# Chapter 2

# Magnetostatic field

## 1- Introduction

When we talk about magnetism, we generally talk about magnets, the north pole and the south pole. We will see in this chapter that magnetic fields can be created by electric currents. But to begin with, everyone knows that we can use a compass to locate north, but why does the little magnetic needle spontaneously orient itself towards north? Let's see that.

### 1-1- Terrestrial magnetism

**Manipulation:** We leave a small magnetic needle; it moves in a preferred direction. If we disturb it a little, after oscillating for a few moments it returns to its initial position.

- The earth behaves like a gigantic magnet, this is due to the convection movements of molten terrestrial rocks around its core.
- As a result, the geographic north pole behaves like a magnetic pole, which is called the magnetic north pole.
- The side of the needle which is oriented according to this pole is also called the north pole by convention, it is generally marked in red; the other side therefore becoming a south pole.

### 1-2- How to locally modify terrestrial magnetism

**1) Using a magnet:**

We then use a magnet, which also has two poles:

**Handling :**

We approach one of the poles of the magnet of the needle, if it repels the north pole of the needle it is because the pole of the magnet approached is a north pole.

If we turn the magnet over, we realize that this time, the north pole of the little needle is attracted, the pole of the magnet is then a south pole.

**Conclusion :**

- Two magnet poles of the same nature repel each other.
- Two magnet poles of different natures attract each other.

**2) Use of a wire carrying a direct current:**

**Oerstedt's experience:**

A magnetic needle is located near a wire which can carry a current. The needle takes the orientation due to terrestrial magnetism. If we establish a direct current in the wire, we notice that the orientation of the needle changes.

## 1-3- Concept of magnetic field

### 1) Its existence:

The magnet and the wire carried by the current modify the magnetic properties around them, we say that they create a magnetic field.

### 2) Its characteristics:

The magnetic field is a vector: B. It therefore has certain characteristics:

_ One direction: that of the axis of the magnetized needle at equilibrium.

_ One direction: from the south pole of the needle to its north pole.

_ A value: B which is given in Tesla (T).

Note: This vector does not have an application point.

### 3) Its measurement:

We use a specific device called a teslameter, it is equipped with a hall effect probe. Some orders of magnitude of field:

- Earth's magnetic field: B = 50*10-6 T
- Field created by a magnet: B = 0.02 T
- Field created by an electromagnet: B = 10 T

### 4) Superposition of two magnetic fields:

Given that the magnetic field is a vector quantity:

If we superimpose two fields, the resulting field is the vector sum of the two.

If we superimpose B1 and B2 then B TOTAL = B1+ B2.

## 1-4- Magnetic spectra and field lines

### 1) Definitions:

**Handling :**

Magnet + iron filings: observation of the magnetic spectrum. Interpretation:

Under the action of the magnetic field, the iron filings behave like a set of small magnetic needles.

They are oriented according to the magnetic field at the point considered.

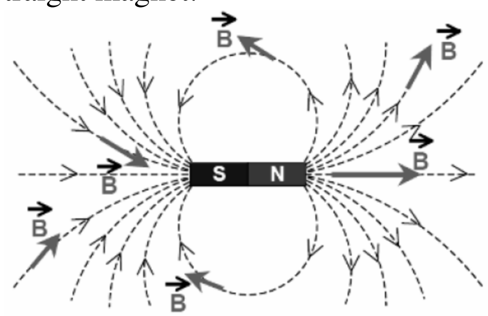
**Conclusion :**

- The figure observed using iron filings is called a **magnetic spectrum**.
- We also observe that the filings are distributed along curved lines around the magnet.

These lines are called **field lines**, at each point of these lines, the magnetic field is tangential.

