University of Djilali BOUNAAMA-Khemis Miliana
Faculty of Matter Sciences and Computer Science
Department of Physics

Course of Geometrical and Physical Optics.

Level: 2nd year Physics

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<u>Chapitre 1</u> : Optique géométrique

1.1- Principes et lois de l'optique géométrique.

1.2- Notions de réfringence.

1.3- Lois de Snell-Descartes, principe de Fermat et construction de Huygens.

1.4- Miroirs sphériques et miroirs plans: formule de position et construction d'images.

1.5- Dioptre plan et dioptre sphérique: formule de conjugaison, grandissement, notions de stigmatisme et construction d'images

1.6- Prisme : formules, déviation et dispersion.

1.7- Lentilles minces :formules de position et construction d'images.

1.8- Instruments optiques : œil, loupe, microscope,…

<u>Chapitre 2</u> : Optique ondulatoire

2.1- Généralités

2.2-Principe de superposition de deux ondes monochromatiques de même fréquence.

2.3- Conditions d'interférence : Notion de cohérence.

2.4- Interférences de deux ondes cohérentes.

2.5-Interférences à ondes multiples :Interféromètres de Michelson et de Pérot-Fabry

2.6- Interférences en lumière polychromatique.

<u>Chapitre3</u> : Diffraction et ses Applications

3.1- Diffraction de Fresnel et diffraction de Fraunhoffer

3.2- Diffraction par une ouverture rectangulaire et diffraction par une ouverture circulaire

<u>Chapitre 4</u> : Polarisation

4.1- Transversalité des ondes

4.2- Structure d'une onde polarisée rectilignement

4.3- Réflexion et réfraction par les corps isotropes transparents

<u>Chapitre 5</u> : Lasers et ses applications.

Chapter 1: Geometrical Optics

1.1- Principles and laws of geometrical optics
1.2- Notions of refringence
1.3- Snell-Descartes laws, Fermat's principle and Huygens' construction
1.4- Spherical mirrors and plane mirrors: position formula and image construction
1.5- Plane and spherical dioptre: conjugation formula, magnification, notions of stigmatism and image construction
1.6- Prism: formulas, deviation and dispersion
1.7- Thin lenses: position formulas and image construction
1.8- Optical instruments: eye, magnifying glass, microscope, ...

Chapter 2: Wave Optics

2.1- Generalities
2.2- Principle of superposition of two monochromatic waves of the same frequency
2.3- Interference conditions: Notion of coherence
2.4- Interference of two coherent waves
2.5- Multiple wave interference: Michelson and Perot-Fabry interferometers
2.6- Interference in polychromatic light

Chapter 3: Diffraction and its Applications

3.1- Fresnel diffraction and Fraunhoffer diffraction
3.2- Diffraction by a rectangular aperture and diffraction by a circular aperture

Chapter 4: Polarization

4.1- Transversality of waves
4.2- Structure of a linearly polarized wave
4.3- Reflection and refraction by transparent isotropic bodies

Chapter 5: Lasers and their Applications

General introduction

General information about optics: Part of physics that is concerned with the study of light phenomena perceived by the eye:

Visual sensation, eye and vision, visible spectrum, colors, ... etc

Light source

Primary Sources




Secondary sources





Optics

Geometrical Optics

Investigates the macroscopic effects of optics such as:

-Rectilinear propagation,

-The reflection and the refraction of light.

Wave optics

Investigates the relationship between light and matter,
It is interpreted by phenomena such as:

- Light Diffraction,

- Interference Phenomena

- Emission and absorption,

Chapter 1: Geometrical Optics

I.1 Definitions:

- Geometric optics is a branch of optics where light is described ***by rays.***
- ***Light rays*** are conceived as geometric lines coming from sources, passing through media and revealed by detectors.
- Their directions represent the paths along which light travels.

I.2. Notion of light ray:

Postulate of geometrical optics:

