Tutorial Session 4 (Solution)

<u>Exercise 1:</u>

1.

a) **G** is simple because it contains no loops or multiple edges.

b) The graph is not regular because the vertices do not have the same degree.

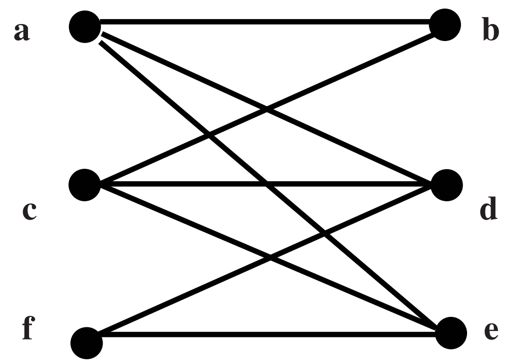
c) The graph is not symmetric because, for example, there is an arc **(a,b)** but not **(b,a)**.

d) The graph is antisymmetric because for every arc of the form **(x,y)**, there is no other arc of the form **(x,y)** or of the form **(y, x)**

e) **G** is not complete because vertices **b** and d are not adjacent.

f) **G** is not a clique because it is not complete.

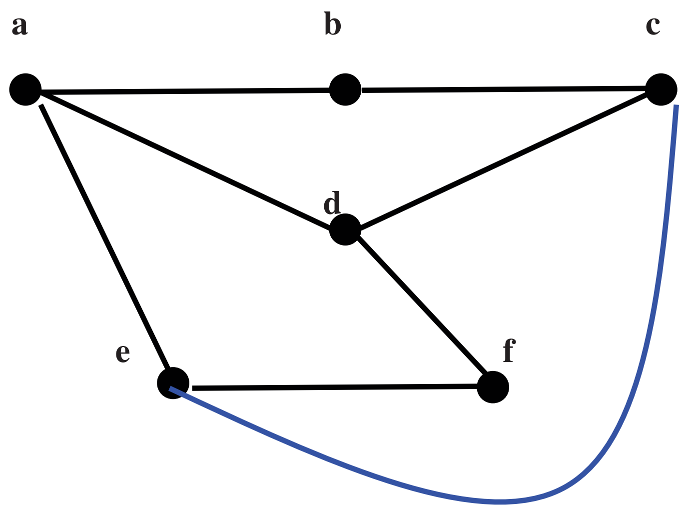
g) **G** is bipartite because its set of vertices can be divided into two classes **X1={a,c,f}** and **X2={b,d,e}** such that no two vertices in the same class are adjacent.



h) **G** is not complete bipartite because vertex **f** is not adjacent to vertex **b**.

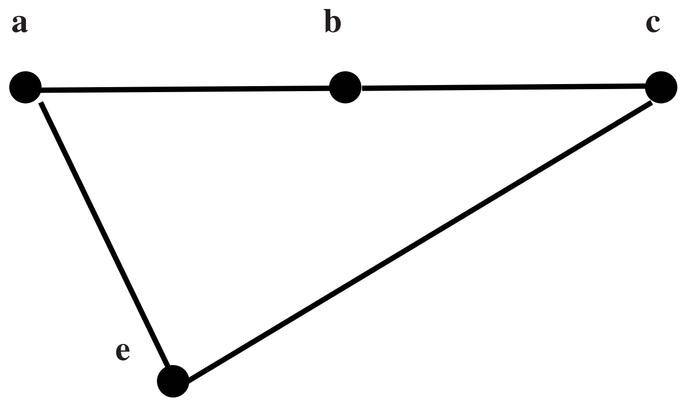
i) **G** is connected because for every pair of distinct vertices **x** and **y**, there exists a chain that links **x** to **y.**

j) **G** is planar because it can be drawn on a plane so that no two edges intersect except at their endpoints.

