***Chapter 3*: Diffraction and its Applications**

Introduction:

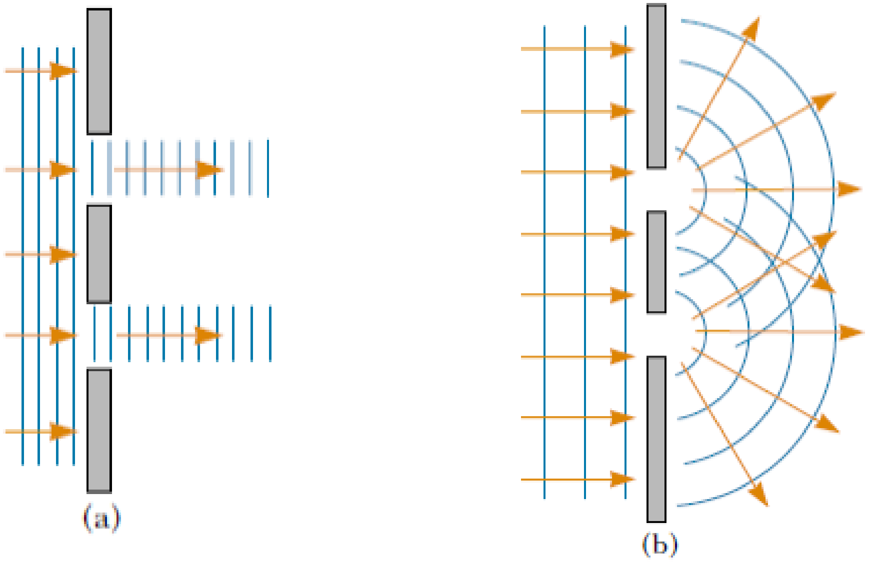
- when light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light.
- This behavior indicates that light, once it has passed through the aperture, spreads beyond the narrow path defined by the aperture into regions that would be in shadow if light traveled in straight lines. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges. This phenomenon, known as diffraction, can be described only with a wave model for light.

Definition:

When waves pass through an aperture or past the edge of an obstacle, they always spread to some extent into the region which is not directly exposed to the oncoming waves. i.e., of the failure of light to travel in straight lines.

This divergence of light from its initial line of travel is called diffraction

In general, diffraction occurs when waves pass through small openings, around obstacles, or past sharp edges, as shown in Figure bellow:



When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an Fraunhofer diffraction, which occurs,for example, when all the rays passing through a narrow slit are approximately parallel to one another. This can be achieved experimentally either by placing the screen far from the opening used to create the diffraction or by using a converging lens to focus the rays once they pass through the opening,

Fresnel And Fraunhofer Diffraction: Diffraction phenomena are conveniently divided into two general classes:

1. Those in which the source of light and the screen on which the pattern is observed are effectively at infinite distances from the aperture causing the diffraction
2. Those in which either the source or the screen, or both, are at finite distances from the aperture.

The phenomena coming under class (1) are called, for historical reasons, Fraunhofer diffraction, and those coming under class (2) Fresnel diffraction

