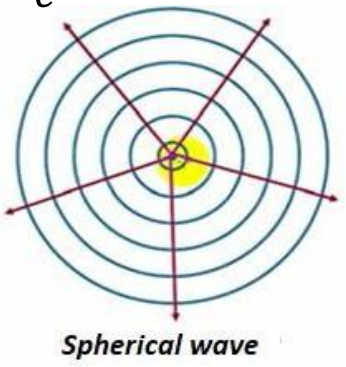
Chapter II: Wave Optics

Electromagnetic description of light:

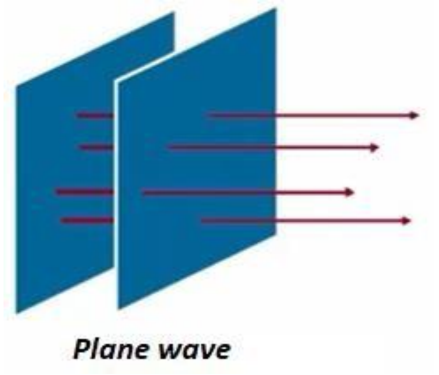
- Light corresponds to the propagation of an electromagnetic field, which is the superposition of an electric field $\vec{E}(\vec{r},\vec{t})$ and a magnetic field $\vec{B}(\vec{r},\vec{t})$, both depending on time and position.
- Light is an electromagnetic wave that propagate at a speed: $v = \frac{n}{c}$
- In general, there are two types of waves:

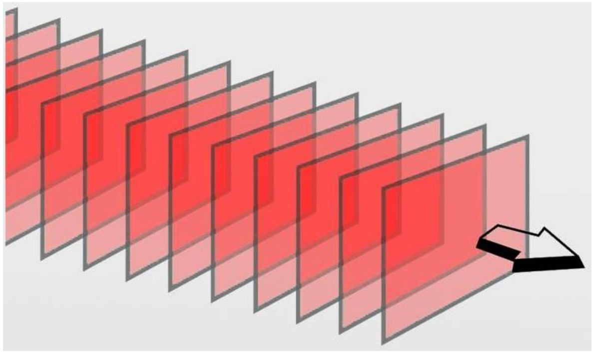
A spherical wave: is a wave whose wavefronts are spheres.

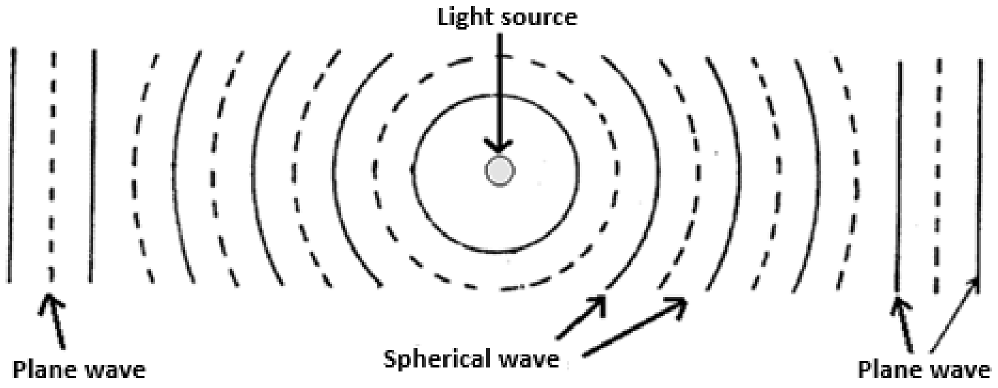

Spherical wave

A plane wave:

is a wave whose wavefronts consist of an infinite set of parallel planes, each perpendicular to the same direction of propagation.


Plane wave





The electric and magnetic fields are not independent, hence Maxwell's equations:

$$div\,\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$div\,\vec{B} = 0 \qquad \overrightarrow{rot}\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\vec{E}$: Electric field

$\vec{B}$: Magnetic field

ρ: Volume charge density

$\vec{J}$: Current density

ε_0: Vacuum Dielectric permittivity

μ_0: Vacuum magnetic permeability

in a vacuum and for an uncharged medium: : $\rho = 0$ et $\vec{J} = \vec{0}$

$$\begin{cases} div\,\vec{E} = 0 & \overrightarrow{rot}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \qquad (1) \\ div\,\vec{B} = 0 & \overrightarrow{rot}\,\vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \qquad (2) \end{cases}$$

$$(1) \Longrightarrow \overrightarrow{rot}\left(\overrightarrow{rot}\vec{E}\right) = -\overrightarrow{rot}\left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\overrightarrow{rot}\vec{B}\right)$$

We assume that the variables x, y, z are independent

$$\Longrightarrow \cancel{\overrightarrow{grad}}\left(div\,\vec{E}\right) - \Delta\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\Longrightarrow \Delta\vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

With the same method we will find:

$$\boxed{\Delta\vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0}$$

$$\begin{cases} \Delta\vec{E} - \mu_0\varepsilon_0 \dfrac{\partial^2\vec{E}}{\partial t^2} = 0 \\ \Delta\vec{B} - \mu_0\varepsilon_0 \dfrac{\partial^2\vec{B}}{\partial t^2} = 0 \end{cases} \quad (I) \quad \Longrightarrow \begin{cases} \Delta E_x - \mu_0\varepsilon_0 \dfrac{\partial^2 E_x}{\partial t^2} = 0 \\ \Delta E_y - \mu_0\varepsilon_0 \dfrac{\partial^2 E_y}{\partial t^2} = 0 \\ \Delta E_z - \mu_0\varepsilon_0 \dfrac{\partial^2 E_z}{\partial t^2} = 0 \end{cases} \text{ and } \begin{cases} \Delta B_x - \mu_0\varepsilon_0 \dfrac{\partial^2 B_x}{\partial t^2} = 0 \\ \Delta B_y - \mu_0\varepsilon_0 \dfrac{\partial^2 B_y}{\partial t^2} = 0 \\ \Delta B_z - \mu_0\varepsilon_0 \dfrac{\partial^2 B_z}{\partial t^2} = 0 \end{cases}$$

This system of wave equations is called the ***D'Alembert equation***: describes the propagation of sound waves in the air in the form of:

$$\Delta p - \frac{1}{v^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Where p is the pressure and v is the speed of sound

By analogy, the speed of propagation of electromagnetic waves in a vacuum is deduced

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cong 3.10^8\, m/s$$

Monochromatic plane waves:

From the equation(I) and (II) we have:

$$\begin{cases} \Delta\vec{E} = \mu_0\varepsilon_0 \dfrac{\partial^2\vec{E}}{\partial t^2} \\ \Delta\vec{B} = \mu_0\varepsilon_0 \dfrac{\partial^2\vec{B}}{\partial t^2} \end{cases}$$

- Each component of $\vec{E}$ and $\vec{B}$ follow an equation of the waves:

$$\Delta A - \frac{n^2}{c^2}\frac{\partial^2 A}{\partial t^2} = 0 \quad (II)$$

With: $A \;=\; E_x, E_y, E_z, B_x, B_y, B_z$ (Light Amplitude)

We take n=1 for the air

Note: *Monochromatic wave = single color = single frequency*

- A monochromatic wave is written in the form: $A(\vec{r},t) = A_m\,(\vec{r})\cos\big(\omega t - \varphi(\vec{r})\big)$

$A_m(\vec{r})$: Amplitude of the wave

ω: Pulsation = $2\pi f\ = 2\pi/T$

$\varphi = \vec{k}.\vec{r}$: Phase (depends on space $\vec{r}$)

$\vec{k}$: wave function

- **On complex notation** (more practical):

$$A(\vec{r},t) = A_m\,(\vec{r})\,e^{i(\omega t - \vec{k}.\vec{r})} = A_m\,(\vec{r})\,e^{i(\vec{k}.\vec{r} - \omega t)} \qquad (III)$$

(cosinus is a pair function)

Verifying that(III) is a solution of the equation(II)):

In Cartesian coordinates: $\vec{k}\begin{pmatrix}k_x\\ k_y\\ k_z\end{pmatrix};\vec{r}\begin{pmatrix}r_x\\ r_y\\ r_z\end{pmatrix} \Longrightarrow \vec{k}.\vec{r} = k_x r_x + k_y r_y + k_z r_z$

$$\Longrightarrow A(\vec{r},t) = A_m\,(\vec{r})\,e^{i(\omega t - \vec{k}.\vec{r})} = A_m\,(\vec{r})\,e^{i(k_x r_x + k_y r_y + k_z r_z - \omega t)}$$

$$\Rightarrow \frac{\partial A(\vec{r},t)}{\partial x} = ik_x A(\vec{r},t) \quad \Rightarrow \frac{\partial^2 A(\vec{r},t)}{\partial x^2} = -{k_x}^2 A(\vec{r},t)\,; \quad \frac{\partial^2 A(\vec{r},t)}{\partial y^2} = -{k_y}^2 A(\vec{r},t)\,; \quad \frac{\partial^2 A(\vec{r},t)}{\partial y^2} = -{k_y}^2 A(\vec{r},t)$$

$$\Rightarrow \Delta A(\vec{r},t) = \frac{\partial^2 A(\vec{r},t)}{\partial x^2} + \frac{\partial^2 A(\vec{r},t)}{\partial y^2} + \frac{\partial^2 A(\vec{r},t)}{\partial y^2} = -\left({k_x}^2 + {k_y}^2 + {k_z}^2\right) A(\vec{r},t) = -k^2 A(\vec{r},t)$$

$$(II) \Rightarrow -k^2 A(\vec{r},t) = \frac{n^2}{c^2} \frac{\partial^2 A(\vec{r},t)}{\partial t^2} = -\frac{n^2}{c^2} \omega^2 A(\vec{r},t) \quad \Rightarrow k^2 = \frac{n^2 \omega^2}{c^2} \quad \boxed{\Rightarrow k = \frac{n\omega}{c}}$$

For a monochromatic plane wave to be a solution of the electromagnetic wave equation, the following condition must be satisfied:

$$\boldsymbol{k = \frac{n\omega}{c} \quad \text{with } (n = 1)}$$

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi c}{n\omega} = \frac{cT}{n} = vT$$: distance traveled in a period

Conclusion: The solution of Maxwell's equations in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$ is given by:

$$E(\vec{r},t) = E_0 \; e^{i(\vec{k}.\vec{r} - \omega t)} \quad \textbf{and} \quad B(\vec{r},t) = B_0 \; e^{i(\vec{k}.\vec{r} - \omega t)}$$

We have:

$$div\,\vec{E} = 0 \qquad div\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = ik_xE_x + ik_yE_y + ik_zE_z = 0 \quad \Rightarrow \vec{k}.\vec{E} = 0 \quad \boxed{\Rightarrow \vec{k} \perp \vec{E}}$$

$$div\,\vec{B} = 0 \qquad div\,\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = ik_xB_x + ik_yB_y + ik_zB_z = 0 \quad \Rightarrow \vec{k}.\vec{B} = 0 \quad \boxed{\Rightarrow \vec{k} \perp \vec{B}}$$

$$\overrightarrow{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \Rightarrow \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = -\left(-\omega\vec{B}\right) \Rightarrow \begin{pmatrix} ik_x \\ ik_y \\ ik_z \end{pmatrix} \wedge \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = -\left(-i\omega\vec{B}\right) \Rightarrow i\vec{k} \wedge \vec{E} = i\omega\vec{B}$$

$$\Rightarrow \vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} \quad \boxed{\Rightarrow \vec{B} \perp \vec{E}}$$

- For a monochromatic plane wave $\left(\vec{E}, \vec{B}, \vec{k}\right)$ form a direct trihedron (rule of the right hand)

$$B = \frac{k}{\omega}E = \frac{n}{c}E$$

- The electric and magnetic fields are in a plane orthogonal to the direction of propagation (**Light is a transverse wave)**

