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Faculty of Science and Technology
Department of Material Science

College year :2023/2024
Module: Math1-S1
Specialization: SM+ST

Exercises serie N3: Real-Valued Function of a Real Variable

Exercise 1 : Find the domain of the function

$1) f(x) = \sqrt[4]{x^2 - 5x}$, $\quad 2) f(x) = \arcsin\left(\dfrac{2e^x - 3}{e^x + 1}\right)$, $\quad 3) \ln(\ln(\ln x))$, $\quad 4) f(x) = \dfrac{\ln(4 - |x - 1|)}{\sqrt[3]{2 - x}}$.

Exercise 2 : determine whether each of the following function is even, odd ,or niether even nor odd

$$f(x) = \sqrt{\frac{1 - |x|}{2 - |x|}} \qquad g(x) = \sin(x) + \frac{1}{2}\cos(2x) \qquad h(x) = \frac{\ln(1+x)}{|x|} \qquad I(x) = x^3 + \tan(x)$$

Exercise 3 : Evaluate the following limits.

$1) \lim\limits_{x\to 0} \dfrac{\arctan x - \frac{\pi}{4}}{x}$ $\quad 2) \lim\limits_{x\to 1}\left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right)$ $\quad 3) \lim\limits_{x\to 0^+} (x)^{\frac{1}{3\ln x}}$ $\quad 4) \lim\limits_{x\to+\infty} \left(e^{2x} + 3x\right)^{\frac{1}{x}}$

$5) \lim\limits_{x\to+\infty}\left(\dfrac{x+1}{x-3}\right)^{-2x+2}$ $\quad 6) \lim\limits_{x\to+\infty}\left(1 + \dfrac{1}{3x}\right)^{x}$ $\quad 7) \lim\limits_{x\to 0^+} (\ln(1+x))^{\frac{1}{\ln x}}$ $\quad 8) \lim\limits_{x\to 3} \dfrac{2^{x-3} - 1}{x - 3}$

Exercise 4 :

1) Find the derivatives of the following functions :

$$f(x) = \left(\sin x + \ln(4 + x^2)\right)^{\frac{3}{7}}, \quad g(x) = (\cosh)^{\cos^2 x}, \quad j(x) = \arctan\left(\frac{1-x}{1+x}\right), \quad l(x) = \ln\left(\sqrt{x^2+1}\sin^4 x\right).$$

2) Find the values of a and b that make the following function differentiable on D_f :

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & x < 0, \\ a & x = 0, \\ ae^{bx} - bx & x > 0. \end{cases} \qquad g(x) = \begin{cases} x\sqrt[3]{ax+b} & x < 0, \\ x^2 + 2x & x \geq 0. \end{cases}$$

Exercise 5 :

1) Let f the function define by $f(x) = \frac{\arctan(x)}{\ln(x+1)}$

   (a) can f admit an extension by continuity in Zero ? justify.

   (b) Is f differentiable at $x = 1$? justify.

   (c) Calculate $f'(x)$.

2) Find the vlues of a and b so that the function is continuous everywhere.

$$f(x) = \begin{cases} -3 & , \quad x \leq 1 \\ ax + b & , \quad 1 < x < 4 \\ 3 & x \geq 4 \end{cases} \qquad g(x) = \begin{cases} \sin(bx) & , \quad x \geq 1 \\ x^2 & , \quad x < 1 \end{cases}$$