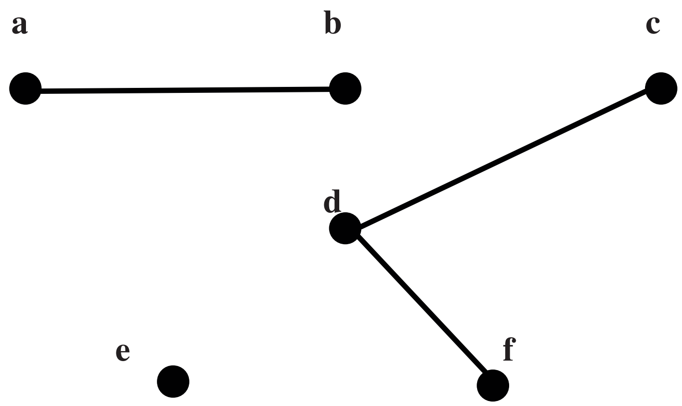


2.

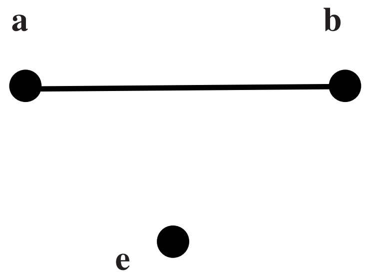
a) Subgraph of **G** generated by **{a, d, c, e}:**



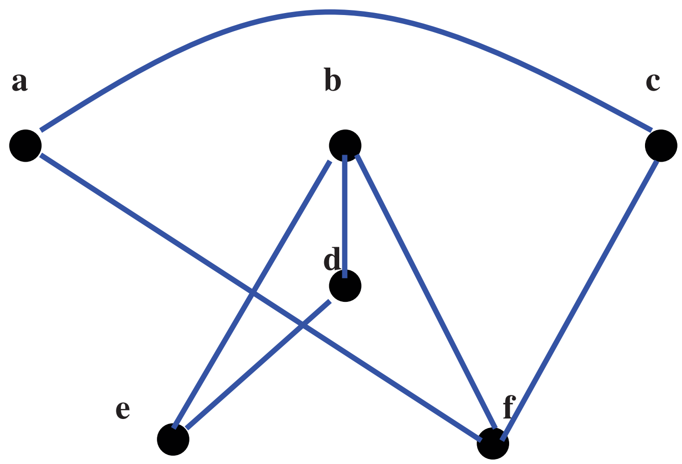
b) Partial graph of **G** generated by { **{a, b}, {c,d}** , **{d,f}** }:



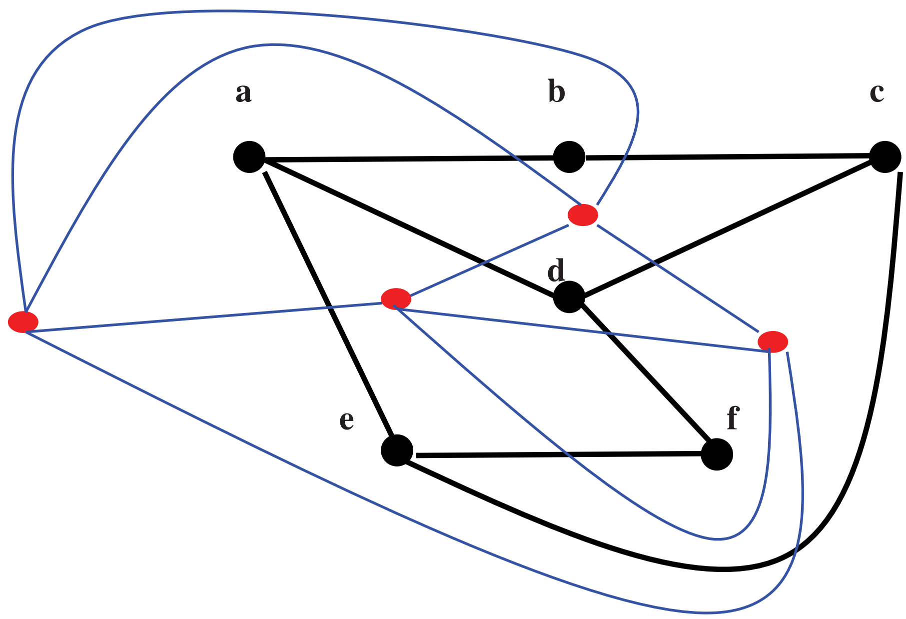
c) Partial subgraph of **G**:



d) Complement graph of **G** (if it exists): **G** is a simple graph; therefore, the complement of **G** exists.



e) Dual graph (if it exists): **G** is planar graph; therefore, the dual of **G** exists.



f) Stable set of **G**: **{a,c,f}** and **{b,d,e}**.