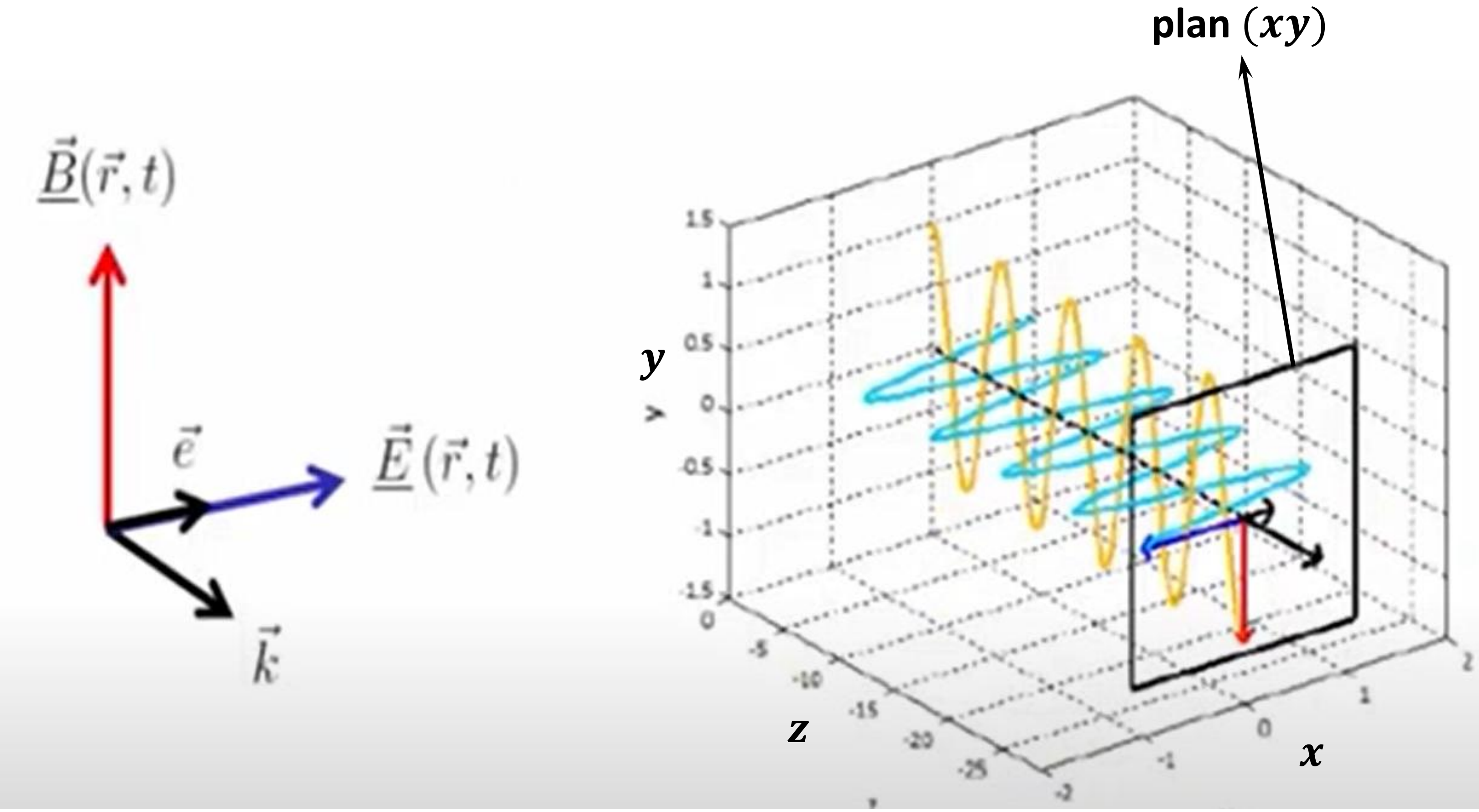
The direction of the electric field is called the polarization direction

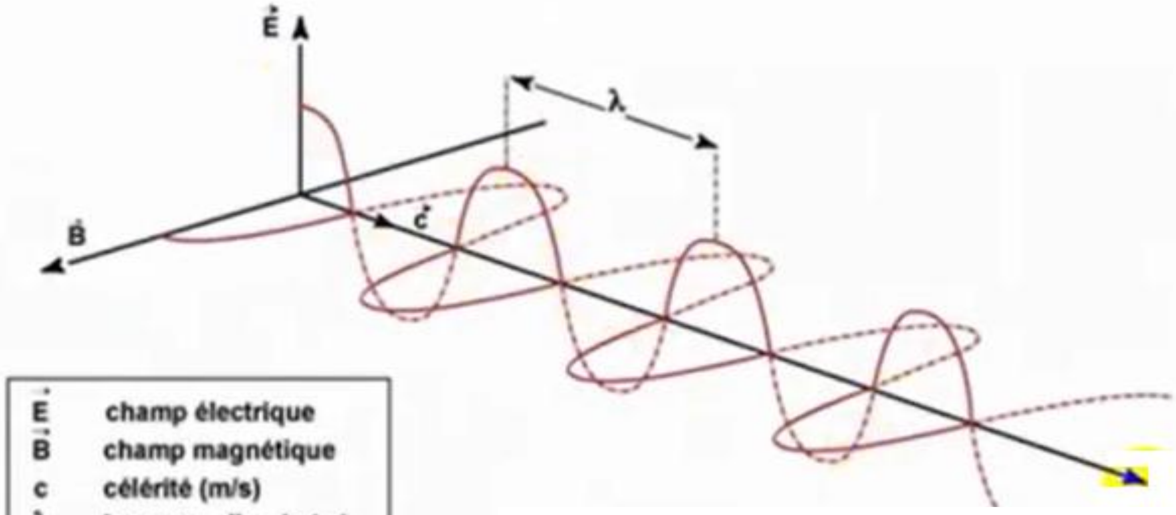
Example:

Dierction of propagation : Oz (ou $\vec{k} \parallel \vec{u}_z$)

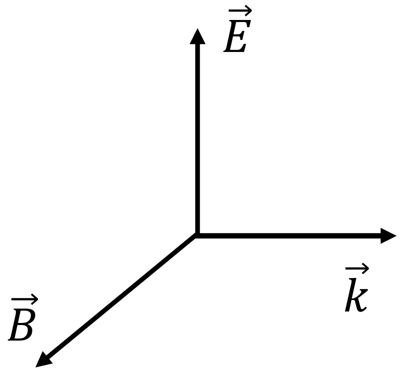
Dierction of polarisation : Ox (ou $\vec{E} \parallel \vec{u}_x$)

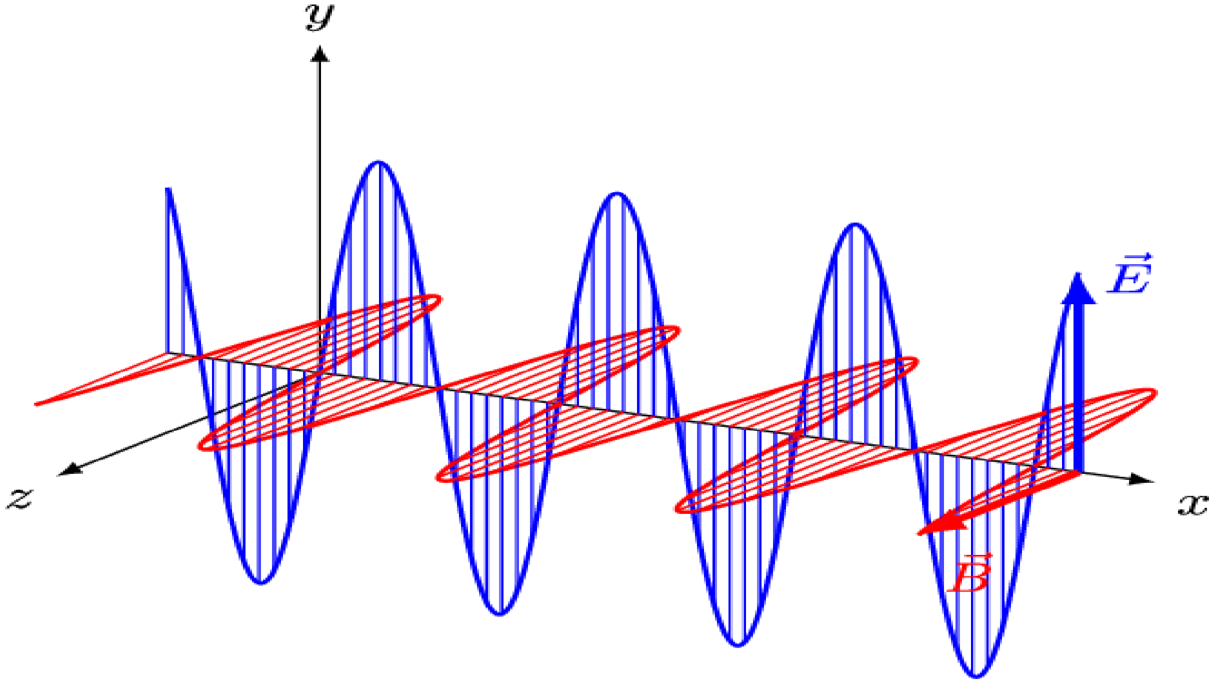
Plane wave: parallel to the plane (xy)





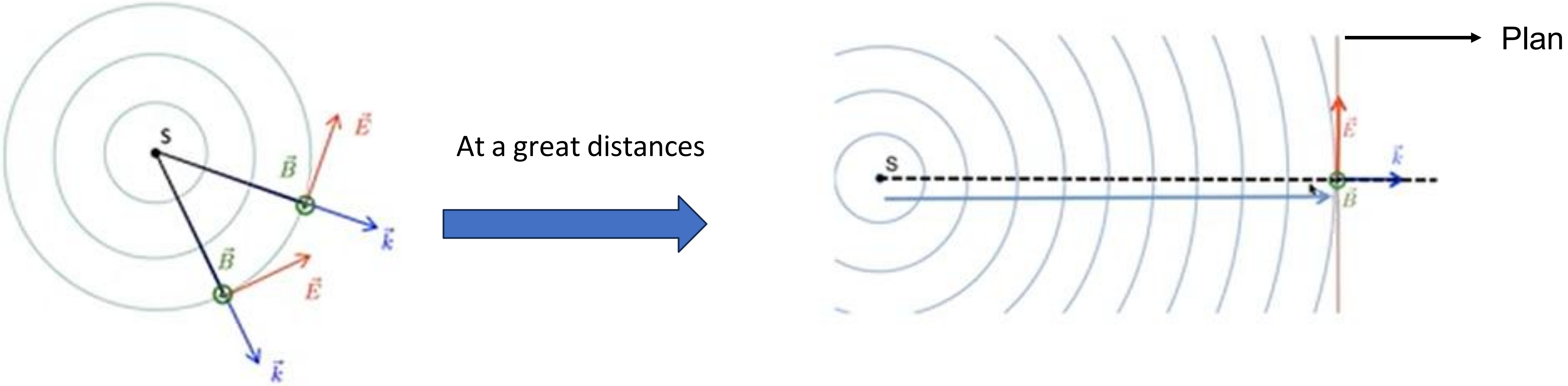
$\vec{E}$	champ électrique
$\vec{B}$	champ magnétique
c	célérité (m/s)
λ	longueur d'onde (m)
T	période = λ/c (s)
f	fréquence = 1 / T (Hz)





Spherical monochromatic waves:

The equation of a monochromatic wave is: $A(\vec{r},t) = A_m\,(\vec{r})\cos\big(\omega t - \varphi(\vec{r})\big)$ With $\left(A_m\,(\vec{r}) = \frac{A_0}{r}\right)$



