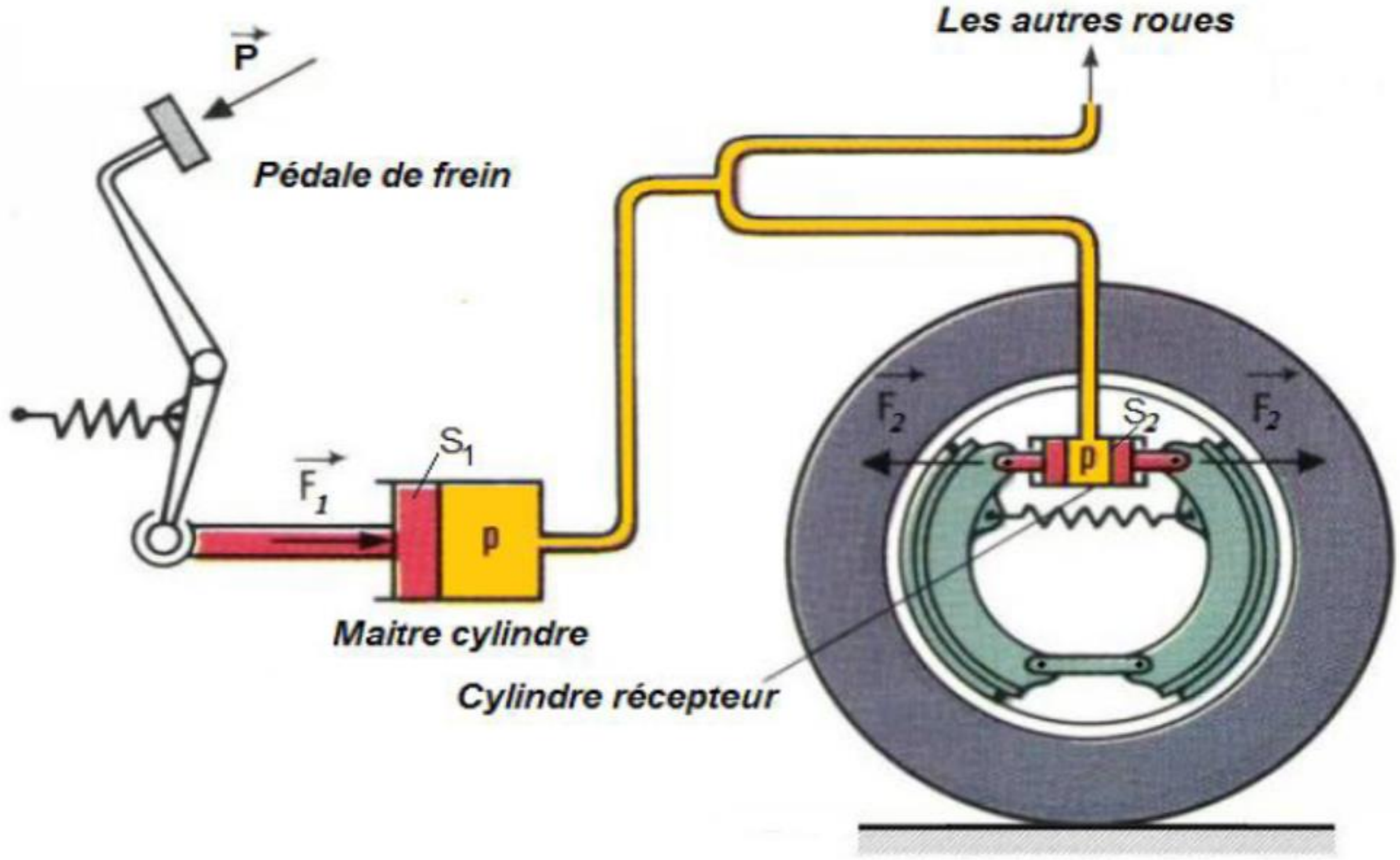


When the two pistons 1 and 2 are on the same level, we have: $P_1 = P_2$

$$\Rightarrow \frac{F_1}{S_1} = \frac{F_2}{S_2} \quad \Rightarrow \frac{F_2}{F_1} = \frac{S_2}{S_1}$$