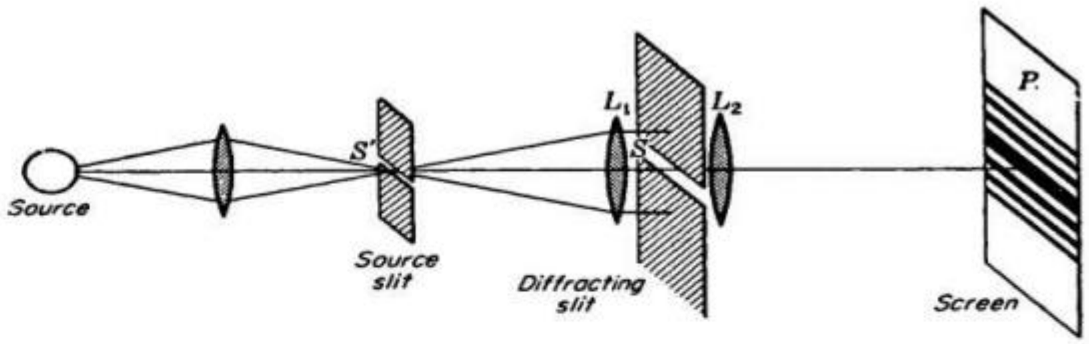


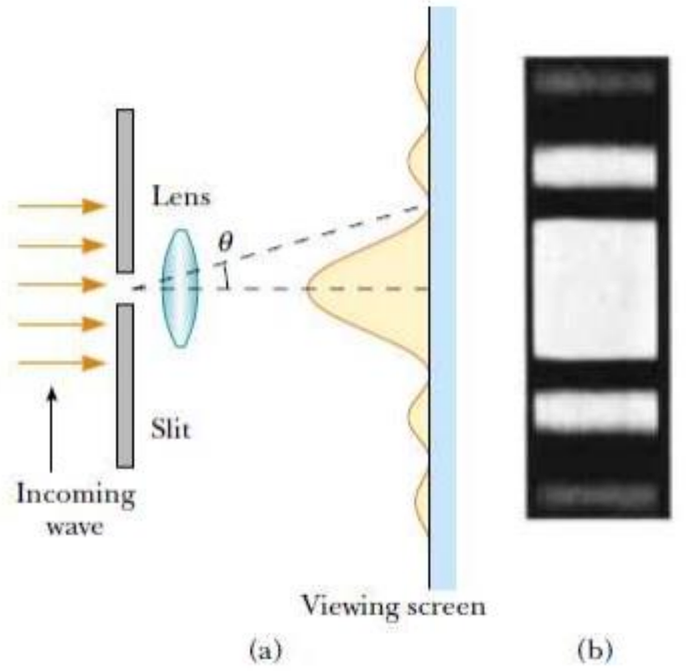
Figure 1: Experimental arrangement for obtaining the diffraction pattern of a single slit; Fraunhofer diffraction.

Fraunhofer diffraction	Fresnel diffraction
1. both of light source and screen are an infinite distance from diffraction element	1. Either the source and the screen or both are finite distance from the diffraction element
2. We are dealing with plane wave	2. We are dealing with spherical wave
3. lenses are required	3. lenses are not required

Diffraction by a single slit(diffraction from narrow slits):

Figure 2 (a): Fraunhofer diffraction pattern of a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes (drawing not to scale).

Figure 2(b): Photograph of a single-slit Fraunhofer diffraction pattern



- Until now, we have assumed that slits are point sources of light. We abandon that assumption and see how the finite width of slits is the basis for understanding Fraunhofer diffraction.

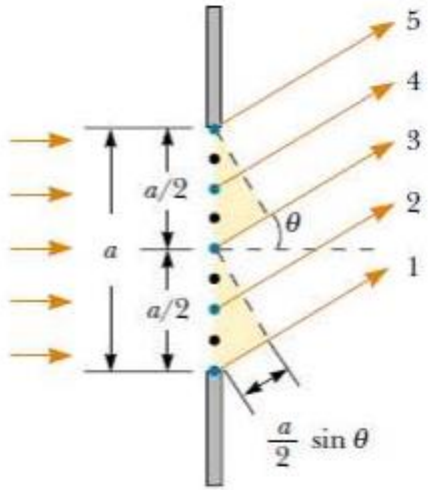
- We can deduce some important features of this phenomenon by examining waves coming from various portions of the slit, as shown in Figure below:

- According to Huygens's principle, each portion of the slit acts the phase difference between two such points is 180 as a source of light

- Each portion of the slit acts as a point source of light waves.

- The path difference between rays 1 and 3 or between rays 2 and 4 is: $(a/2)\sin\theta$

- Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half:



fig(2): Diffraction of light by a narrow slit of width a

$$\left(\frac{a}{2}\right)\sin\theta = \frac{\lambda}{2} \quad \Longrightarrow \sin\theta = \frac{\lambda}{a}$$

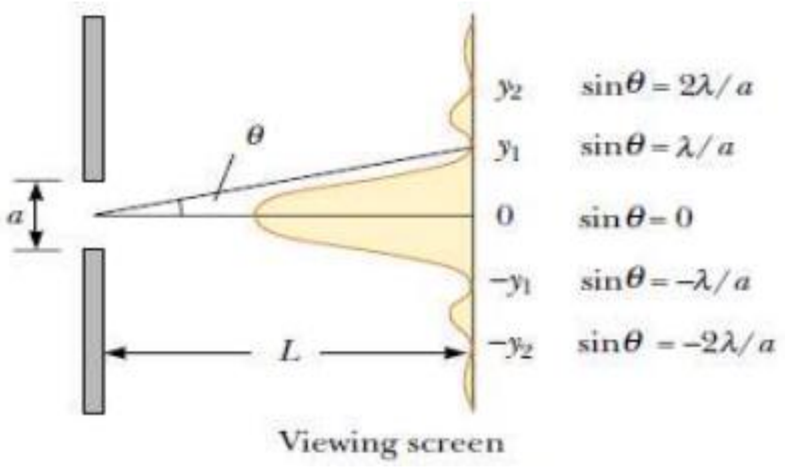
In general:

$$\sin\theta = m\frac{\lambda}{a}\ ;\ \ m = \pm1, \pm2, \pm3, \dots\dots$$

Condition for destructive interference

- Broad central bright fringe is observed;
- this fringe is flanked by much weaker bright fringes alternating with dark fringes.