2) **Different spectra:**

- Magnetic spectrum created by a straight magnet:




- Magnetic spectrum created by a U-shaped magnet:

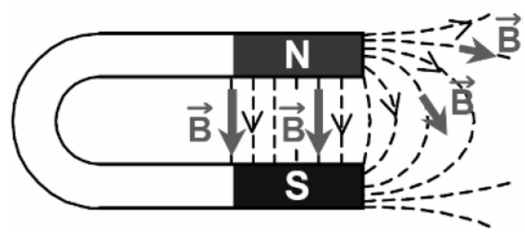



Figure 2.1- movement of the magnetic field (a) straight magnet, (b) U-shaped magnet.

Noticed :

- The field lines close on themselves.
- We see that the field lines located between the two branches of the U-shaped magnet are parallel: the magnetic field vectors have the same meaning, same direction and same value.

This is the characteristic of a uniform magnetic field.

**1-5- Properties of the magnetic field created by a current**

1) **The field created by a wire:**

We saw with Oerstedt's experiment that a wire carrying a direct current creates a magnetic field. We will study here the properties of such a field:

- Field lines are circles centered on the wire.

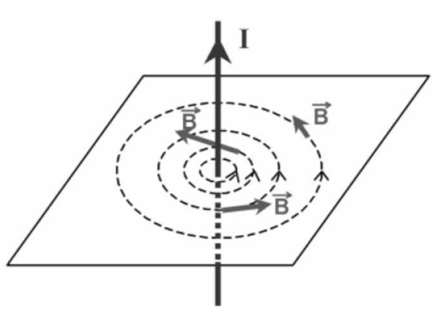



Figure 2.2- magnetic field line.

- Their meaning is given by the corkscrew rule.

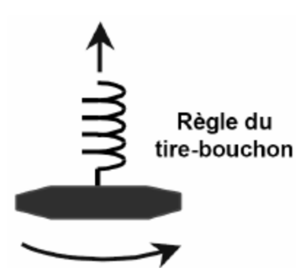


Figure 2.3- Corkscrew rule in a magnetic field.

- The magnetic field vector B(M) created at a point M by a rectilinear wire is in the plane containing M and perpendicular to the wire.

**Field value:**

This is proportional to the intensity of the current passing through the wire: $B = k\,x\,I$

The constant k (expressed in $T.A^{-1}$) depends on the point where the field is measured.

2) **if the field is created by a solenoid:**

- What is a solenoid?

It is made up of a helical winding of turns on a cylindrical support. By definition, the radius of the cylinder must be small compared to its length.

- Field lines:

_ We determine their meaning using the corkscrew rule.

_ They enter through the south face of the inductance and exit from the north face. (This allows you to locate the north face of the inductance, you can also use the right hand rule)

_ Inside the solenoid the field lines are parallel straight lines, the field is therefore uniform.

_ On the outside of the solenoid, the field lines resemble those of a bar magnet.

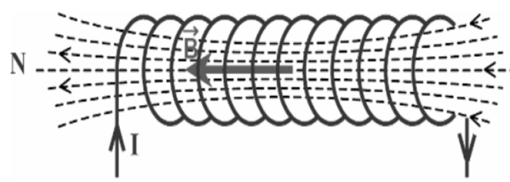



Figure 2.4- magnetic field line in a solenoid.

**Noticed :**

The right-hand rule makes it possible to determine the position of the north face of an inductance: If we wind the current with the palm of the right hand, the direction indicated by the push gives the location of the north face, therefore also the direction of the field lines.

- Value of the field inside a solenoid:

It is given by the formula: $B = n\,x\,I\,x\,\mu_0$ (2.1)

Where

B: value of the magnetic field in Tesla (T).

I: current intensity in Ampere (A)

n: number of turns per unit length (m-1)

$\mu_0 = 4\,x\,\pi\,x\,10^{-7}$ $T.m.A^{-1}$

### 3) Conclusion

The value of a magnetic field created by a current depends on the geometry of the current, its intensity and the position of the measuring point.

- An immobile electric charge creates an electric field only.
- A moving charge (a current) creates an electric field and a magnetic field.

**Definition:** magnetostatics is the study of static magnetic phenomena, generated by constant currents only (direct current).