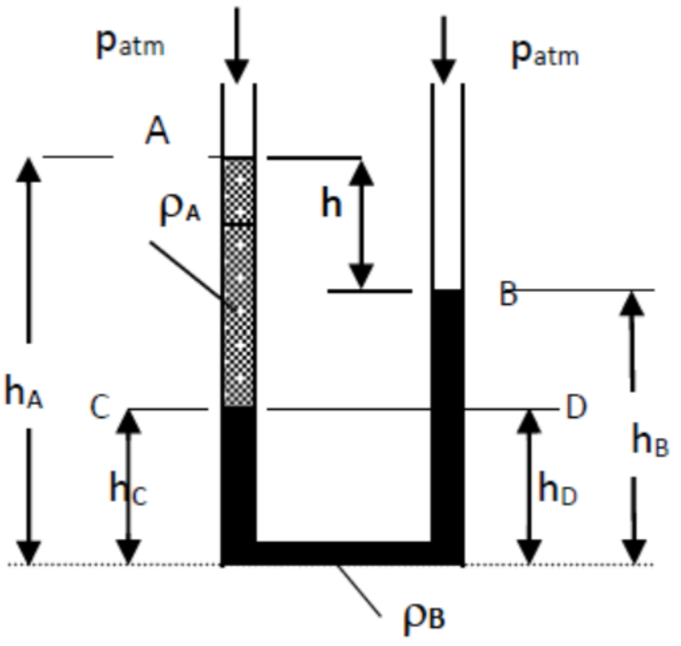
So if: $\boldsymbol{S_2 \gg S_1 \Rightarrow F_2 \gg F_1}$

The principle of car braking:



Equilibrium of two immiscible fluids:

- A U-shaped tube filled with a liquid of density (ρ_B), if in one of the branches another liquid immiscible to the first and of density (ρ_A) is poured, a difference in level $h = (h_A - h_B)$ is observed between the two liquids.



- The two free surfaces are at atmospheric pressure.
- According to Pascal's principle, it is possible to write the following equations:

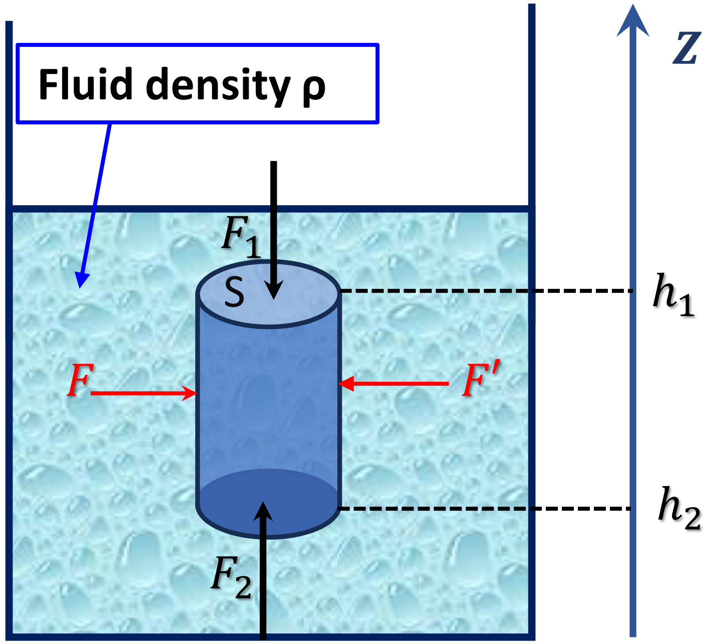
$$P_D = P_{atm} + \rho_B g(h_B - h_D)$$
$$P_C = P_{atm} + \rho_A g(h_A - h_C)$$
$$\Longrightarrow P_{atm} + \rho_B g(h_B - h_D) = P_{atm} + \rho_A g(h_A - h_C)$$

and since $h_D = h_C$(same horizontal plane of the same fluid) $\Longrightarrow \rho_B g(h_B - h_C) = \rho_A g(h_A - h_C)$

$$\Longrightarrow \rho_B = \rho_A \frac{h_A - h_C}{h_B - h_C}$$

- Simply measuring the heights of the two fluids can determine the density of a fluid.
- This concept is also used for pressure control with liquid column pressure gauges or differential pressure gauges.