- The positions of two minima on each side of the central maximum are labeled(drawing not to scale).



- The diffraction pattern that appears on a screen when light passes through a narrow vertical slit. The pattern consists of a broad central bright fringe and a series of less intense and narrower side bright fringes

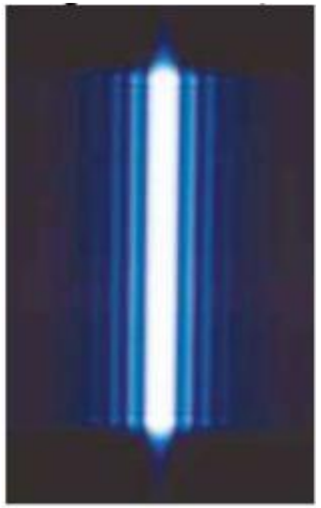


Fig 3 :Intensity distribution for a Fraunhofer diffraction pattern from a single slit of width "a"

Example 1: Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution:

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.80\times10^{-7}\text{ m}}{0.300\times10^{-3}\text{ m}} = \pm1.93\times10^{-3}$$

$$\sin\theta \approx y_1/L.$$

$$y_1 \approx L\sin\theta = \pm L\frac{\lambda}{a} = \pm3.87\times10^{-3}\text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to: $2|y_1| = 7{,}74\times10^{-3}m = 7{,}74\ cm$

Intensity of Single-Slit Diffraction Patterns

- ***Fraunhofer diffraction by a single slit:***

The light intensity at point P is the resultant of all the incremental electric field magnitudes from zones of width Δy.

where the phase difference $\Delta\beta$ is related to the path difference $\Delta y \sin\theta$ between adjacent zones by the expression:

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

The total phase difference between waves from the top and bottom portions of the slit is At this point, the vector sum is zero, and so corresponding to the first minimum on the screen.

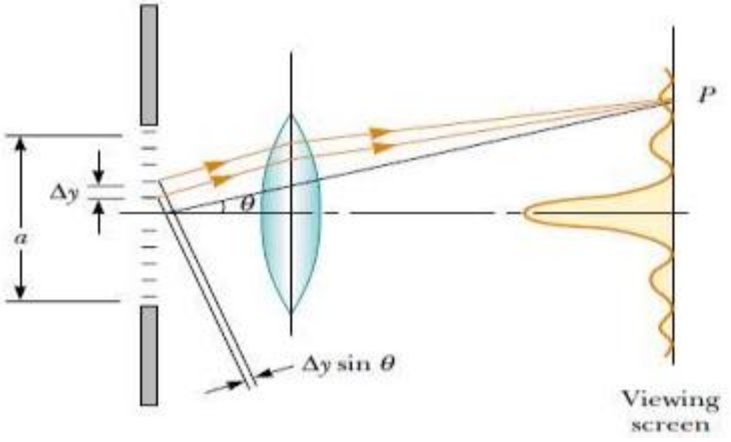


*Fig 5: Fraunhofer diffraction by a single slit. The light intensity at point **P** is the resultant of all the incremental electric field magnitudes from zones of width Δy*

To find the magnitude of the total electric field on the screen at any angle θ we sum the incremental magnitudes ΔE due to each zone.

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{1}{2} ka \sin\theta$$

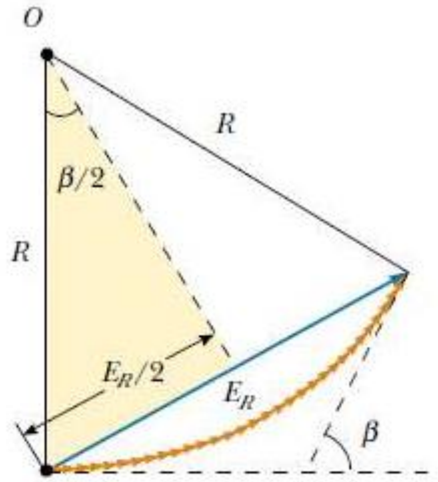
where $a = N\Delta y$ is the width of the slit.

$$\beta = N\Delta\beta = 2\pi$$

$$2\pi = \frac{2\pi}{\lambda} a \sin\theta$$

$$\sin\theta = \frac{\lambda}{a}$$

We can obtain the total electric field magnitude E_R and light intensity I at any point P on the screen in Figure 3 by considering the limiting case in which dy becomes infinitesimal is the radius of curvature. But the arc length E_0 is equal to the product E_R, where β is measured in radians,