Exercise 2:

1.

a) The order: **| G | = 6**. The size: **||G|| = 8**.

b) The multiplicity: **G** is a **1**-graph .

c) The outer demi-degree, inner demi-degree and the total degree for each vertex of the graph **G**:

Vertex x	**a**	**b**	**c**	**d**	**e**	**f**
$d_G^+(x)$	2	0	2	2	1	1
$d_G^-(x)$	1	2	1	1	2	1
$d_G(x)$	3	2	3	3	3	2

d) The set of successors of vertex **d:** $\Gamma_G^+(d) = \{a, c\}$.

The set of predecessors of vertex **d**: $\Gamma_G^-(d) = \{f\}$.

The set of neighbors of vertex **d:** $\Gamma_G(d) = \Gamma_G^+(d) \cup \Gamma_G^-(d) = \{a, c\} \cup \{f\} = \{a, c, f\}$.

e) The multiplicity of (a, b): $m_G^+(a, b) = 1$.

f) The set of arcs incident to the set **{a,d,c}**:

$\omega_G(\{a, d, c\}) = \omega_G^+(\{a, d, c\}) \cup \omega_G^-(\{a, d, c\})$

$= \{(a, b), (a, e), (c, b), (c, e)\} \cup \{(f, d)\}$

$\{(a, b), (a, e), (c, b), (c, e), (f, d)\}$.

g) A path between vertices **f** and **e: .(f,d,c,e)**. The path is simple (does not use the same edge twice). Is it also an elementary path (it does not visit any vertex more than once).

h) A circuit: **(a,e,f,d,a)**.

i) A cocycle: $\omega_G(\{a, d, c\}) = \{(a, b), (a, e), (c, b), (c, e), (f, d)\}$.

2. a) vertex-vertex incidence matrix:

	a	**b**	**c**	**d**	**e**	**f**
a	0	**1**	0	0	**1**	0
b	0	0	0	0	0	0

c	0	**1**	0	0	**1**	0
d	**1**	0	**1**	0	0	0
e	0	0	0	0	0	**1**
f	0	0	0	**1**	0	0

b) Vertex-arc incidence matrix:

	(a,b)	**(a,e)**	**(c,b)**	**(c,e)**	**(d,a)**	**(d,c)**	**(e,f)**	**(f,d)**
a	**1**	**1**	0	0	**-1**	0	0	0
b	**-1**	0	**-1**	0	0	0	0	0
c	0	0	**1**	**1**	0	**-1**	0	0
d	0	0	0	0	**1**	**1**	0	**-1**
e	0	**-1**	0	**-1**	0	0	**1**	0
f	0	0	0	0	0	0	**-1**	**1**

3. The condensed forms of the vertex-vertex incidence matrix:

a) Compressed form of the vertex-vertex incidence matrix:

x	y	$m_G^+(x, y)$
a	**b**	**1**
a	**e**	**1**
c	**b**	**1**
c	**e**	**1**
d	**a**	**1**
d	**c**	**1**
e	**f**	**1**
f	**d**	**1**

b) The IFV matrix:

Arc	IV	TV
1	**a**	**b**
2	**a**	**e**
3	**c**	**b**
4	**c**	**e**
5	**d**	**a**
6	**d**	**c**
7	**e**	**f**
8	**f**	**d**

4. **a)** **G** is a simply connected graph because, for every pair of distinct vertices **x** and **y**, there exists a chain connecting **x** to **y**.

b) **G** is not a strongly connected graph because there is a path from **a** to **b**, but no path from **b** to **a**.