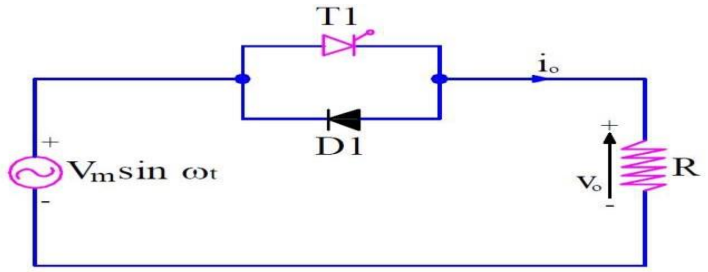
Chapter 3

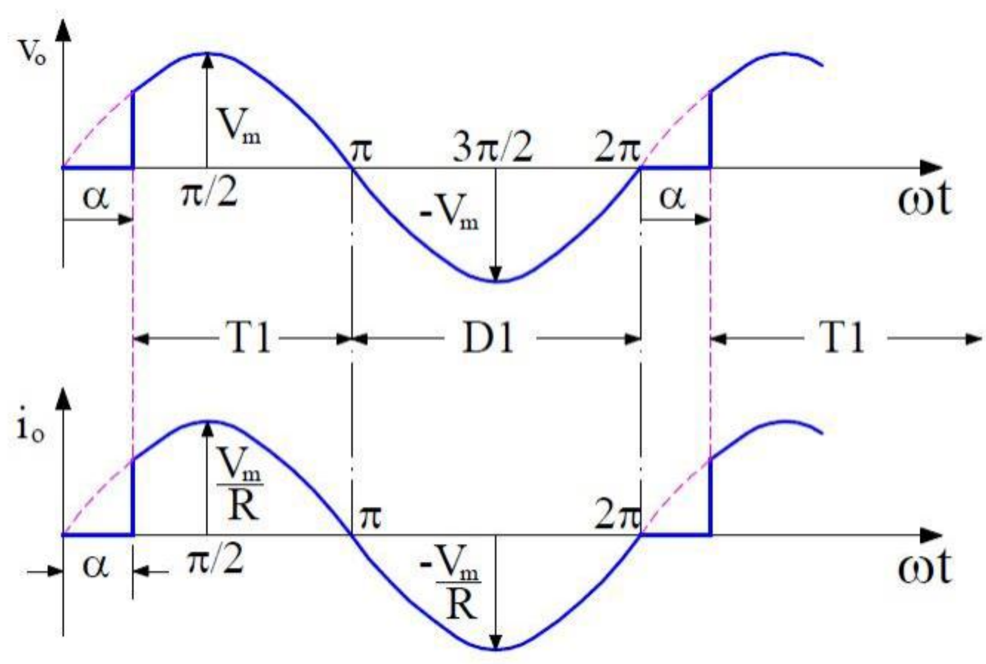
AC TO AC Converter

3.1 Single phase controller (phase control)

3.1.1 Unidirectional controller

A single phase half wave AC voltage controller comprises of a thyristor connected in anti-parallel with a power diode. The circuit diagram is shown in figure below.





$$if\ \ input\ \ voltage,\ v_s = V_m \sin\omega t = \sqrt{2}\,V_s \sin\omega t$$
$$delay\ \ angle\ \ of\ \ thyristor\ \ T_1,\ \ \omega t = \alpha$$

the rms output voltage,

$$V_o = \left\{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} 2V_s^2 \sin^2\omega t\ d(\omega t) + \int_{\pi}^{2\pi} 2V_s^2 \sin^2\omega t\ d(\omega t)\right]\right\}^{1/2}$$
$$= \left\{\frac{2V_s^2}{4\pi}\left[\int_{\alpha}^{\pi}(1-\cos 2\omega t)\ d(\omega t) + \int_{\pi}^{2\pi}(1-\cos 2\omega t)\ d(\omega t)\right]\right\}^{1/2}$$
$$= V_s\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

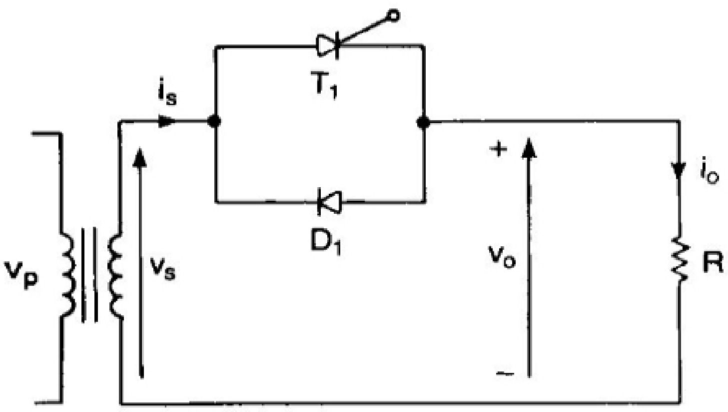
the average value of output voltage,

$$V_{dc} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi}\sqrt{2}\,V_s \sin\omega t\ d(\omega t) + \int_{\pi}^{2\pi}\sqrt{2}\,V_s \sin\omega t\ d(\omega t)\right]$$
$$= \frac{\sqrt{2}\,V_s}{2\pi}(\cos\alpha - 1)$$

$$If,\ \alpha = 0 \rightarrow \pi\ :\ V_o = V_s \rightarrow \frac{V_s}{\sqrt{2}},\ \ V_{dc} = 0 \rightarrow \frac{-\sqrt{2}V_s}{\pi}$$