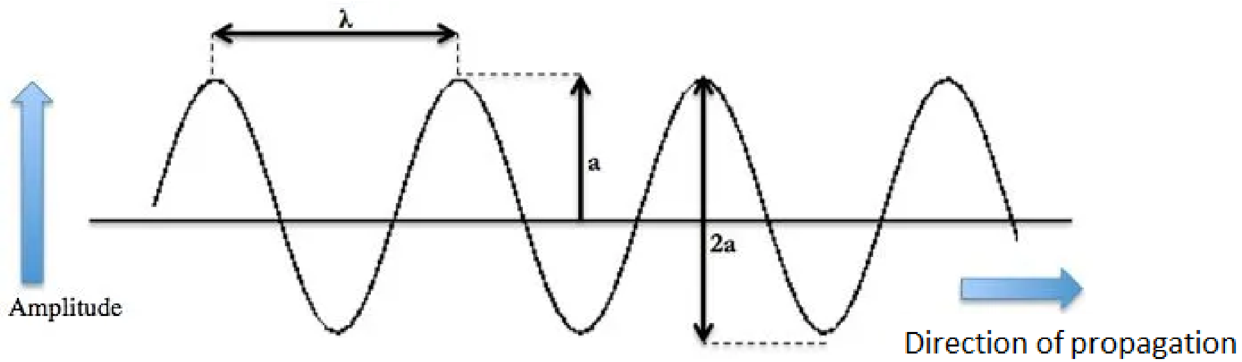
- Light ***(Light Energy)*** is described by a set of independent light rays.
- These light rays are characterized by a propagation direction $\vec{u}$ and a propagation velocity $\boldsymbol{v}$
- These light rays propagate in a straight line in any homogeneous medium at a velocity that depends on the medium.
- In a vacuum, all light propagates in a straight line at the velocity of $\boldsymbol{c \cong 3.10^8 m.s^{-1}}$.

Geometric optics is interested in the path taken by light based on the properties of the environments (milieux) it passes through.

Concept of electromagnetic spectrum:

In geometric optics, light is considered to be an electromagnetic wave (wave vibration) that propagates in all directions of space.

Electromagnetic wave is characterized by:



a: Amplitude λ: wavelength

- **Wavelength λ (m) :**

  Movement of the wave during a cycle of vibrations.

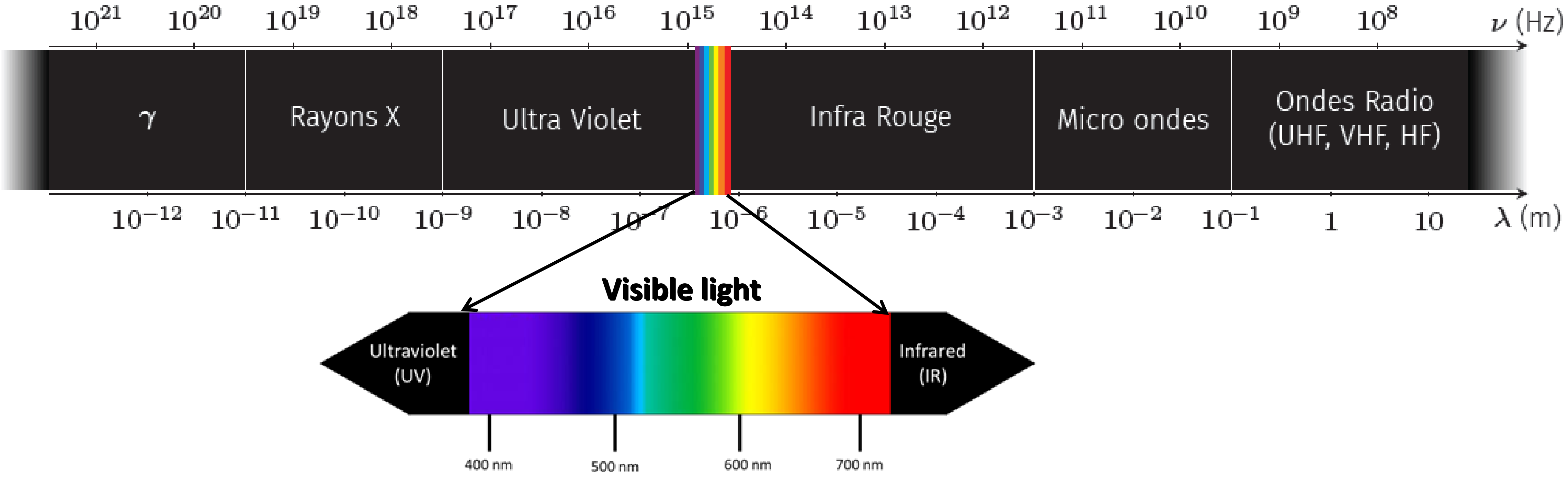
- **Frequency $\nu(s^{-1})$:** Number of cycles per second.

Frequency and wavelength are related by : $\lambda = \frac{v}{\nu}$ where v is the wave propagation velocity.