## 2- Ampere's law

The Danish physicist Hans C. Oersted (1777 – 1851), by noticing the deviation of a compass placed near a conductor crossed by a current, was the first to observe the magnetism created by an electric current.

### 2-1- straight conductor

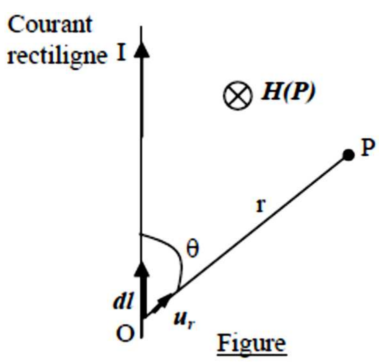



Figure 2.5- straight conductor.

$$H = \int_{-\infty}^{+\infty} \frac{I dl \wedge u_r}{4\pi r^2} \quad (2.2)$$

H : magnetic field

r = OP ;

$u_r$: unit vector of r.

$$B = \int_{-\infty}^{+\infty} \frac{\mu_0 I dl \wedge u_r}{4\pi r^2} \quad (2.3)$$

B : Magnetic Induction

2-2- closed conductor

$$B = \oint \frac{\mu_0 I dl \wedge u_r}{4\pi r^2}$$

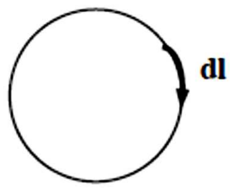



Figure 2.6- circular current

With

$\mu_0$ : magnetic permeability (vacuum, air, etc.) its value is cited at the top.

$B = \mu_0 H$

Units

[B] = Tesla T ; [H]= A /m

2-3- case of volume current

J current density (A/m2) ;

$J = I / S$,

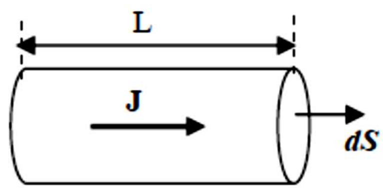



Figure 2.7- volume conductor

Let $\boldsymbol{I} = \boldsymbol{J}\,\boldsymbol{S}$,

Or generally :

$$I = \int J dS \Rightarrow I dl = \int J dS dl = J S dl = J dV \qquad (2.4)$$

The magnetic field of a cylindrical (volumic) current is given by:

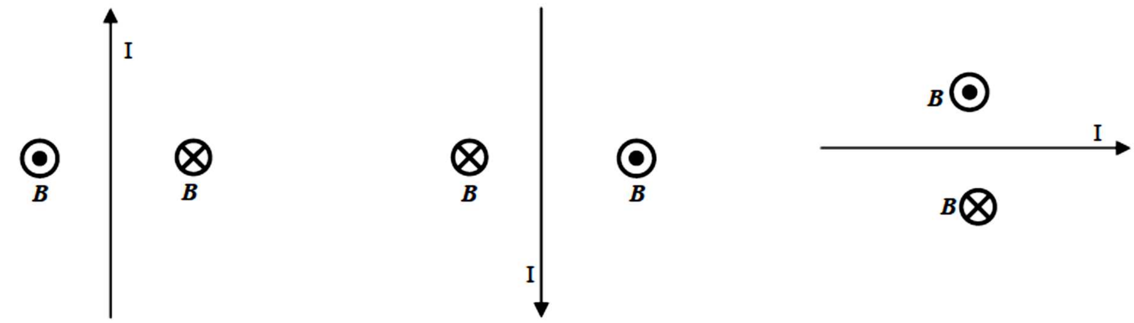
$$H = \int \frac{J \wedge u_r}{4\pi r^2} dv \qquad (2.5)$$

So either :

$$B = \mu \times H = \mu_0 \int \frac{J \wedge u_r}{4\pi r^2} dv \qquad (2.6)$$

3- Magnetic field direction (right hand rule)

3-1- Straight wire: (Right hand rule)




3-2- Spire: (Screwdriver rule)