At a great distances

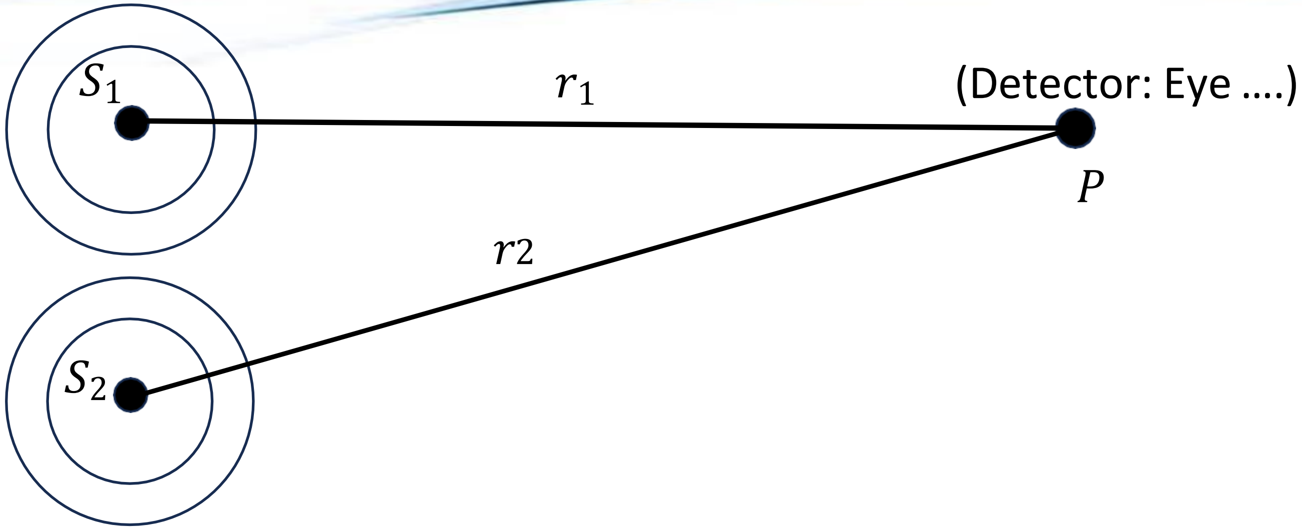
Plan

- At a great distance, the wavefront of the spherical wave approaches a plane and the term $1/r$ decreases slowly.

- We can therefore, as a first approximation, consider that at a great distance the spherical wave can be approximated by a plane wave.

Interference by Two waves:

Superposition of two monochromatic waves:



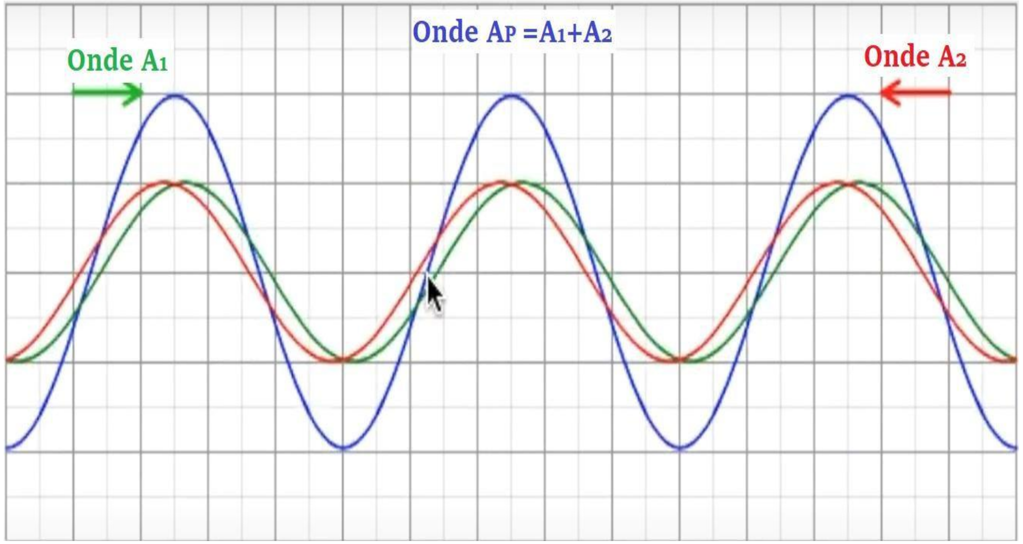
- Let us first consider two sources S_1 and S_2 producing two monochromatic waves:

$$A_1 = A_{01}\, e^{i(k_1 r_1 - \omega_1 t)}$$

$$A_2 = A_{02}\, e^{i(k_2 r_2 - \omega_2 t)}$$

($\vec{r}$ and $\vec{k}$ *are* collinear)

- The principle of superposition states that when two or more waves overlap, the resulting wave is the algebraic sum of the individual waves.



- The resultant wave equation is given by:

$$A_P = A_1 + A_2 = A_{01}\, e^{i(k_1 r_1 - \omega_1 t)} + A_{02}\, e^{i(k_2 r_2 - \omega_2 t)}$$

$$= A_{01}\, e^{i(k_1 r_1 - \omega_1 t)} + A_{02}\, e^{i(k_2 r_2 - \omega_2 t)}$$

Conditions set:

- Sources of the same magnitude: $A_{01} = A_{02} = A_0$

$$\boxed{\Longrightarrow \boldsymbol{A_P = A_0\left(e^{i(k_1 r_1 - \omega_1 t)} + e^{i(k_2 r_2 - \omega_2 t)}\right)}}$$

$$\boxed{\Longrightarrow A_P = A_0\left(e^{i(k_1r_1-\omega_1 t)} + e^{i(k_2r_2-\omega_2 t)}\right)}$$

$$\Longrightarrow A_P = A_0\left[\cos(k_1r_1-\omega_1 t) + \cos(k_2r_2-\omega_2 t) + i\left(\sin(k_1r_1-\omega_1 t) + \sin(k_2r_2-\omega_2 t)\right)\right]$$

We are only interested in the real part:

$$\boxed{A_P = A_0\left(\cos\underbrace{(k_1r_1-\omega_1 t)}_{x} + \cos\underbrace{(k_2r_2-\omega_2 t)}_{y}\right)}$$

We know that: $\cos x + \cos y = 2\cos\frac{x+y}{2}\times\cos\frac{x-y}{2}$

$$\Longrightarrow A_P = A_0\, 2\cos\frac{(k_1r_1-\omega_1 t)+(k_2r_2-\omega_2 t)}{2}\times\cos\frac{(k_1r_1-\omega_1 t)-(k_2r_2-\omega_2 t)}{2}$$

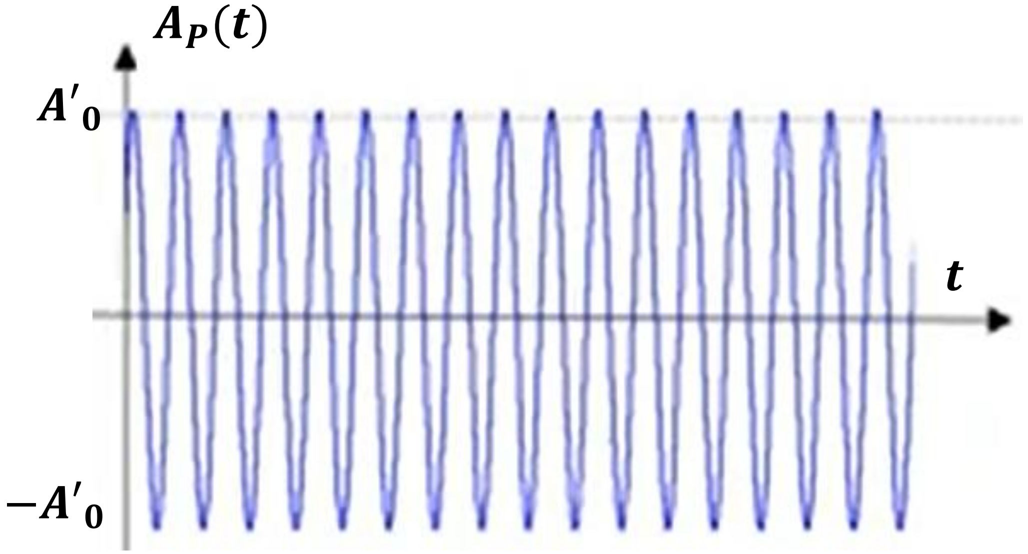
$$= A_0\, 2\cos\left(\frac{k_1r_1+k_2r_2}{2} - \frac{(\omega_1+\omega_2)t}{2}\right)\times\cos\left(\frac{k_1r_1-k_2r_2}{2} - \frac{(\omega_1-\omega_2)t}{2}\right)$$

- ☐ *For sources of the same frequencies (same pulsations):* $\omega_1 = \omega_2 = \omega$:

$$A_P = 2A_0\underbrace{\cos\left(k\frac{r_1-r_2}{2}\right)}_{\text{Depend on space}}\times\underbrace{\cos\left(k\frac{r_1+r_2}{2}-\omega t\right)}_{\text{Depends on time and space}}$$

- ☐ *Consider* $A'_0 = 2A_0\cos\left(k\frac{r_1-r_2}{2}\right)$ *:New amplitude after interference*

$$\boxed{A_P = A_0{}'\times\cos\left(k\frac{r_1+r_2}{2}-\omega t\right)}$$



Light intensity (illuminance): The amount of energy transport by the light wave per unit area per unit time,

For a wave with complex amplitude A, light intensity can be expressed mathematically as:

$$\boldsymbol{I} =< \boldsymbol{A}.\boldsymbol{A}^* >$$

where:

- $A^*$is the complex conjugate of A,
- $\langle\cdot\rangle$ denotes the time average over a period much longer than the wave period T.

$$\Longrightarrow I =< A_{02}\ e^{i(k_2r_2-\omega_2t)}.A_{02}\,e^{-i(k_2r_2-\omega_2t)} > \ = {A_0}^2 < e^0 > = {A_0}^2$$

❖ **Mathematical reminder:**

The time-averaged of a periodic function $f(\,t\,)$, of period T, noted by $< \ f(t) >$ is defined by: $< f(t) > = \dfrac{1}{T}\displaystyle\int_0^T f(t)dt$

➢ **In real notation, Light intensity is given by:**

$$I_P =< A_P(\vec{r},t).A_P^*(\vec{r},t) > =< {A_P}^2(\vec{r},t) > \ =< \left(2A_0\cos\left(k\frac{r_1-r_2}{2}\right)\times\cos\left(k\frac{r_1+r_2}{2}-\omega t\right)\right)^2 >$$

$$= 4{A_0}^2\,cos^2\left(k\frac{r_1-r_2}{2}\right) < \left(\cos\left(k\frac{r_1+r_2}{2}-\omega t\right)\right)^2 >$$

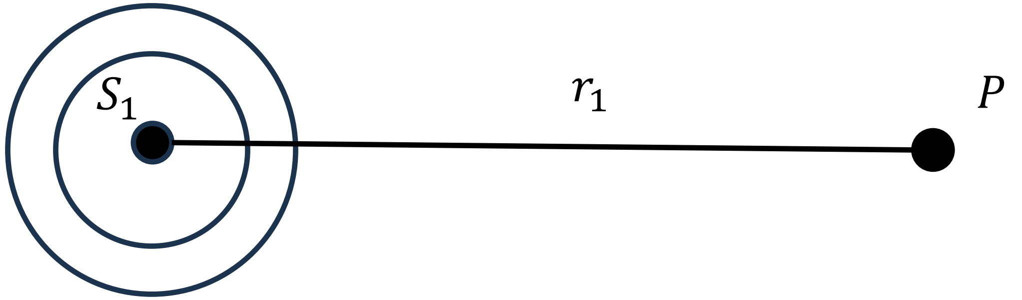
We now that: $< \sin(\omega t) > = 0 \;; < \cos(\omega t) > = 0 \;; < sin^2(\omega t) > = \frac{1}{2} \;; < cos^2(\omega t) > = \frac{1}{2}$

$$\Rightarrow I_P = 4{A_0}^2\, cos^2\left(k\frac{r_1 - r_2}{2}\right) \overset{\frac{1}{2}}{< \left(\cos\left(k\frac{r_1 + r_2}{2} - \omega t\right)\right)^2 >} = 2{A_0}^2\, cos^2\left(k\frac{r_1 - r_2}{2}\right)$$

$$\boxed{\boldsymbol{I_P = 2{A_0}^2\, cos^2\left(k\frac{r_1 - r_2}{2}\right) \ldots . (\mathrm{I})}}$$

