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**Exercise Series of Geometrical and Physical Optics, 2024-2025**
**Exercise Series Number 2: Physical Optics_Part 1_Solution**

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## **Solution1 :**

The frequency $f$ of the wave being conserved. We have :

$$f = \frac{v}{\lambda}$$

Where $v$ is the velocity of the wave in the medium under consideration :

$v = \frac{C}{n}$ ($C$ is the velocity of the wave in a vacuum and $n$ is the refractive index of the medium).

The wavelength$\lambda_1$ being given in a vacuum ($n_1 = 1$), we have :

$$f = \frac{C}{\lambda_1} = \frac{C}{n_2 \lambda_2} \Longrightarrow \lambda_2 = \frac{\lambda_1}{n_2} = 0.6328151\ \mu m$$

The precision on the value of $\lambda_2$ is obtained by differentiating the logarithm of the above expression (logarithmic derivative):

$$\frac{d\lambda_2}{\lambda_2} = \frac{d\lambda_1}{\lambda_1} - \frac{dn_2}{n_2}$$

We passed from differentials to uncertainty $\Delta$ by summing the absolute values:

$$\frac{\Delta\lambda_2}{\lambda_2} = \frac{\Delta\lambda_1}{\lambda_1} + \frac{\Delta n_2}{n_2} \Longrightarrow \Delta\lambda_2 = \lambda_2 \left(\frac{\Delta\lambda_1}{\lambda_1} + \frac{\Delta n_2}{n_2}\right) = 10^{-6}$$

## ***Solution 2 :***

1. A point source emits ***a spherical wave*** in an isotropic homogeneous medium: the wave surfaces are spheres with a center O.
2. The amplitude of the wave depends on the position of the point $M$ on the wavesurface under consideration.
   Let us note $E_M$, the amplitude of the electric field on the wave surface under consideration. The phase of the wave corresponds to the propagation of the wave (The phase is written $(\omega t - kr)$ for a spherical wave).

   If $\varphi$ denotes the phase difference between the point considered and the source, the sought wave surface has the equation: $\varphi = kr$. Finally, the electric field is written :

$$E(M,t) = E_M \cos(\omega t - \varphi)$$

3. When placed very far from the source, spherical wave surfaces have a very large radius of curvature; they can therefore be assimilated locally to straight lines: we find the wavefronts of a plane wave. This is what happens, for example, with the light waves emitted by the Sun at the level of the Earth. A more rigorous way to generate a plane wave from a point source is to place the source in the focal

plane object of a converging lens : The beam emerging from the lens is a beam of parallel rays corresponding to a plane wave.

## Solution 03

We have the wave is generally written in the form of:

$$E_x(z,t) = E_0 \sin(k_z.z - \omega t) \text{ ou } E_x(z,t) = E_0 \sin(\vec{k}.\vec{r} - \omega t)$$

1-

- The amplitude: $E_0 = 10^2\ V/m\ ;\ k = \frac{2\pi}{\lambda}\ ;\ v = \frac{\omega}{k}$
- The direction of propagation corresponds to the direction Oz: $\vec{k}\begin{pmatrix}0\\0\\k_z\end{pmatrix}$
- The direction of bolarization corresponds to the direction Ox: $E_y(z,t) = E_z(z,t) = 0$

2- The vlocity $v = \frac{\omega}{k} = 9.\frac{10^{14}}{3.10^6} = 3.10^8 m/s$

The wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi.3.10^6} = 0.66\ \mu m$

The Frequency $\nu = \frac{C}{\lambda} = 4.5.10^{14}\ Hz$, it is the color red (between 400 and 180 THz)

3- On utilise les équations de Maxwell :

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}; \qquad \vec{B}\begin{pmatrix}B_x\\B_y\\B_z\end{pmatrix}$$

$$rot\vec{E} = \vec{\nabla} \times \vec{E} = \begin{pmatrix}\frac{\partial}{\partial x}\\\frac{\partial}{\partial y}\\\frac{\partial}{\partial z}\end{pmatrix} \wedge \begin{pmatrix}E_x\\0\\0\end{pmatrix} = \begin{pmatrix}0\\\frac{\partial E_x}{\partial z}\\0\end{pmatrix}$$

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Longrightarrow \begin{pmatrix}0\\\frac{\partial E_x}{\partial z}\\0\end{pmatrix} = \begin{pmatrix}\frac{\partial B_x}{\partial t}\\\frac{\partial B_y}{\partial t}\\\frac{\partial B_z}{\partial t}\end{pmatrix}$$