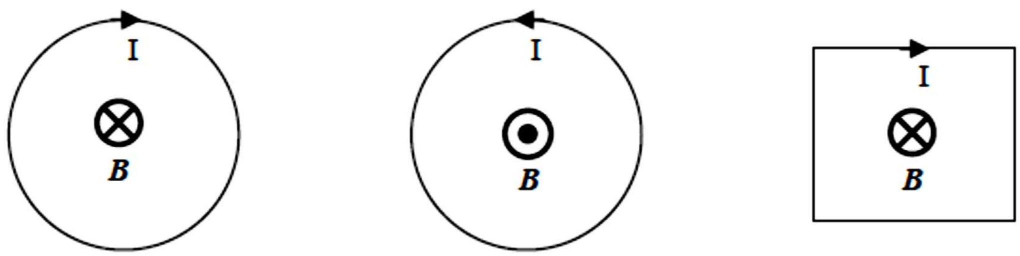



Figure 2.7- magnetic field (a) straight wire, (b) spire.

4- Magnetic potential

As q is a scalar, which produces a scalar electric potential V; By analogy with electrostatics: The element *Idl* is a vector, produces a vector magnetic potential A.

B=rotA

$$A = \frac{\mu_0}{4\pi}\int \frac{J}{r} dV \qquad (2.7)$$

Which represents the expression of potential A.

5- Ampere theorem

5-1- Ampere theorem

$$\oint Hdl = ? \qquad H = \frac{1}{2\pi r}u$$

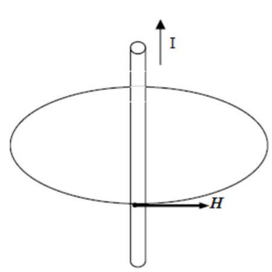



Figure 2.8- Ampere theorem

$$\oint Hdl = \oint \frac{I}{2\pi r} udl'$$

(2.8)

**Reminder :**

$$Au_x = (A_x u_x + A_y u_y + A_z u_z)u_x = A_x \qquad (2.9)$$

i.e. the component of A along the x axis. By analogy*: dl.u = dl'* is the component of *dl* following *u*.

As moreover, ⊥ ***ur***, i.e. ***u*** ⊥ ***r***, therefore also ***dl'*** ⊥ ***r***; dl' therefore represents an arc of circle of radius r ⇒ ***dl'=r dθ.***
By consequent:

$$\oint Hdl = \frac{I}{2\pi}\oint \frac{rd\theta}{r} = \frac{I}{2\pi}\oint d\theta = \frac{I}{2\pi}\theta\binom{2\pi}{0} = I \qquad (2.10)$$

So

$$\oint Hdl = I$$

Which represents the Ampere theorem.
Important note : I is a current circulating inside the closed contour.

5-2- Differential form

$$\oint Hdl = I$$

Is the integral form of Ampere theorem.
As

$$\oint Hdl = \int rotH.dS$$

And that

$$I = \int JdS$$

It can be possible to write:

$$\int_s rotH.dS = \int_s J.dS$$

Therefore: which represents the differential form of Ampère's theorem.
**Conclusion:** rot H=J implies that the magnetic field is rotational, that is to say that the field lines are closed, unlike the electric field lines.
**Noticed :** - The magnetic field lines are closed curves because unlike the electric field which has electric charges as its source (starts from the positive charge and arrives at the negative charge), there are no magnetic charges.

6- Magnetic flow

$$\emptyset_m = \int B.ds \qquad (2.11)$$

Unit [Φm]=Weber(Wb) ;

6-1- Unclosed surface

Flow: represents the quantity of field lines passing through the surface.

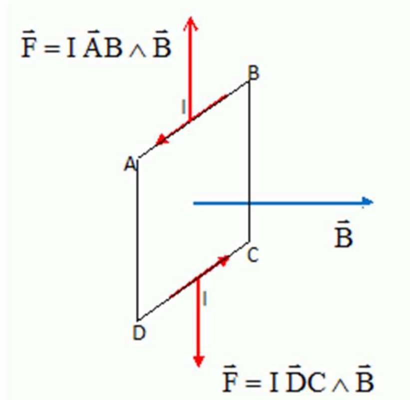



Figure 2.9- The flow in an Unclosed surface.