(For two sources of the same frequency)

For a single source, the light intensity is given by:



$$I_{P1} =< A_{P1}(\vec{r}, t)^2 > = < (A_0 \cos(kr_1 - \omega t))^2 > \boxed{\boldsymbol{= \frac{{A_0}^2}{2} = I_0}}$$

$$\Rightarrow \boldsymbol{{A_0}^2 = 2I_0}$$

We replace in (I) $\boxed{\boldsymbol{\Rightarrow I_P = 4I_0 cos^2\left(k\frac{r_1 - r_2}{2}\right)}}$

$$-1 < \cos x < 1 \Rightarrow 0 < cos^2 x < 1 \qquad \boxed{\Rightarrow I_P \in [0, 4I_0]}$$

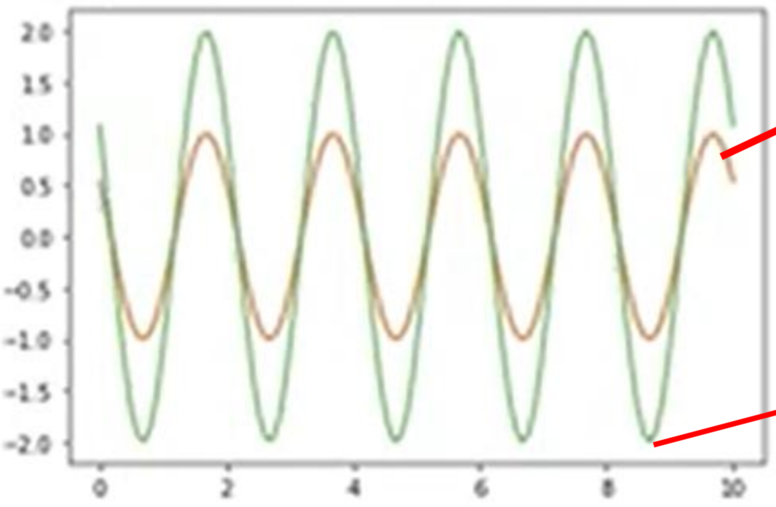
The intensity depends on the position of the point P (detector):

$$\boldsymbol{I_P = 4I_O cos^2\left(k\frac{r_1 - r_2}{2}\right)}$$

$With\ I_P \in [0, 4I_0]$

- ☐ $I_P = O$: destructive interference (light waves are in opposite phase)
- ☐ $I_P = 4I_0$: constructive interference (light waves are in the same phase)

In phase = Constructive interference

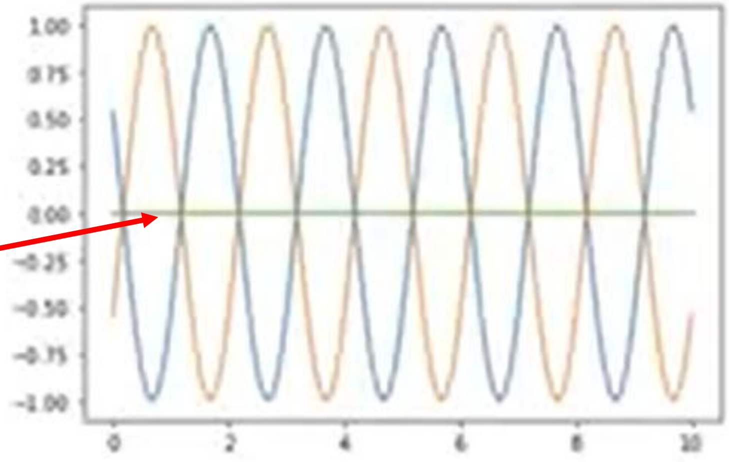


The two signals are identical (in phase)

The signal obtained in P

$$\boldsymbol{I_P = 4I_0}$$

In opposite phase = Destructive interference

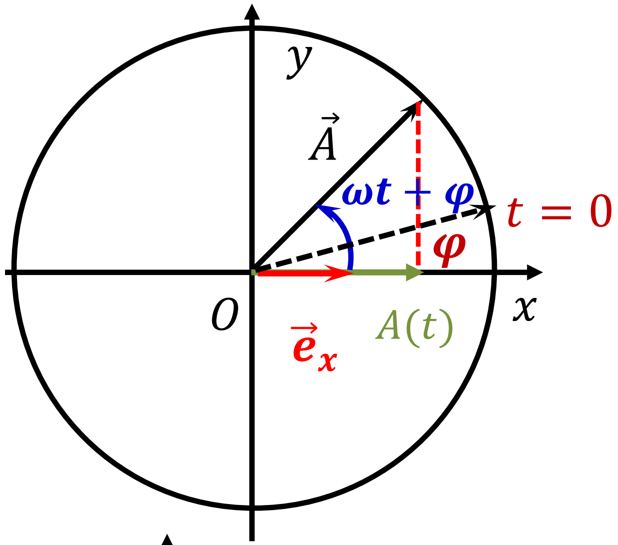


$$\boldsymbol{I_P = 0}$$

Addition of two sinusoidal functions by the Fresnel method

To any sinusoidal function $A(t) = A_m \cos(\omega t + \varphi)$, we associate the vector $\vec{A}$ of the plan (xOy) such as:

- $\left\|\vec{A}(t)\right\| = A_m$ The magnitude of $\vec{A}$ corresponds to the amplitude of the signal.
- $\left(\vec{e}_x, \vec{A}\right) = \omega t + \varphi$
- $\vec{A}$ is a vector rotating at constant angular velocity ω.
- φ is the angle between $\vec{e}_x$ and $\vec{A}$ at $t = 0$.
- The light intensity is proportional to the square of the amplitude: $I \propto {A_m}^2$
- In the International System(SI), The unit of light intensity is: *the candela (cd).*

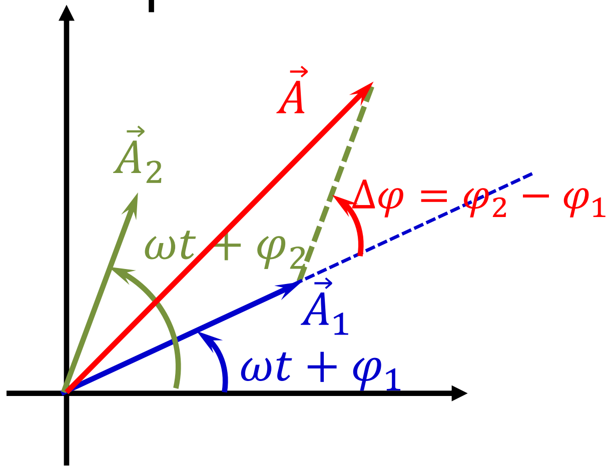


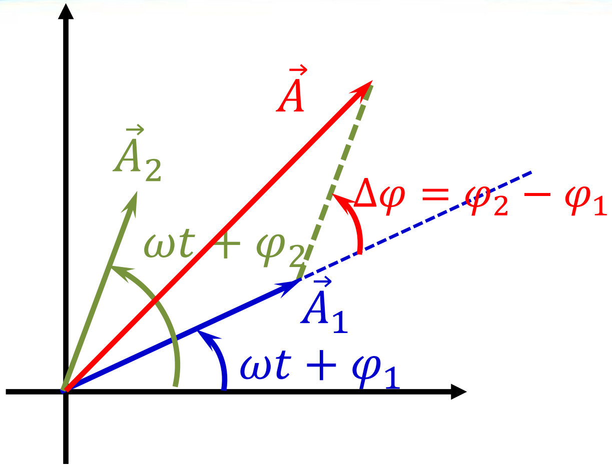
- For two sinusoidal signals s1 and s2, of the same pulsation, we have:

$$A_1(t) = A_{01} \cos(\omega t + \varphi_1),$$

$$A_2(t) = A_{02} \cos(\omega t + \varphi_2),$$

$A(t) = A_1(t) + A_2(t)$ is presented by the vector $\vec{A} = \vec{A}_1 + \vec{A}_2$





- The vectors $\vec{A}, \vec{A}_1$ *and* $\vec{A}_2$ rotate at the same angular velocity ω
- We calculate the amplitude A_0 of the vector $\vec{A}$, corresponding to the amplitude of the resulting signal

$$\|\vec{A}\|^2 = \|\vec{A}_1 + \vec{A}_2\|^2 = \|\vec{A}_1\|^2 + \|\vec{A}_2\|^2 + 2\|\vec{A}_1\|\|\vec{A}_2\|\cos\Delta\varphi$$

$$\Longrightarrow A_m{}^2 = A_{01}{}^2 + A_{02}{}^2 + 2A_{01}A_{02}\cos\Delta\varphi$$

The light intensity (illuminance) is proportional to the square of the amplitude, which leads to:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi \qquad \text{Avec } \Delta\varphi = \varphi_2 - \varphi_1$$

Cas où $A_{01} = A_{02} = A_0$: $A_m = A_0\sqrt{2(1+\cos\Delta\varphi)} = 2A_0\cos\frac{\Delta\varphi}{2}$ $\qquad (1 + \cos x = 2\,cos^2\frac{x}{2})$

Consequently we have: $I_1 = I_2 = I_0$ and the intensity I is given by: $I = 2I_0(1+\cos\Delta\varphi)$ $= 4I_0 cos^2\frac{\Delta\varphi}{2}$

- If the two waves are in phase$(\Delta\varphi = 0)$: $I = 4I_0$ $\Rightarrow$ **Constructive interference**
- If the two waves are in opposite phase $(\Delta\varphi = \pi)$: $I = 0$ $\Rightarrow$ **Destructive interference**

Interference from two synchronous sources:

1- Notion of coherence:

The S_1 and S_2 transmitters are assimilated to point sources that emit sinusoidal signals of ***the same pulsation*** and ***constant phase shift***: we say that ***the sources are coherent***.

$$A_1(t) = A_{01}\cos(\omega t + \varphi_1), \qquad A_2(t) = A_{02}\cos(\omega t + \varphi_2),$$

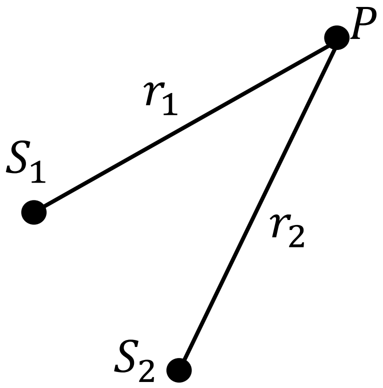
The two sources are coherent, their phase shift is constant : $\Delta\varphi = \varphi_2 - \varphi_1 = kr_2 - kr_1 = Cts$

Remark: In many cases, the signals have the same amplitude $A_{01} = A_{02} = A_0$ and are in phase ($\varphi_2 = \varphi_1$ therefore $\Delta\varphi = 0$), we then say that ***the sources are synchronous***.

2- Notion of the path difference :

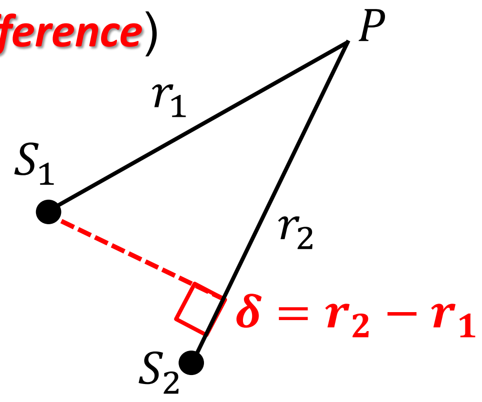
Let us consider the vibrations arriving at a pointP :

- ✓ The one from S_1 has traveled the distance r_1 = $S_1\ P$
- ✓ The other from S_2 has traveled the distance r_2 = $S_2\ P$



The vibration from S_1 does not take the same time to arrive at P as that from S_2 .