Combining this information with the previous expression gives:

$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]$$

Because the resultant light intensity I at point P on the screen is proportional to the square of the magnitude E_R , we find that:

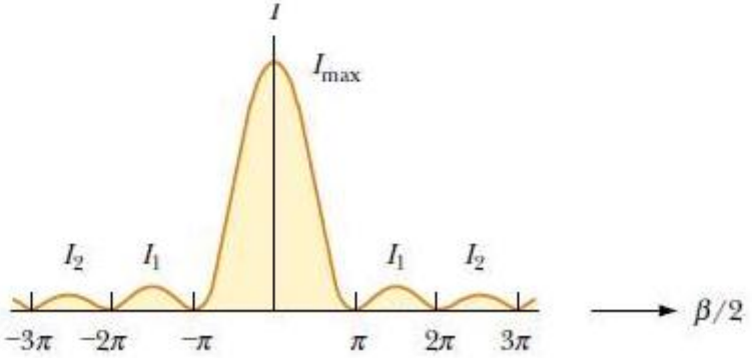
$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

Intensity of a single-slit Fraunhofer diffraction pattern

$$I_{max} = {E_r}^2 \ or\ A^2$$

Were A: Amplitude, $E_r = A = \frac{ab}{y}$

$$I = I_{max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]$$



From this result, we see that minima occur when

$$\frac{\pi a \sin\theta}{\lambda} = m\pi$$

or

$$\sin\theta = m\frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ldots$$

Condition for intensité minima

Intensity of Two-Slit Diffraction Patterns:

To determine the effects of both interference and diffraction, we simply combine Equation: $I = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

Become:

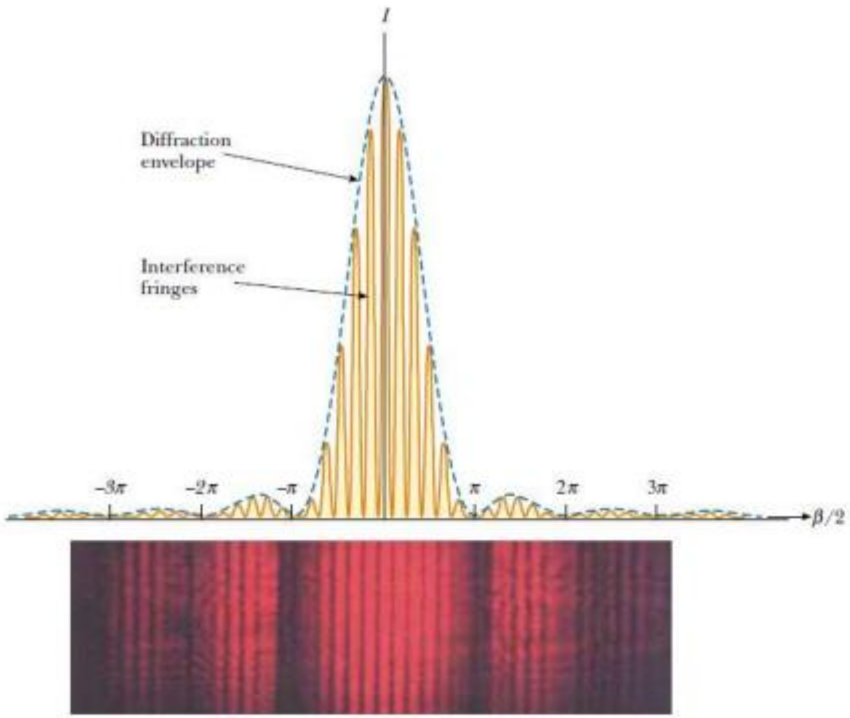
$$I = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

interference maxima as $d\sin\theta = m\lambda$ … (1) , where d is the distance between the two slits

Specifies that the first diffraction minimum occurs when $a\sin\Theta = m\lambda$ … (2), where a is the slit width m=1.

Dividing Equation(2) by Equation (1) (with allows us to determine which interference maximum coincides with the first diffraction minimum

$$\frac{dsin\,\theta}{asin\,\theta} = \frac{m\lambda}{\lambda} \Rightarrow \frac{d}{a} = m$$



<u>Example:</u> $d/a = 18\ \mu m/3.0\ \mu m = 6$

The sixth interference maximum (if we count the central maximum $m = 0$ as is aligned with the first diffraction minimum and cannot be seen.