6-2- Closed surface

$$\oint BdS = \int divBdV = \int div(rotA)dV = 0$$

$$\oint BdS = 0 \qquad (2.12)$$

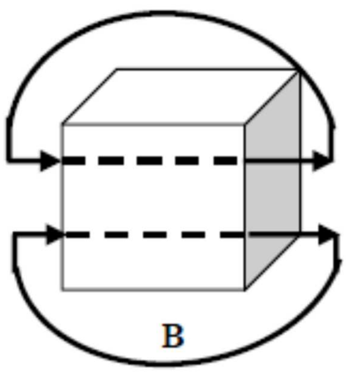



Figure 2.10- the flow in the closed surface.

**Differential Form:**

$$\oint BdS = 0$$

Is the integral form of this law.

$$\oint BdS = \int divBdv = 0 \Rightarrow divB = 0$$

$$divB = 0 \qquad (2.13)$$

Is the differential form.

## 7- Magnetic force

### 7-1- Lorentz force

An electric charge animated with a speed v and placed in an electric and magnetic field, undergoes the following force:

$$F = q(E + v \wedge B)$$

$F = qE + qv \wedge B = F_e + F_m$ (2.14)

With :

$\boldsymbol{Fe = qE}$ is the electric force;

If $q = 0 \Rightarrow Fe = 0$

The electric force is canceled if the charge is zero.

Fm=q(v∧ B) is the Magnetic Force.

The magnetic force is canceled if the charge is zero or immobile. Magnetic induction only exerts force on a moving charged particle (or a current).

**Conclusion:**

The magnetic force only acts on a moving charge, or a conductor passing through a current.

### 7-2- Laplace force

Consider a cylindrical conductor crossed by a current I.

Either:

n': number of charged particles passing through the conductor;

e: elementary charge of a particle.

The charge crossing the conductor is then: q=n'e

By setting : n = n' / V

n: number of particles/unit volume;

V: conductor volume.

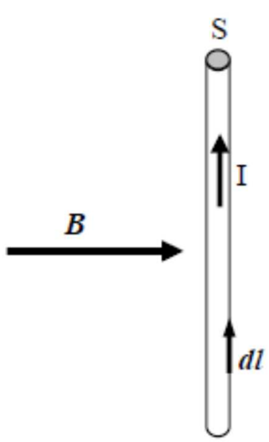


Figure 2. 11- Laplace force in an cylindrical conductor.

We obtain :

$$I = \frac{dq}{dt} = \frac{d}{dt}(n'e) = \frac{d}{dt}(neV) = ne\frac{dV}{dt} = neS\frac{dl}{dt} = neSv \qquad (2.15)$$

With :

v : particle movement speed.

By consequent:

$$J = \frac{I}{S} = \frac{neSv}{S} = nev \Rightarrow J = nev$$

This equality is also valid in vector notation:

$$J = nev \qquad (2.16)$$

In other hand, we reporting the charge per volume unit $= ne$ Lorentz's law, we obtain :

$$Fm = q(v \wedge B) = nev \wedge B = J \wedge B \qquad (2.17)$$

**For an elementary volume $dV$ :**

$$dFm = (J \wedge B)dV \qquad (2.18)$$

**For the entire volume V :**

$$Fm = \int (J \wedge B)dV = \int (JdV \wedge B) \qquad (2.19)$$

As $JdV = Idl$ , we arrive at the expression of the Laplace Force.

**Remark:**

Si $I = 0 \Rightarrow Fm = 0$

The magnetic force therefore only acts on a conductor through carrying a current

## 8- Magnetic energy Wm

We consider the example of a toroidal coil comprising n turns.

Determine the energy stored when the current in the coil increases from 0 to I.

Consider a circuit formed by an inductor.