EXAMPLE 1:-

A single-phase ac voltage controller in figure has a resistive load of R=10Ω and the input voltage is V_s=120V, 60Hz.
The delay angle of thyristor T_1 is $\alpha=\pi/2$. Determine,
(a) The *rms* value of output voltage V_o
(b) The input power factor *PF*
(c) The average input current



$$R = 10\Omega,\ V_s = 120\,\text{V},\ V_m = \sqrt{2}\times 120 = 169.7\,\text{V},\ \alpha = \frac{\pi}{2}$$

(a) *the rms value of output voltage*,

$$V_o = V_s\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = 120\sqrt{\frac{3}{4}} = 103.92\,\text{V}$$

(b) *the rms load current*,

$$I_o = \frac{V_o}{R} = \frac{103.92}{10} = 10.392\,\text{A}$$

the load power, $P_o = I_o^2 R = 10.392^2 \times 10 = 1079.94\,\text{W}$

$$VA = V_s I_s = V_s I_o = 120 \times 10.392 = 1247.04\ \text{VA}$$

the input power factor,

$$PF = \frac{P_o}{VA} = \frac{V_o}{V_s} = \left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = \sqrt{\frac{3}{4}} = \frac{1079.94}{1247.04}$$

$$= 0.866\ (lagging)$$

(c) *the average output voltage*,

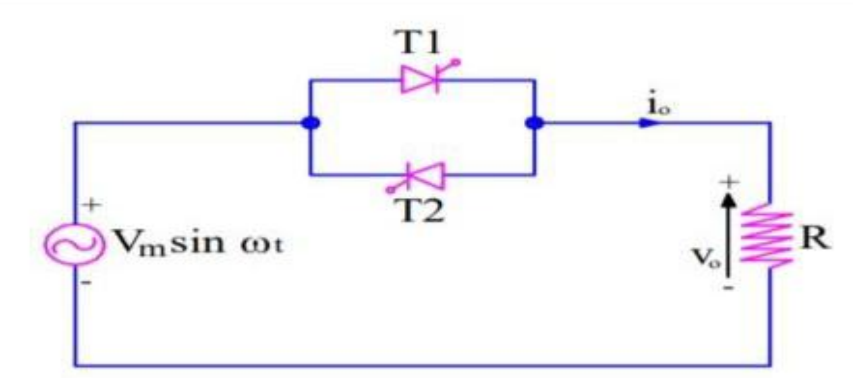
$$V_{dc} = \frac{\sqrt{2}\,V_s}{2\pi}(\cos\alpha - 1) = -120 \times \frac{\sqrt{2}}{2\pi} = -27\ \text{V}$$