The delay of one vibration in relation to the other in P is: $\Delta t = \frac{r_2 - r_1}{c} = \frac{\Delta r}{c} = \frac{\delta}{c}$ (δ is the ***Path difference***)

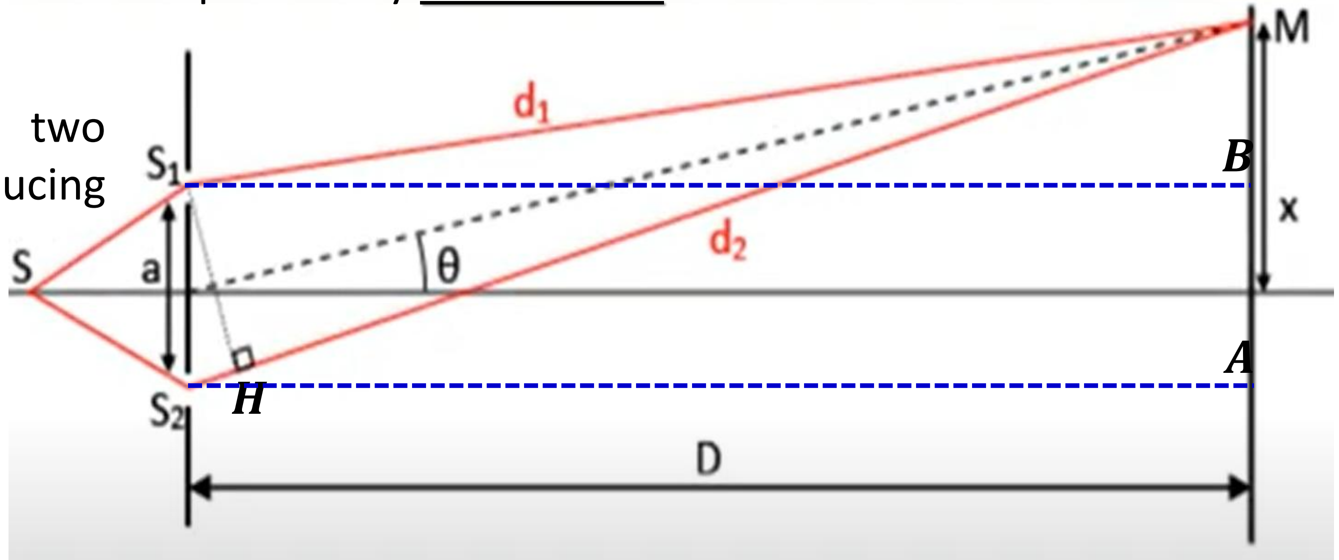


For two synchronous sources, the phase difference at a point P (phase shift) is given by:

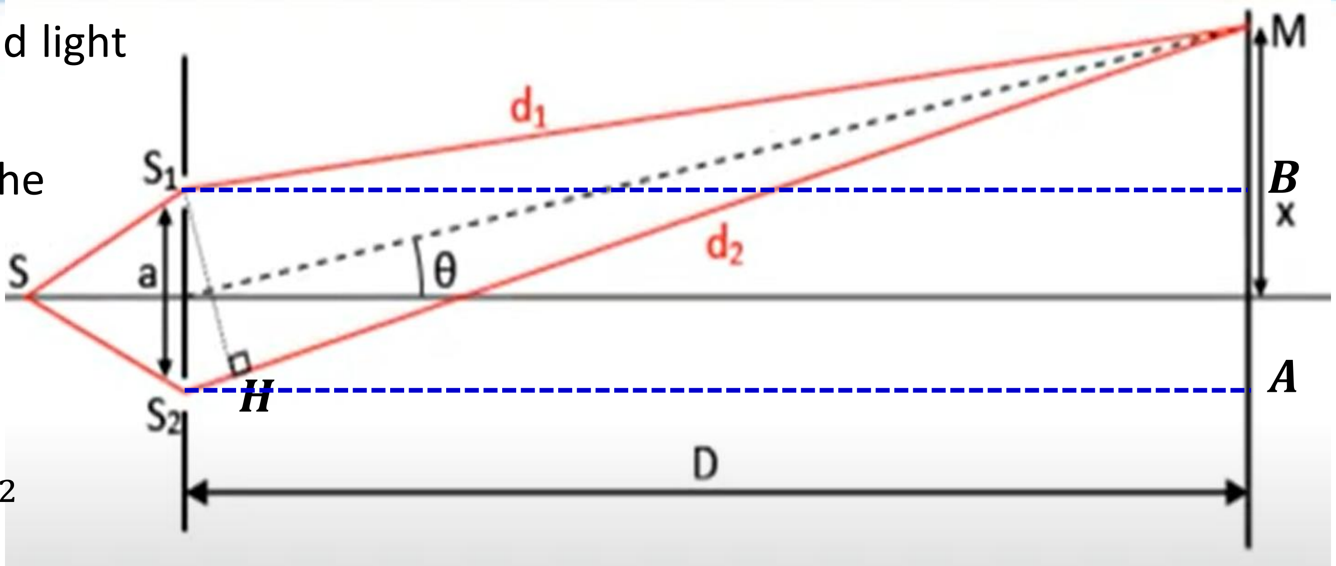
$$\Delta\varphi = \varphi_2 - \varphi_1 = k(r_2 - r_1) = \frac{2\pi}{\lambda}\delta$$

Young's Double Slit Experiment (Interference) (1801):

- In 1801, Thomas Young carried out an experiment that highlighted the wave-like nature of light.
- The device consists of illuminating, with the help of a monochromatic point source S, a screen pierced with two identical holes S1 and S2 relatively close and equidistant from S separated by a distance a.



- These two holes diffract the light and behave like two secondary point sources vibrating in phase and producing interference fringes on a screen placed at the distance D.
- In the interference field, we observe that:



1. The light is distributed in a system of alternately dark and light fringes that are almost rectilinear.
2. This interference phenomenon disappears when one of the holes is masked.

The path difference is given by:

$$\delta = S_2H = S_2M - S_1M \quad = d_2 - d_1$$

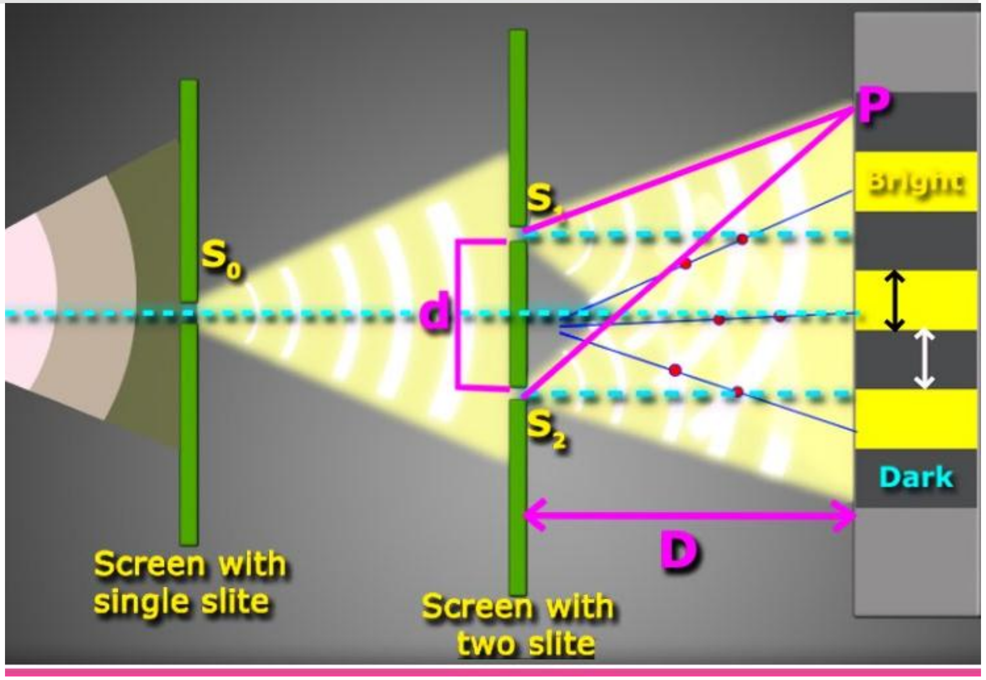
- ➢ <u>From the triangle S_2AM, we have:</u> $d_2{}^2 = D^2 + \left(x + \frac{a}{2}\right)^2$
- ➢ <u>From the triangle S_1BM, we have:</u> $d_1{}^2 = D^2 + \left(x - \frac{a}{2}\right)^2$

$$d_2{}^2 - d_1{}^2 = \left(D^2 + \left(x + \frac{a}{2}\right)^2\right) - \left(D^2 + \left(x - \frac{a}{2}\right)^2\right)$$

$$= D^2 + x^2 + \frac{a^2}{4} + xa - D^2 - x^2 - \frac{a^2}{4} + xa \quad \boxed{= 2xa}$$

$$d_2{}^2 - d_1{}^2 = 2ax \implies (d_2 - d_1)(d_2 + d_1) = 2ax$$

Since x et $a \ll D \quad \implies d_2 + d_1 \cong 2D$ $\boxed{\implies d_2 - d_1 = \delta = \frac{2ax}{d_2 + d_1} = \frac{ax}{D}}$

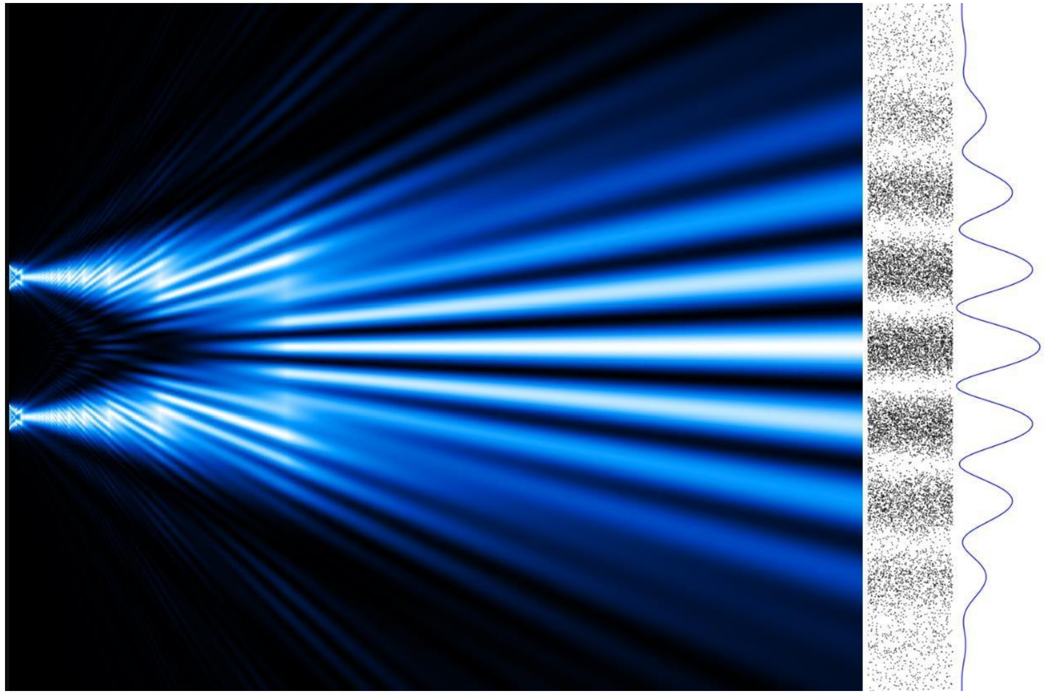


Remark:

