Djilali Bounaama- Khemis Miliana-University
B03 Electrical Engineering
Electromagnetic field theory **2023 /2024**

Series 4

Variable speed - Maxwell's equations

Exercise 1:

The vectors are located in the Cartesian frame, we consider a progressive monochromatic plane wave (PMPW), propagating in a lhi (linear, homogeneous, isotropic) medium, of pulsation ω and wave vector $\vec{k}$. the electric and magnetic fields are given in complex notation by: $\vec{E} = E_0\, e^{j(\omega t - \vec{k}\vec{r})}$ and $\vec{B} = B_0\, e^{j(\omega t - \vec{k}\vec{r})}$, we recall that Maxwell's equations in linear, homogeneous and isotropic material media are given by:

$\vec{\nabla}.\vec{E} = \rho/\varepsilon$ (Loi de Gauss) (4.23)

$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Loi de Faraday) (4.24)

$\vec{\nabla}.\vec{B} = 0$ (Absence de charges magnétiques) (4.25)

$\vec{\nabla} \times \vec{B} = \mu\vec{j} + \mu\varepsilon\frac{\partial \vec{E}}{\partial t}$ (Loi d'ampère) (4.26)

1- Establish the wave equation for the electric field $\vec{E}$.
2- What happens to this equation far from the sources.
3- Deduce the wave equation in a vacuum.
4- Deduce the dispersion relation (relation between k and ω) in vacuum.

Exercise 2:

We study an electromagnetic wave whose electric field is:

$$\vec{E} = E_x\overrightarrow{u_x} + E_y\overrightarrow{u_y}$$

$$E_x = E_0 e^{[i(\frac{k}{3}(2x+2y+z)-\omega t)]}$$

The wave propagates in a vacuum and its wavelength is $\lambda = 6 \times 10{-}7\text{m}$.

a- Calculate the frequency of the wave.
b- In what area of the electromagnetic spectrum is this wave located?
c- Calculate the numerical value of the constant k.
d- Establish the Cartesian equation of a wave plane.
e- Express Ey according to Ex.
f- Calculate the magnetic field $\vec{B}$.

Prepared by: Dr KARA MOSTEFA KHELIL.C

Exercise3:

We consider a plane, progressive and sinusoidal electromagnetic wave of pulsation ω, propagating in a vacuum (characterized by the constant of Coulomb's law). space is related to a Cartesian reference frame Oxyz with an orthonormal base. the wave propagates in the OY direction. The electric field vector:

$\vec{E} = E_0 \cos(\omega t - \vec{k}\vec{r})\overrightarrow{u_x}$ of amplitude E0 is parallel to Ox.

1- Write in real notation the components of the wave vector $\vec{k}$ then those of the electric field vector $\vec{E}$ at the point M with coordinates (x,y,z) such that $\vec{r} = \overrightarrow{OM}$ and at time t.
2- Using Maxwell's equation in a vacuum, establish the equation for the propagation of $\vec{E}$ in a vacuum. Deduce the dispersion relation of this wave in a vacuum.
3- Using Maxwell's equations in a vacuum, express the components of the magnetic field vector of wave $\vec{B}$ at point M. Specify in particular the expression of the amplitude B0 of the magnetic field.
4- Represent the vectors $\vec{E}$, $\vec{B}$ and $\vec{k}$ on a clear diagram. Is the electromagnetic wave studied longitudinal or transverse? justify your answer.
5- Calculate the volume density of electromagnetic energy U=Ue + Um. Express its time average value <U> as a function of E0 and ε0.