At time t we have: U=LdI / dt

By multiplying the two members by i dt so as to reveal the energies involved during dt:

$$Uidt = Lidi = d\,1\,((1/2)\,Li^2) \qquad (2.20)$$

The term U i dt represents the energy supplied by the generator, the term $dW = d\,((1/2)\,Li^2)$

Corresponds to the energy supplied to estabilish the current i, energy stored in the inductance.

**Demonstration :**

By analogy with electrostatics where the density of electrostatic energy $we = \left(\frac{1}{2}\right).\varepsilon_0.E^2$, demonstrate that the magnetic energy density is $wm=(1/2)\,\mu_0\,H^2$ For this, let us consider an elementary induction tube

Let $dV = S\,dl$

The magnetic energy located in the volume element $dV$ is:

$$dW = \frac{1}{2}\pi H^2 dV = \frac{1}{2}\pi H^2 Sdl \qquad (2.21)$$

Taking into account that the induction flux is constant in the tube: $\Phi = \int B.dS = B.S$

And the Ampere theorem: $\oint Hdl = I$

We obtain:

$$W = \frac{1}{2\pi}B^2 Sdl = \frac{1}{2\pi}BS\int Bdl = \frac{1}{2}BS\int Hdl = \frac{1}{2}\emptyset I \qquad (2.22)$$

As: $\emptyset = LI$

$$W = \frac{1}{2}\emptyset I = \frac{1}{2}LI^2 \qquad (2.23)$$

**Conclusion :** the magnetic field stores the energy of density $w_m=(1/2)\,\mu_0\,H^2$

**9- Summary of regime stationary laws**

**9-1- Gauss Theorem**

$$\oint EdS = \frac{q}{\varepsilon_0} \qquad (2.24)$$

$$divE = \frac{\rho}{\varepsilon_0} \qquad (2.25)$$

$$rotE = 0 \qquad (2.27)$$

$$\oint Edl = 0 \qquad (2.26)$$

**9-2- Ampere Theorem**

$$\oint Hdl = I \qquad (2.28)$$

$$rotH = J \qquad (2.29)$$

**9-3- Magnetic flow Theorem**

$$\oint BdS = 0 \quad (2.30)$$

$$divB = 0 \quad (2.31)$$

**10- Analogy between electrostatics and magnetostatics**

| ELECTROSTATIQUE | MAGNETOSTATIQUE |
|---|---|
| Loi de Coulomb (champ électrique | Loi de Biot & Savart (champ magnétique) |
| $q \rightarrow \boldsymbol{E} = \frac{q}{4\pi\varepsilon r^2}\boldsymbol{u}$ | $I\boldsymbol{dl} \rightarrow \boldsymbol{H} = \oint \frac{I\boldsymbol{dl} \wedge \boldsymbol{u_r}}{4\pi r^2}$ |
| Déplacement électrique | Induction magnétique |
| $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ | $\boldsymbol{B} = \mu \boldsymbol{H}$ |
| Potentiel électrique | Potentiel magnétique |
| $V = \frac{q}{4\pi\varepsilon r}$ | $\boldsymbol{A} = \frac{\mu}{4\pi}\int \frac{\boldsymbol{J}}{r} dv$ |
| $\boldsymbol{E} = -gradV$ | $\boldsymbol{B} = rot\boldsymbol{A}$ |
| $\oint \boldsymbol{E.dl} = 0$ | $\oint \boldsymbol{H.dl} = I$ |
| $rot\boldsymbol{E} = 0$ | $rot\boldsymbol{H} = \boldsymbol{J}$ |
| $\oint \boldsymbol{E.dS} = \frac{q}{\varepsilon}$ | $\oint \boldsymbol{B.dS} = 0$ |
| $div\boldsymbol{E} = \frac{\rho_v}{\varepsilon}$ | $div\boldsymbol{B} = 0$ |
| $w_e = \frac{1}{2}\varepsilon E^2$ | $w_m = \frac{1}{2}\mu H^2$ |
| $E = 0$ dans un conducteur | $H \neq 0$ dans le conducteur |