Archimedes' principle: Let be a cylinder of length L and cross-section S, immersed in a fluid of density ρ in the Earth's gravity field,



This cylinder is subjected to several forces:

- Radial pressure forces exerted on the vertical wall that are opposite and cancel each other two by two (F and F')
- A normal vertical force exerted on the lower surface S, directed upwards and of intensity $F_2 = P_2.S$.
- A normal vertical force exerted on the upper surface S, directed upwards and of intensity $F_1 = P_1.S$

By definition, Archimedes' thrust is the result of all these forces:

$$\sum \vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 = -P_1.S\vec{k} + P_2.S\vec{k} = (P_2 - P_1)S\vec{k} = \Delta P.S\vec{k} = -\rho_{fluid} g \Delta h S\vec{k} = -\rho_{fluid} g(h_2 - h_1)S\vec{k} = \rho_{fluid} g(h_1 - h_2)S\vec{k}$$

$(h_1 - h_2)$ is the height of the cylinder, so: $(h_1 - h_2)S = V_{cyl} = V_{immersed}$: $\Rightarrow \sum \vec{F}_{ext} = \rho_{flui} V_{imm} g\vec{k} = -\vec{w}_{deplaced\ fluid}$

Archimedes' principle is a force that directed in the opposite direction of the gravity field and is announced as follows:

"Any body that is totally immersed in a fluid is subjected to a downward force equal to the weight of the fluid displaced, i.e. corresponding to the volume of the immersed body."

Exercise 1: cork stopper (bouchon en liège).

A cork is held at the bottom of a container filled with water. It is then released.

1) What will the cork do?
2) We study the cork system. The latter is subject to two forces. Which? You will give their name and characteristics.
3) The cork has a volume of 0.250 dm3, the density of cork is 0,2 kg.L-1 and that of water is 1 kg. L-1 . Remember that 1 L = 1 dm3.

   a) Calculate the mass of the cork.

   b) Deduct its weight. We remind that g = 9,81 N.kg-1.

   c) Calculate the magnitude of Archimedes' thrust(force).

   d) Represents, on a diagram, a cork in the water and the two forces experienced by the plug. We will take 1 cm for 0.5 N as a scale.

4) Are the values found for the intensity of each force in agreement with the answer to question 1?

Gas Static:

Gas statics is a branch of fluid mechanics that studies gases at rest, i.e. in the absence of macroscopic motion. It is interested in the properties of gases, such as pressure, temperature, density, and their distribution in an equilibrium system.

1. Pressure in a gas: is a force exerted by the gas per unit area. It is due to the collisions of the gas molecules against the walls of the container.

The pressure is uniform in a gas at rest (in the absence of external forces such as gravity).

2. Law of ideal gases:

An ideal gas is a thermodynamic model to describe the behavior of a gas that performs few collisions between the molecules of the gas and has a weak electrical interaction between the molecules of the gas Molar mass.

For an ideal gas, the relationship between pressure P, volume V, temperature T and quantity of matter n is given by:

$$PV = nRT$$

where:

- P : pressure (in Pascals, Pa),
- V : volume (in cubic metres), m³),
- n : amount of substance (in moles)$(n = \frac{m}{M} = \frac{mass}{Molar\ mass})$
- R : Ideal Gas Constant (R=8,314 SI),
- T : Absolute temperature (in Kelvins, K).

In the presence of gravity, the pressure in a gas varies with altitude. This variation is described by the law of hydrostatic equilibrium:

$$dP = -\rho(z)g dz \implies \frac{dP}{dz} = -\rho(z)g$$

We have: $PV = nRT = \frac{m}{M}RT \implies \frac{PM}{RT} = \frac{m}{V} = \rho(z)$

$$\implies \frac{dP}{dz} = -\frac{PM}{RT}g \quad \implies \frac{dP}{P} = -\frac{Mg}{RT}dz \implies \int_{P_0}^{P}\frac{dP}{P} = -\frac{Mg}{RT}\int_0^z dz \implies \ln\frac{P}{P_0} = -\frac{Mg}{RT}z$$

$$\boldsymbol{\implies P = P_0 e^{-\frac{Mg}{RT}z} = P_0 e^{-\frac{z}{H}}}$$

où :

P_0 : pressure at reference altitude (often at ground level),

H : Height Scale ($H = RT/Mg$, where M is the molar mass of the gas).

Mariotte's Law (or Boyle-Mariotte's Law)

is a fundamental law of thermodynamics that describes the behavior of ideal gases during an isothermal transformation (i.e. at a constant temperature).

It relates the pressure P and the volume V of an ideal gas.

Statement of Mariotte's Law

For a given quantity of ideal gas at a constant temperature, the product of the pressure P and the volume V remains constant.

Mathematically, this is expressed by: $PV = Constant$

or again: $$P_1V_1 = P_2V_2$$

P_1 and V_1 are the initial pressure and volume,

P_2 and V_2 are the pressure and the final volume.