If we are working in a medium with index n:

$$\boxed{\delta = n(d_2 - d_1) = n\frac{ax}{D}}$$

3- Interfrange $i(m)$:



It is the distance between two successive bright fringes (Two successive dark fringes)

$$\delta = \frac{ax}{D} \quad (n = 1) \text{ So : } x = \frac{\delta D}{a}$$

$\boldsymbol{\Delta\varphi = 2\pi k}$: a multiple of 2π) → waves in phase → bright fringe.

$$\Delta\varphi = 2\pi k = \frac{2\pi}{\lambda}\delta \quad \Longrightarrow \delta = k\lambda$$

- $\delta = k\lambda$ for a bright fringe (with k=0,1,2,….) :

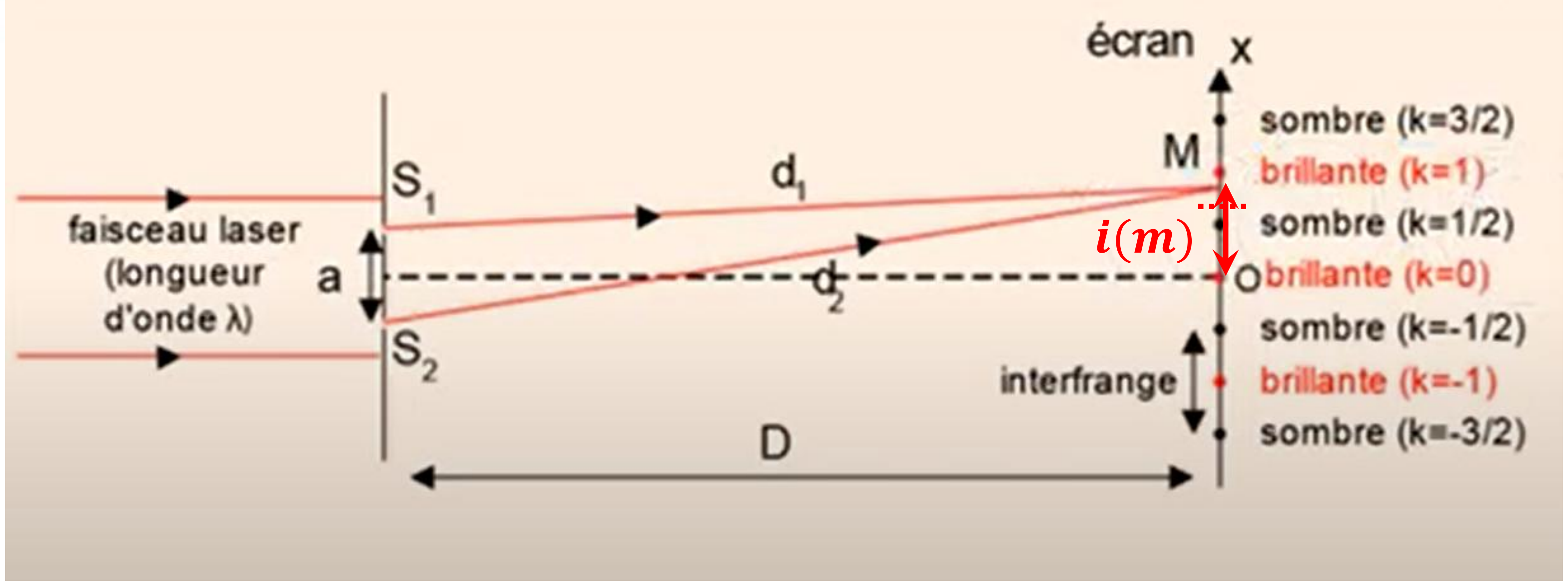
$$\boxed{\Longrightarrow x = \frac{k\lambda D}{a}}$$ (Abscice a bright fringe)

- For dark fringe: $\Delta\varphi = (2k+1)\pi = \frac{2\pi}{\lambda}\delta \quad \Longrightarrow \delta = \left(\frac{2k+1}{2}\right)\lambda \quad \Longrightarrow \boldsymbol{\delta = \left(k + \frac{1}{2}\right)\lambda}$

$$x = \frac{k\lambda D}{a}$$

- For $k = 1$: $x_1 = \frac{\lambda D}{a}$
- For $k = 0$: $x_0 = 0$

$$\boxed{\Longrightarrow i = x_1 - x_0 = \frac{\lambda D}{a}}$$



In general, the interfringe in this case is:

$$i(m) = x_{k+1} - x_k = \frac{(k+1)\lambda D}{a} - \frac{k\lambda D}{a} = \boldsymbol{\frac{\lambda D}{a}}$$

<u>Parameters that influence the interfringe</u>

- $\boldsymbol{\lambda}$ **:Wavelength**: if $\lambda \nearrow$ then $i \nearrow$
- $\boldsymbol{D}$ **: Distance between the source and the screen:** $D \nearrow$ then $i \nearrow$
- $\boldsymbol{a}$ **:Distance between the two sources (Young's slits):** $a \nearrow$ the $i \searrow$

Light intensity (Illuminance) at a point M:

$$I(M) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi$$

If $I_1 = I_2 = I_0$: $I(M) = 2I_0(1+\cos\Delta\varphi)$; $\Delta\varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda}\delta \implies I(M) = 2I_0\left(1 + \cos\left(\frac{2\pi\delta}{\lambda}\right)\right)$

We have: $\cos 2\theta = 2cos^2\theta - 1$ $\implies I(M) = 2I_0\left(1 + 2cos^2\left(\frac{\pi\delta}{\lambda}\right) - 1\right)$

$$\boxed{\implies I(M) = 4I_0\, cos^2\left(\frac{\pi\delta}{\lambda}\right)}$$

Or: $\boxed{I(M) = 4I_0\, cos^2\left(\frac{\pi a x}{\lambda D}\right)}$ by replacing δ

Or: $\boxed{I(M) = 4I_0\, cos^2\left(\frac{\pi x}{i}\right)}$ by replacing with i

Multiple wave interference:

- The Young's slit experiment allowed us to illustrate the phenomenon of two-waves interference.
- There are also interference devices that rely on multiple interferences.

The superposition of N waves in phase:

Let us suppose that at a point M in space, N harmonic waves of pulsation ω, phase φ_i and amplitude A_i are superimposed $i = \{1 \ldots N\}$. The wave state in M is written:

$$A_P(\vec{r},t) = \sum_{i=1}^{N} A_i \cos(\omega t + \varphi_i)$$

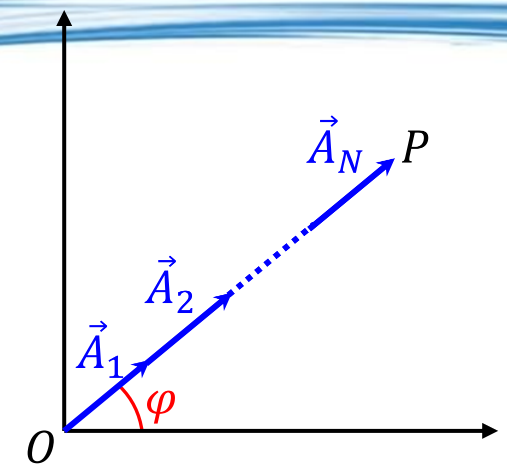
We assume that all waves oscillate in phase, that is, that $\varphi_i = \varphi$ for all i. In this case, we find:

$$A_P(\vec{r},t) = \cos(\omega t + \varphi) \sum_{i=1}^{N} A_i$$

The corresponding light intensity is: $$I_P = < {A_P}^2(\vec{r},t) > \;= < \left(\cos(\omega t + \varphi) \sum_{i=1}^{N} A_i\right)^2 > \;= \frac{1}{2}\left(\sum_{i=1}^{N} A_i\right)^2$$

- ☐ In terms of phases, this superposition can be represented by putting N aligned Fresnel vectors end to end
- ☐ The intensity being proportional to the square length($\propto (\vec{A}_P)^2$) of the resulting vector $\vec{A}_P$.
- ☐ The intensity is maximum in the case where all the waves oscillate in phase

$\Longrightarrow$ ***Constructive interference phenomenon.***



Fresnel construction corresponding to the superposition of N waves in phase.

- ☐ For simplicity, all waves are considered to have the same amplitude Ai = A, we obtain:

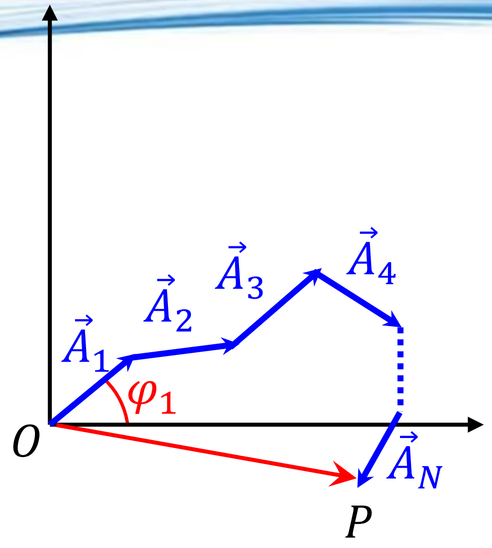
$$A_P(\vec{r},t) = NA\cos(\omega t + \varphi) \quad \text{et} \quad I_P = N^2 I_0$$

N presents the number of waves in phase

Superposition of N Incoherent waves:

- ➢ Let us now consider the case where the waves come from N independent quasi-monochromatic sources.
- ➢ To simplify, let's assume that all waves oscillate with the same amplitude ($I_0 = A^2/2$). The phase φi of each wave varies randomly with independent sources.

- If we describe the resulting wave by the Fresnel geometric construction, we obtain a random path.
- the resulting vector can be seen as the displacement of a point that has made N steps of amplitude A in random directions.
- The intensity is then related to the average square distance of a random path difference of N steps : $I = {}^{1}/_{2}\langle OP^2\rangle$.