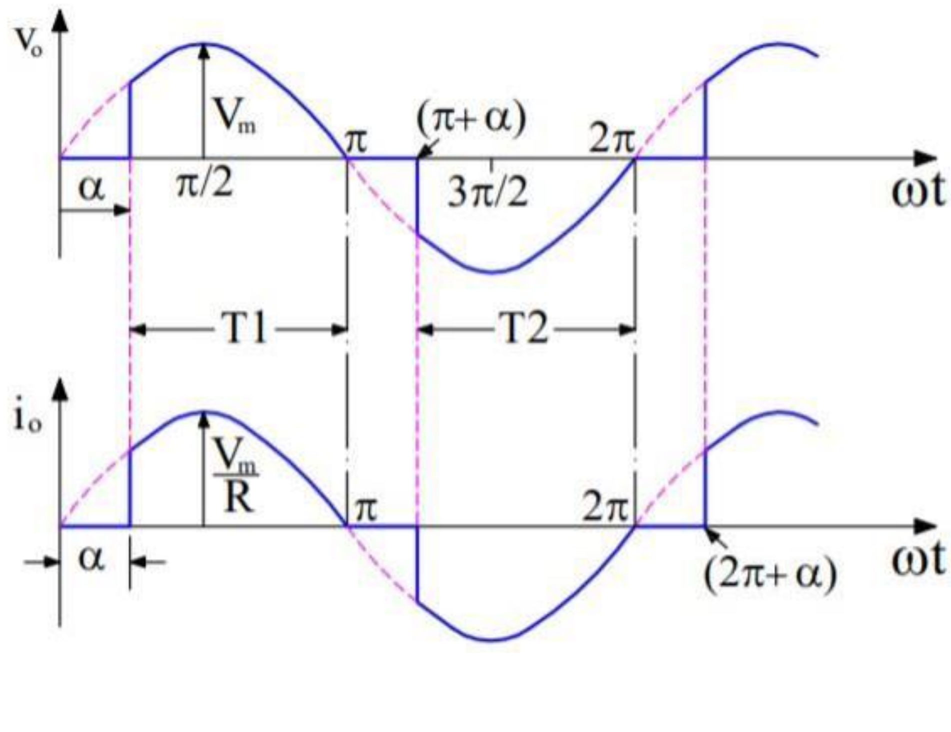
the average input current

$$I_D = \frac{V_{dc}}{R} = -\frac{27}{10} = -2.7\ \text{A}$$

3.2.2. Bidirectional controller

A single phase full wave AC voltage controller comprises of two thyristor connected in anti-parallel. The circuit diagram is shown in figure below.





$$\text{if input voltage, } v_s = V_m \sin\omega t = \sqrt{2}\, V_s \sin\omega t$$

$$\text{delay angle of thyristor } T_1 \text{ and } T_2,\ \alpha_1 = \alpha_2 = \alpha$$

the rms output voltage,

$$V_o = \left[\frac{2}{2\pi}\int_\alpha^\pi 2V_s^2 \sin^2\omega t \; d(\omega t)\right]^{1/2} = \left[\frac{4V_s^2}{4\pi}\int_\alpha^\pi (1-\cos 2\omega t)\; d(\omega t)\right]^{1/2}$$

$$= V_s\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$\text{If},\ \alpha = 0 \rightarrow \pi \ :\ V_o = V_s \rightarrow 0$$

EXAMPLE 2:-

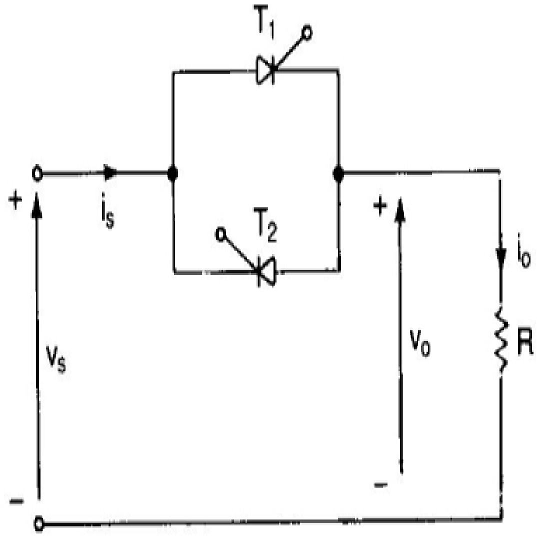
A single-phase full-wave ac voltage controller in figure has a resistive load of R=10Ω and the input voltage is V_s=120V(rms), 60Hz. The delay angle of thyristors T_1 and T_2 are equal : $\alpha_1 = \alpha_2 = \pi/2$. Determine,

(a) The *rms* output voltage V_o

(b) The input power factor *PF*

(c) The average current of thyristors I_A

(d) The RMS current of thyristors I_R



$$R = 10\Omega,\ V_s = 120\,\text{V},\ V_m = \sqrt{2}\times 120 = 169.7\,\text{V},\ \alpha = \frac{\pi}{2}$$

(a) *the rms value of output voltage*,

$$V_o = V_s\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = \frac{120}{\sqrt{2}} = 84.85\,\text{V}$$

(b) *the rms load current*,

$$I_o = \frac{V_o}{R} = \frac{84.85}{10} = 8.485\,\text{A}$$

the load power, $P_o = I_o^2 R = 8.485^2 \times 10 = 719.95\,\text{W}$

$$VA = V_s I_s = V_s I_o = 120 \times 8.485 = 1018.2\ \text{W}$$

the input power factor,

$$PF = \frac{P_o}{VA} = \frac{V_o}{V_s} = \left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = \frac{1}{\sqrt{2}} = \frac{719.95}{1018.2}$$

$$= 0.707\ \ (lagging)$$

(c) *the average thyristor current*,

$$I_A = \frac{1}{2\pi R}\int_{\alpha}^{\pi} \sqrt{2}\,V_s \sin\omega t\ d(\omega t) = \frac{\sqrt{2}\,V_s}{2\pi R}(\cos\alpha + 1)$$

$$= \sqrt{2}\times\frac{120}{2\pi\times 10} = 2.7\ \text{A}$$

(d) *the rms value of the thyristor current*,

$$I_R = \left[\frac{1}{2\pi R^2}\int_{\alpha}^{\pi} 2V_s^2 \sin^2\omega t\ d(\omega t)\right]^{1/2}$$

$$= \left[\frac{V_s}{4\pi R^2}\int_{\alpha}^{\pi}(1 - \cos 2\omega t)\ d(\omega t)\right]^{1/2}$$

$$= \frac{V_s}{\sqrt{2}\,R}\left[\frac{1}{\pi}(\pi - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= \frac{120}{2\times 10} = 6\ \text{A}$$