$\frac{\partial B_x}{\partial t} = \frac{\partial B_z}{\partial t} = 0 \Longrightarrow B_x\ et\ B_z$ are constants do not correspond to progressive waves

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_x}{\partial z} = -kE_0\cos(k_z.z - \omega t) \Longrightarrow \boldsymbol{B_y(z,t) = \frac{k}{\omega}E_0\sin(k_z.z - \omega t) = \frac{E_x(z,t)}{v}}$$

## Solution 04:

1. $\psi = \psi_1 + \psi_2 = a\cos\omega t + a\cos(\omega t + \varphi) = a(\cos\omega t + \cos(\omega t + \varphi))$

We have : $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$$\Rightarrow \psi = 2a\cos\left(\frac{\omega t + \omega t + \varphi}{2}\right)\cos\left(\frac{\omega t - \omega t - \varphi}{2}\right) = \boldsymbol{2a\cos\left(\frac{\varphi}{2}\right)\cos\left(\omega t + \frac{\varphi}{2}\right)}$$

In the form of: $\psi = A\cos(\omega t + \alpha)$ avec $A = 2a\cos\left(\frac{\varphi}{2}\right)$; $\alpha = \frac{\varphi}{2}$

**<u>The complex representation</u>** :

$$\psi = \psi_1 + \psi_2 = ae^{i\omega t} + ae^{i(\omega t+\varphi)} = ae^{i\omega t}(1 + e^{i\varphi})$$

$$= ae^{i\omega t}e^{i\varphi/2}(e^{-i\varphi/2} + e^{i\varphi/2}) = a(e^{-i\varphi/2} + e^{i\varphi/2})e^{i\left(\omega t+\frac{\varphi}{2}\right)}$$

On a : $e^{-i\varphi/2} + e^{i\varphi/2} = cos\left(\frac{\varphi}{2}\right) - i\sin\left(\frac{\varphi}{2}\right) + cos\left(\frac{\varphi}{2}\right) + i\sin\left(\frac{\varphi}{2}\right) = 2\cos\frac{\varphi}{2}$

$$\Rightarrow \boldsymbol{\psi = 2a\cos\frac{\varphi}{2}.e^{i\left(\omega t+\frac{\varphi}{2}\right)}}$$

**2.**

$$\boldsymbol{I = 4a^2 cos^2\frac{\varphi}{2}}$$

a. The two waves are in phase: $\frac{\varphi}{2} = \boldsymbol{2\pi n}$ $(\boldsymbol{n = 0, 1, 2, 3, \ldots\ldots}) \Rightarrow \boldsymbol{I = I_{max} = 4a^2}$

**b.** The two waves are in phase opposition: $\boldsymbol{I = 0} \Rightarrow \boldsymbol{cos\frac{\varphi}{2} = 0}$

**c.** two waves are in phase quadrature: : $\frac{\varphi}{2} = \frac{\pi}{2}(\boldsymbol{2n + 1}) \Rightarrow \boldsymbol{I = 4a^2\left(\frac{\sqrt{2}}{2}\right)^2 = 2a^2}$

**Solution 05 :**

We have:

$a = 0{,}2.10^{-3}m$, $D = 1{,}0\ m$, $x_{Bright\ gringe}(k = 3) = 9{,}49.10^{-3}\ m.$

The equation of the 3rd bright fringe is given by

$$x_{FB}\ (k = 3) = 3\frac{\lambda D}{a} \Rightarrow \lambda = \frac{a.x_{FB}(k = 3)}{3D}$$

$$\Rightarrow \lambda = \frac{0{,}2.10^{-3}.9.49.10^{-3}}{3.1} = 0.633.10^{-3}\ m = 633\ nm$$

**Solution 06 :**

1. Let us recall the formula of the interfringe:

$$i = \frac{\lambda D}{a} \Rightarrow \lambda = \frac{ia}{D}$$
$$i = \frac{7.5}{3} = 2.5\ mm; a = 0.2\ mm; D = 1m = 1000mm$$
$$\Rightarrow \lambda = 0.5\mu m$$

2. The distance to the central fringe of the 3rd dark fringe being 2,5, we have :

$$i' = \frac{7.5}{2.5} = 3\ mm$$
$$\lambda' = \frac{i'a}{D} = 0.6\ \mu m$$