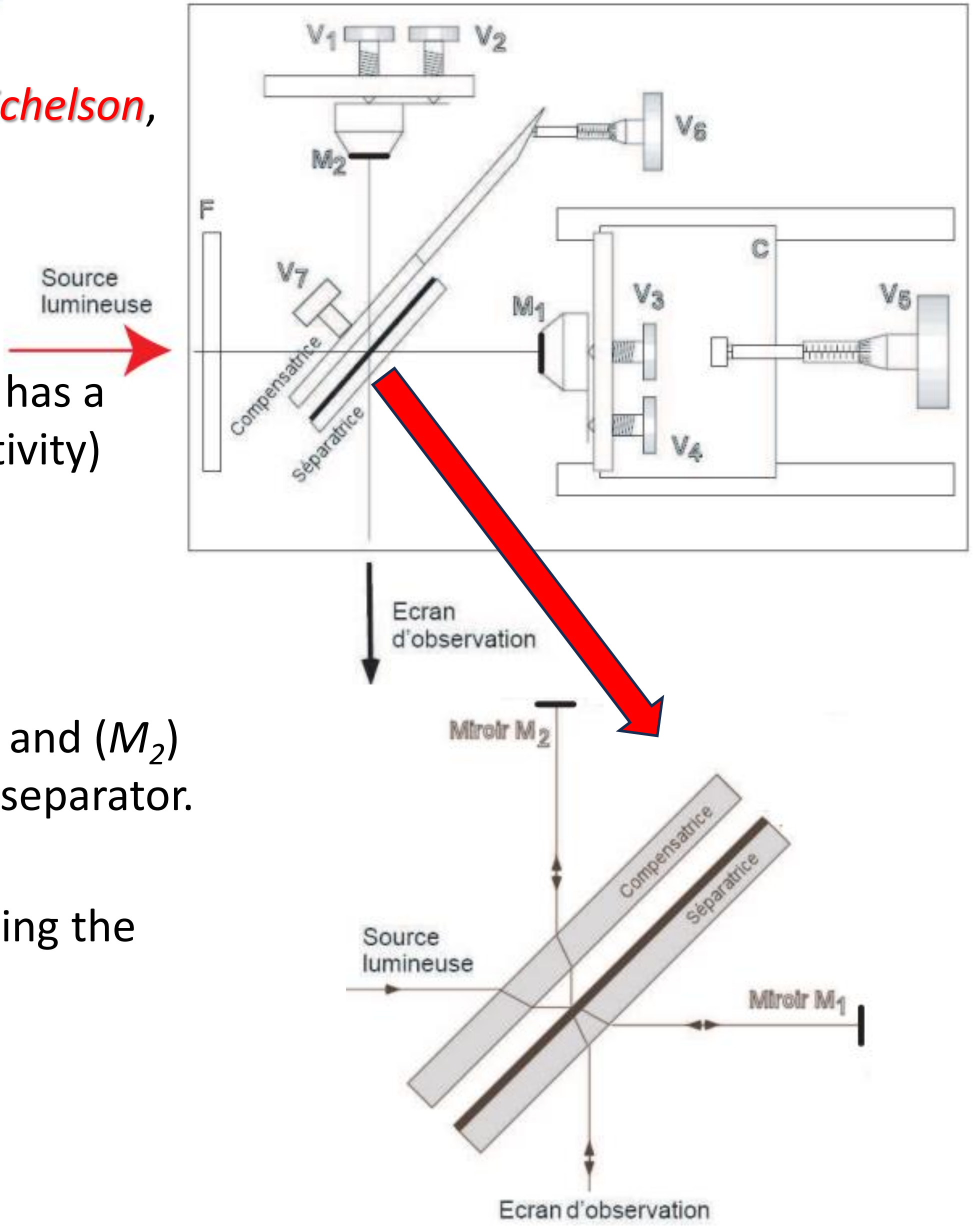
- In statistical mechanics, the average squared distance end-to-end is $\langle OP^2\rangle = NA^2$ where A is the amplitude of each wave.
- We therefore obtain: $I = {}^{1}/_{2}\,NA^2 = NI_0$

Conclusion:

- ❑ N incoherent sources of the same amplitude produce a phenomenon whose intensity is proportional to N, whereas in the case where these N waves vibrate in phase, the phenomenon is proportional to N^2 .
- ❑ Light can therefore be significantly amplified by multiple interferences.

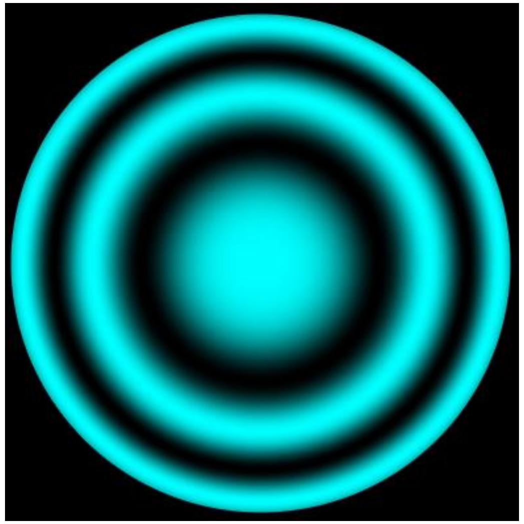
Michelson interferometers:

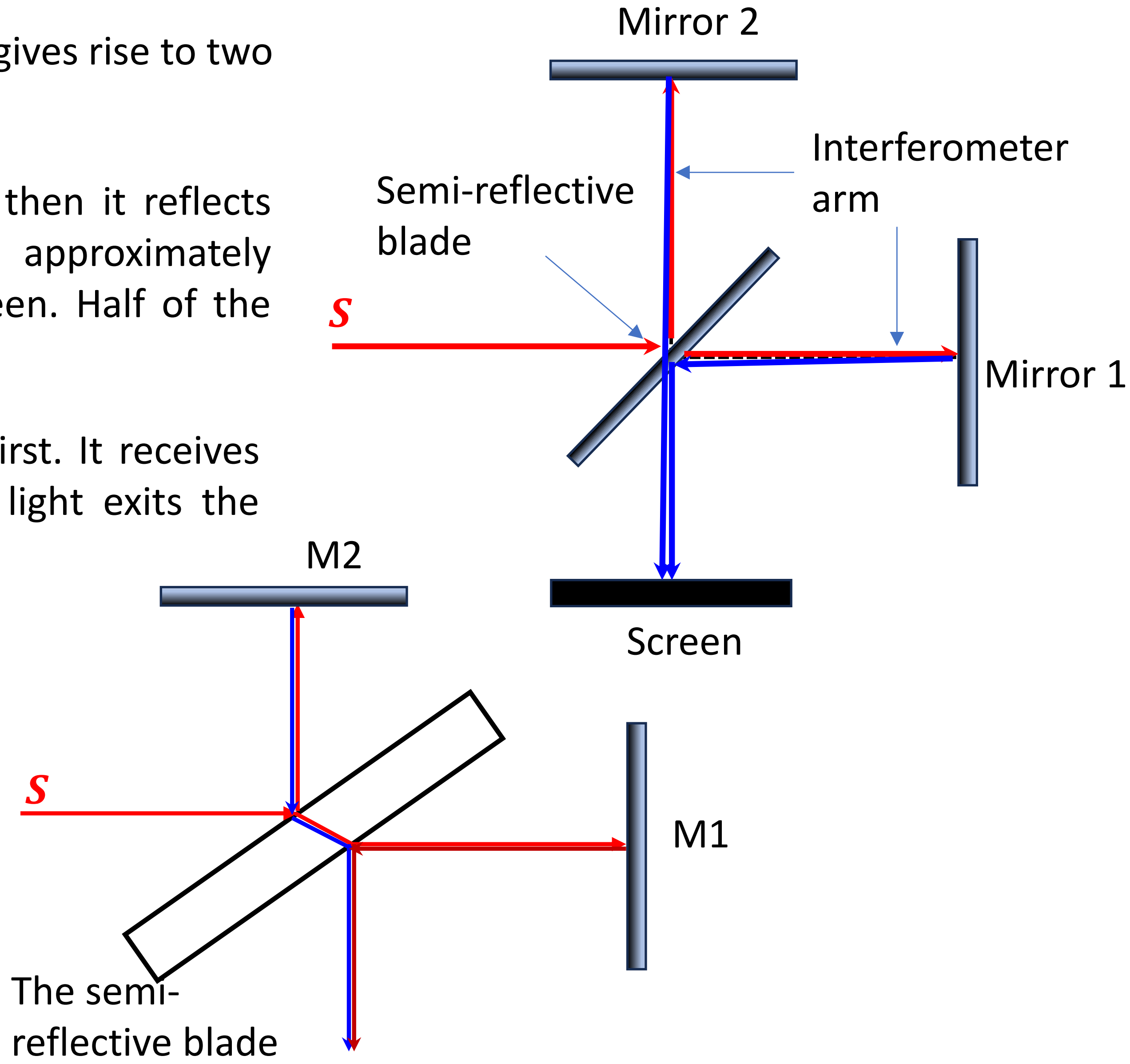
- ❑ Michelson interference is an optical device invented by *Albert Abraham Michelson*, producing interference and allowing to measure very small displacements.
- ❑ It is an invention that has had a major impact on physics:
  - ✓ He demonstrated through his experiments with *Edward Morley* that light has a speed independent of the frame of reference (Einstein's Principle of Relativity)
  - ✓ Detection of gravitational waves.
  - ✓ Allows the determination of the refractive index of a gaseous medium
- ❑ The Michelson interferometer consists essentially of two plane mirrors(M_1) and (M_2) placed at about90° one from the other and a semi-reflective blade called a separator.
- ❑ Each mirror can rotate around two orthogonal axes using two screws, allowing the relative tilt of the two mirrors to be adjusted.



How Michelson Interference Works:

- A light wave from a source (S), incident on the separator, gives rise to two waves of the same amplitude.
- The light transmitted by the slide meets the mirror M_1 then it reflects back through the semi-reflective blade in a direction approximately perpendicular to the incident direction towards the screen. Half of the intensity goes back to the light source.
- The second mirror M_2 is arranged perpendicular to the first. It receives the light reflected by the separator. On reflection, This light exits the device after passing through the separator to the screen.



