

## Series 2

### Magnetostatic Field

#### Exercise 1: Earth's magnetic field

A solenoid comprising  $N = 1000$  contiguous turns has a length  $L = 80$  cm. It carries a current of intensity  $I$ . a) Make a diagram in which you represent:

- the magnetic spectrum of the solenoid
- the North and South faces
- the magnetic field vector at the center of the solenoid.

We assume the solenoid is long enough to be comparable to a solenoid of infinite length.

b) What is the expression for the intensity of the magnetic field at the center of the solenoid?

A.N. Calculate  $B$  if  $I = 20$  mA.

The axis of the solenoid is placed perpendicular to the plane of the magnetic meridian. In the center of the solenoid we place a small movable compass around a vertical axis.

c) What is the orientation of the compass for  $I = 0$ ?

When the current of intensity  $I = 20$  mA passes through the solenoid, the compass rotates by an angle  $\alpha = 57.5^\circ$ .

Deduce the intensity  $B_h$  of the horizontal component of the earth's magnetic field.

#### Exercise 2: Magnetic field created by a turn

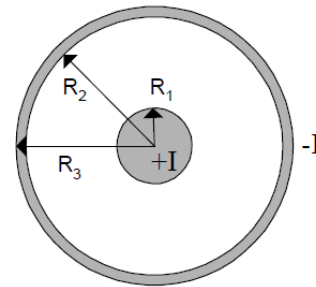
Using the Biot and Savart formula, determine the characteristics of the magnetic field created at the center of a flat coil of  $N$  turns, of radius  $R$  and carried by a current  $I$ . Numerical application:  $R = 5$  cm,  $N = 100$  and  $I = 100$ mA.

#### Exercise 3: Magnetic field created by a cable

We consider a cable of radius  $R$ , of infinite length, traversed by a current of intensity  $I$  evenly distributed in the conductor section. Using Ampère's theorem, determine the intensity of the magnetic field at a point located at the distance  $r$  from the axis of the cable. Draw the curve  $B(r)$ .

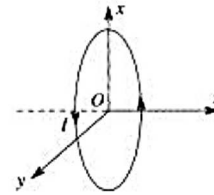
**Exercise 4: Magnetic field created by a coaxial cable**

We consider an infinite cylindrical coaxial cable of radii  $R_1$ ,  $R_2$  and  $R_3$ . The current of total intensity  $I$  passes in one direction in the inner conductor and returns in the other direction through the outer conductor.



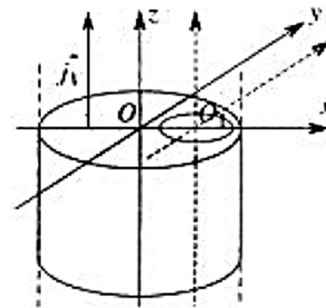
**Exercise 5:**

Turn carrying a filiform current of intensity  $I$ . Consider a turn of radius  $a$  and axis  $(Oz)$  traversed by a current of intensity  $I$ . What are the symmetries and invariances of this distribution?



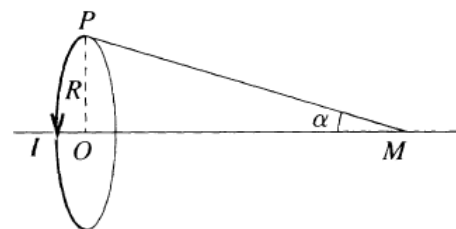
**Exercise 6:**

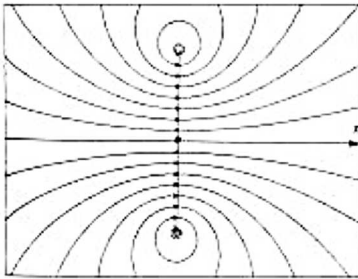
Cylinder with cavity carrying a volume density of currents. An infinite cylinder with a circular base is traversed by a uniform volume current parallel to its generators. In this cylinder there is a cylindrical cavity with a circular base and generators parallel to the previous cylinder. Study the symmetries and invariances of this current distribution.