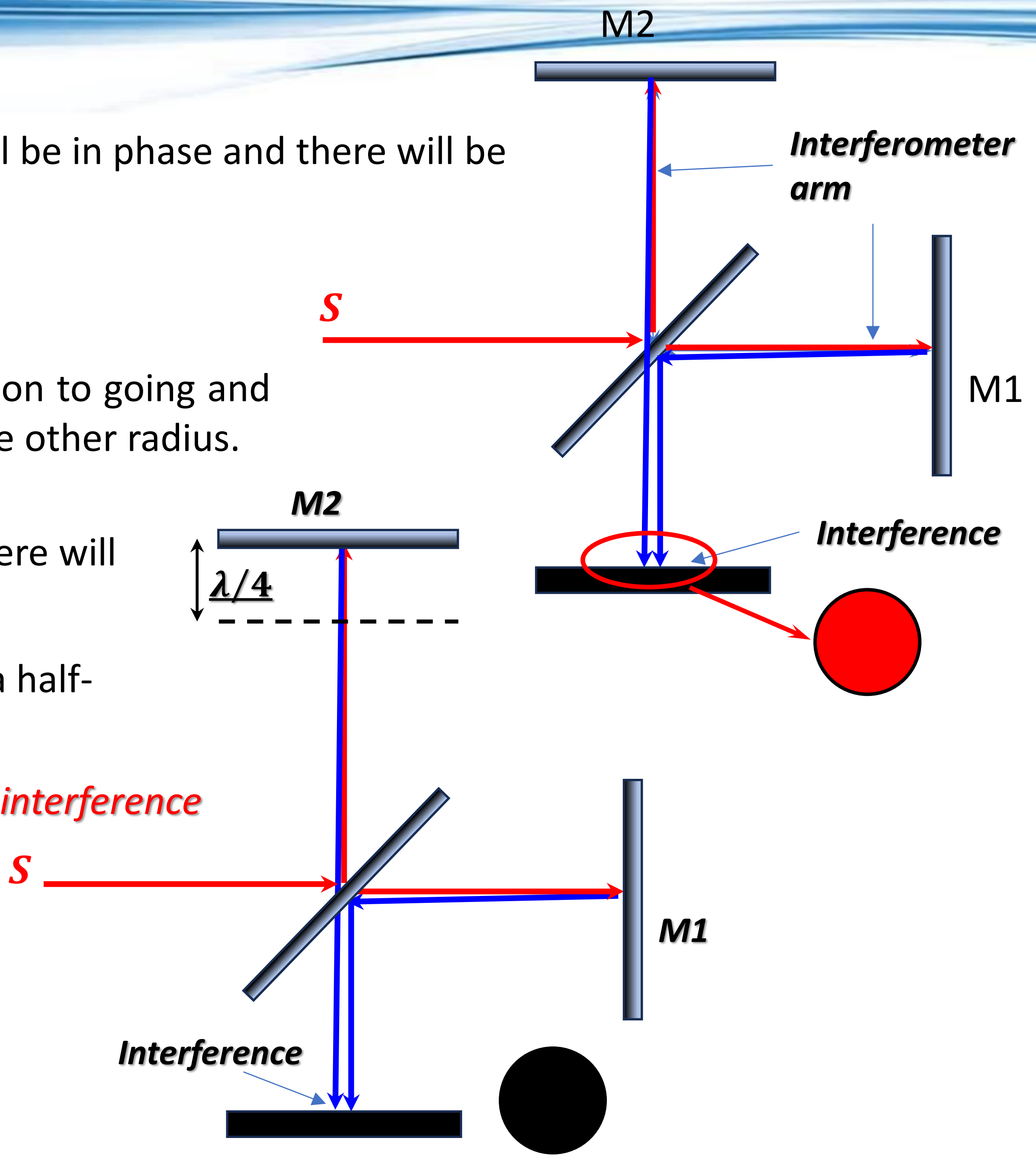


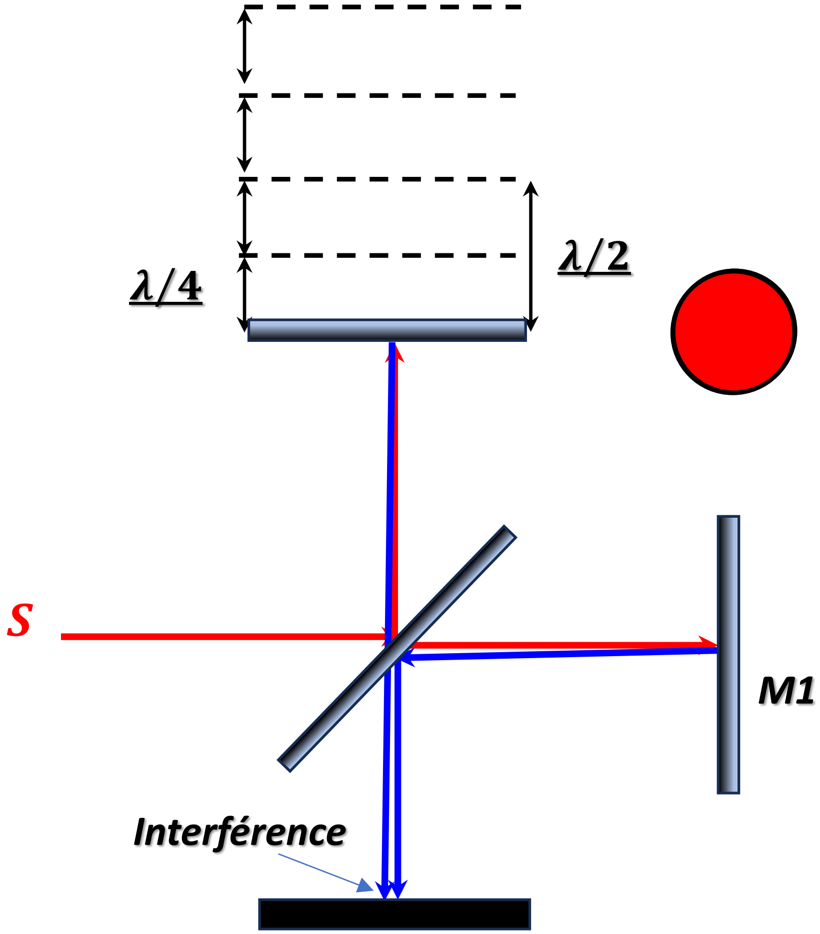
Cases where the interferometer arms are equal:

- ☐ The two rays will travel the same distance. The two waves will be in phase and there will be *constructive interference* (creation of a clear fringe)

If one of the mirrors is going to shift by a distance of $\lambda/4$:

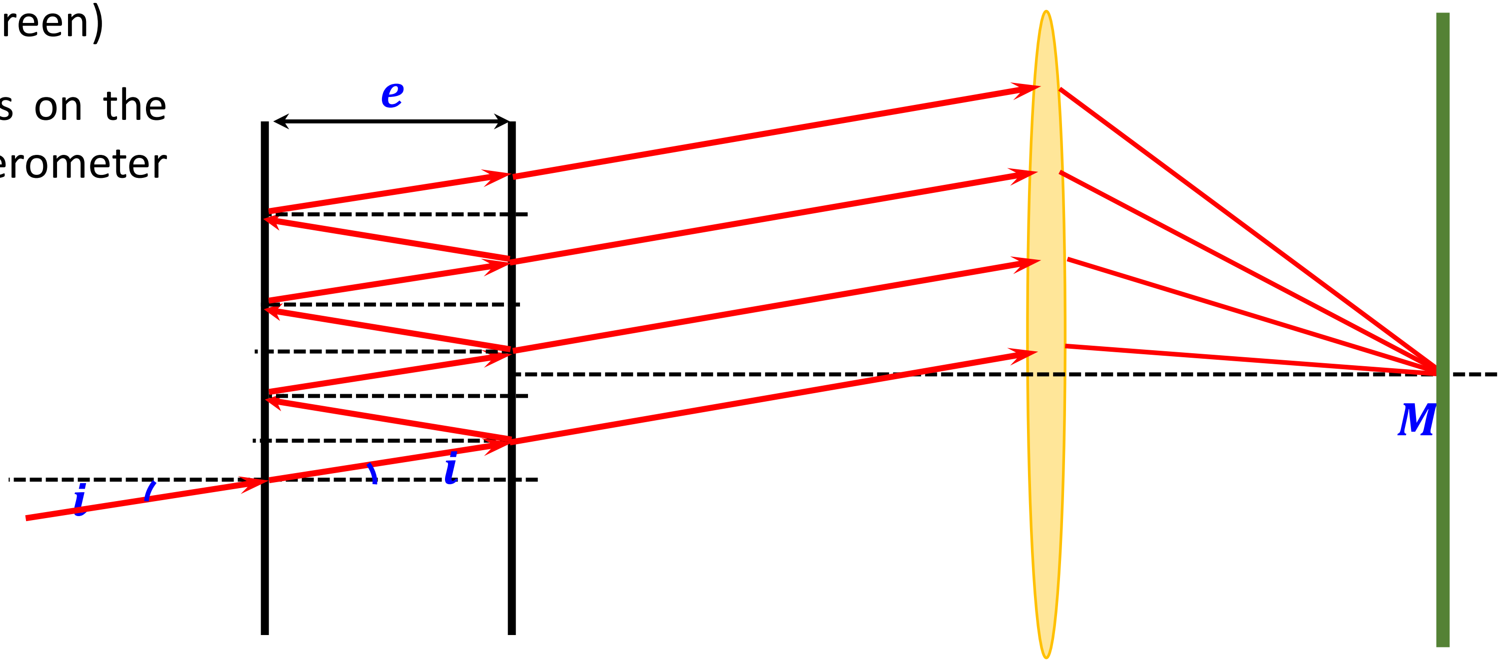
- ☐ One of the rays of light will travel a distance of $\lambda/4$ in addition to going and $\lambda/4$ in return. Dans le total, a distance of $\lambda/2$ compared to the other radius.
- ☐ They will no longer arrive on the screen at the same time, There will be a lag of $\lambda/2$
- ☐ Each shift of the mirror in quarter-wavelengths will produce a half-wavelength increase in the path of light
- ☐ The two waves are out of phase and we will have ***destructive interference***
- ☐ We will then go from a light fringe to a dark fringe or from a dark to a light fringe.



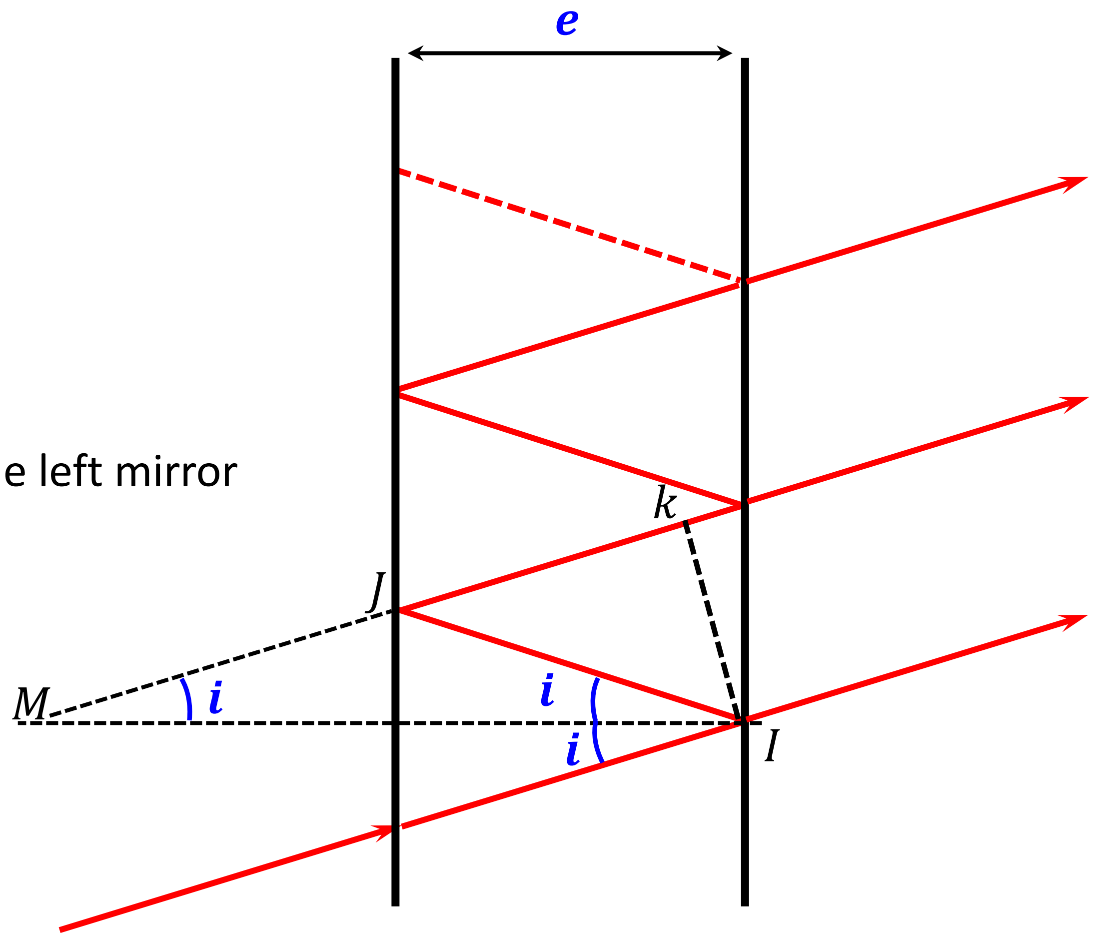

λ/4
λ/2
S
M1
Interférence

The Pérot-Fabry interferometer

- The Perot-Fabry interferometer (PF) is a multi-wave interferometer, unlike the Michelson interferometer where only two waves interfere
- It consists of two parallel semi-reflective mirrors delimiting a cavity of thickness e filled with a medium of index n (usually air)
- Suppose that a monochromatic plane wave enters the PF with an angle of incidence i. The penetrating light can undergo multiple internal reflections at the same angle of incidence i before exiting from it on the other side.
- The different rays are transmitted parallel and can interfere infinitely (In practice, they are collected by a lens and interfere with the focal plane of the lens on a screen)
- The intensity of the transmitted light depends on the angle of incidence i, thickness e of the interferometer and the wavelength λ of light.



Phase shift between successive emergent rays:



- We do the calculation on the first two emerging rays.
- Let be K the orthogonal projection of I on the upper radius.
- *The path difference to be taken into account is* $\boldsymbol{\delta = \overline{IJ} - \overline{KJ}}$

Or $\boldsymbol{\delta = \overline{IJ} + \overline{JK}}$

- We consider the point M, the symmetry of K with respect to the left mirror so that the triangle IJM is isosceles.

Then: $\overline{IJ} = \overline{MJ} \quad \Longrightarrow \delta = \overline{MJ} + \overline{JK} = \overline{MK}$

The triangle IKM is rectangle in K, we then have:

$$\overline{MK} = \overline{MI}\cos i = 2e\cos i \quad \boxed{\boldsymbol{\Longrightarrow \delta = 2e\cos i}} \quad (n = 1)$$

Generally speaking, in monochromatic light, the phase shift between two successive rays is: $\boldsymbol{\Delta\varphi = \frac{2\pi}{\lambda_0} 2e\cos i}$