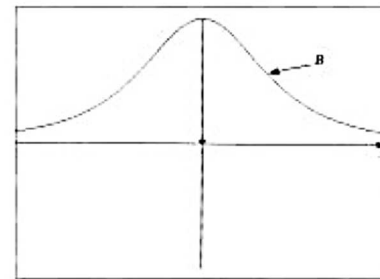
**Exercise 7:**

Field created by a circular turn on its axis. 1. Calculate the magnetostatic field created by a turn of radius  $R$ , traversed by a current of intensity  $I$ , at a point  $M$  of its axis  $(Ox)$ , the turn being seen at angle  $\alpha$  from  $M$ . 2. Interpret the following figures obtained with Maple:





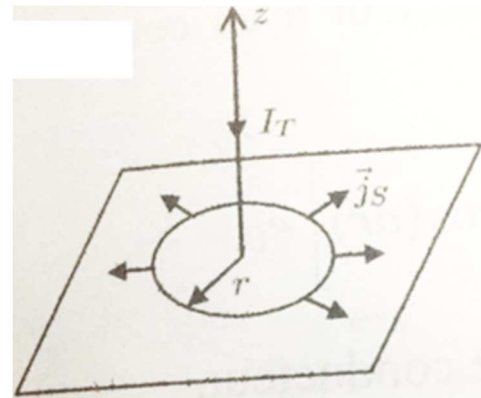
Lignes de champ magnétique d'une spire.



Champ sur l'axe d'une spire.

**Exercise 8:**

A current of intensity  $IT$  travels a filament along the  $z$  axis and penetrates a thin conductive layer at  $z=0$ . Express the surface current density vector  $j_s$  for this sheet.



**Exercise 9:**

Find the intensity of the current which passes through the surface of a sphere of radius  $r$  centered at the origin, knowing that the current density vector is given by:

$$j = I_0 \frac{\sin(\theta)}{r^2} e_r$$

**Exercise 10:**

In a cylindrical conductor with a radius of 2mm, the current density varies with the distance from the axis according to the relation  $j = 10^3 e^{-400r} (Am^{-2})$ . Find the total current intensity.

**Exercise 11:**

Calculate the magnetic field  $B$  in the region surrounding an infinitely long rectilinear wire-shaped current, of intensity  $I$ . Deduce the vector potential  $A$ .

**Exercise 12:** Using Ampere's theorem, calculate the magnetic field  $B$  created by a solid cylindrical conductor, of radius  $a$ , carried by a current of intensity  $I$ , uniformly distributed across the straight section.

**Exercise 13:**

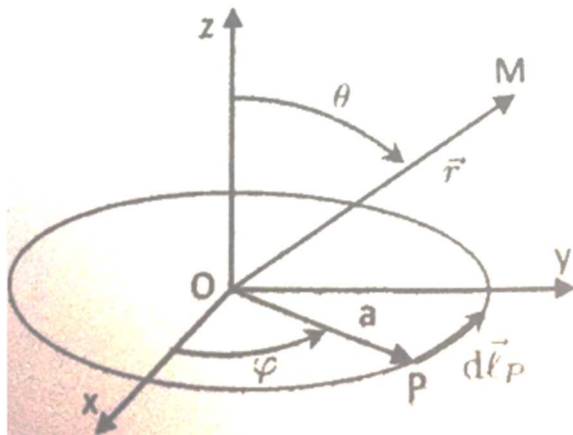
In a cylindrical coordinate system, the current density is:

$$j = \begin{cases} 4.5e^{-2r} (Am^{-2}) & 0 < r < 0.5m \\ 0 & \text{partout ailleurs} \end{cases}$$

- Find the magnetic field  $B$  by Ampère's theorem.

**Exercise 14:**

We have a circular turn with center  $O$ , radius  $a$ , axis  $(Oz)$ , traversed by a current  $I$ . We want to calculate the magnetic field created at a point  $M$  at a long distance from the turn (i.e.  $OM \gg a$ ).



1. Calculate the vector potential  $dA$  created at point  $M$  by an element  $d\ell_P$  of the turn located at a current point  $P$ .
2. Deduce the components of the vector potential  $A(M)$  created by this turn.
3. Deduce the components of the magnetic field  $B$  created by this turn.
4. Compare the form of these expressions with the expressions for the electric field created by an electrostatic dipole.