- **The two quantities v and λ depend on the medium through which the wave propagates**

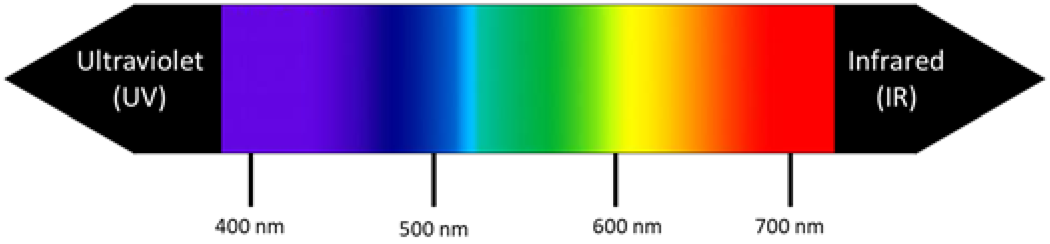
v (Air or vacuum)	v(water)	v(glass)
300 000 km/s	225 000 km/s	200 000km/s

Electromagnetic Spectrum:



- The electromagnetic spectrum is almost completely invisible to the human eye, except for a small portion called:

"the visible spectrum"



- The visible spectrum extends from red to violet, including all the colors of the rainbow (red, orange, yellow, green, blue, indigo, and violet).

- The color of light mainly depends on how the eye perceives it and the brain interprets it

- It will be noted that the visible spectrum corresponds to the interval $\mathbf{400\ nm < \lambda_{visible} < 700nm}$

- Light is said to be **<u>polychromatic</u>** when it is made up of several wavelengths, **<u>monochromatic</u>** when it is made up of a single wavelength

- **<u>Monochromatic sources</u>** in the strict sense of the term do not exist, but some ***<u>lasers</u>*** produce light with a very narrow spectrum. They are therefore considered monochromatic sources.

I.3. Refractive Index:

Definitions:

a- Homogeneous medium: Any medium in which light propagates with a constant velocity v.

b- Inhomogeneous medium (non-homogeneous) : Any medium in which light propagates with a variable velocity v.

c- Transparent medium: Any medium that lets light through.

Example: water, glass, cellophane,…

d- Opaque medium: any medium that totally stops the light.

Example: wood, steel, marble…

- In a transparent, homogeneous and isotropic medium (***having the same optical properties in all directions***), light propagates in a straight line at a velocity:

$$v = \frac{c}{n}$$

where the scalar n is a dimensionless quantity, called the **refractive index**. It describes the behavior of light in the medium.

- In a vacuum, the velocity of light is given by: $c = \lambda_0 \nu$
- In a medium, the celerity is given by: $v = \lambda_{medium} \nu$

Therefore, the refractive index is defined by: $n = \frac{c}{v} \geq 1$

$$\Rightarrow n = \frac{\lambda_0}{\lambda_{medium}}$$

Remarks:

I. The refractive index depends on the wavelength of the measurement but also on the characteristics of the environment (**in particular pressure and temperature**).

Medium	Air	Water	Sea	Glass	Polyester	Diamond
Index	**1,0003 ≅ 1**	**1,33**	**1,34**	**1,5 - 1,8**	**1,58**	**2,42**

II. Refractive index n of a homogeneous medium is constant ($n = c/v$).

III. Refractive index n of an inhomogeneous medium is variable, in this medium ($n = c/v$).

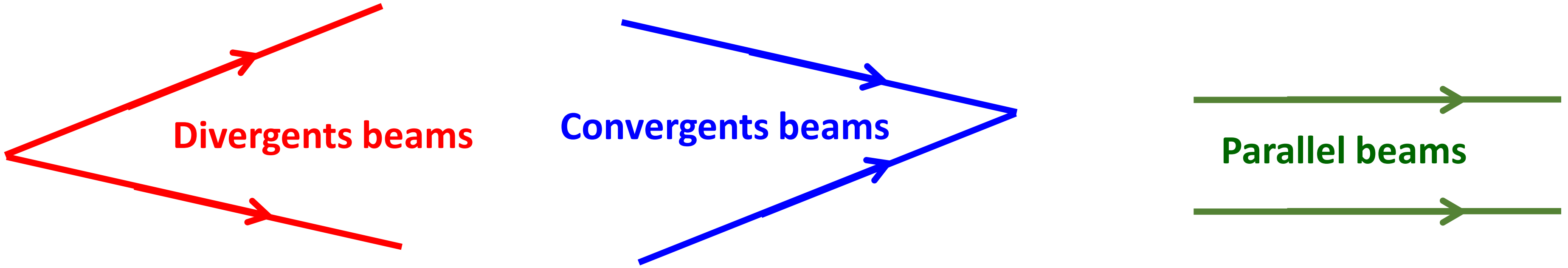
I.2.Concepts of refringence:

Reminders:

- A ray of light is represented by a straight line AB on which an arrow is placed indicating the direction of propagation of light.

- In practice, an isolated light ray does not exist.

- The rays are always grouped into bundles, forming what we call light beams: There are 3 types of beams



- ***Rectilinear propagation principle :*** In a transparent, isotropic and homogeneous medium, light propagates in a straight line.

- ***Reverse light return principle:*** If we reverse its direction of propagation, a ray of light follows the same path.

- ***Principle of independence of light rays:*** No interaction exists between two light rays. One ray cannot deflect (dévier) another.

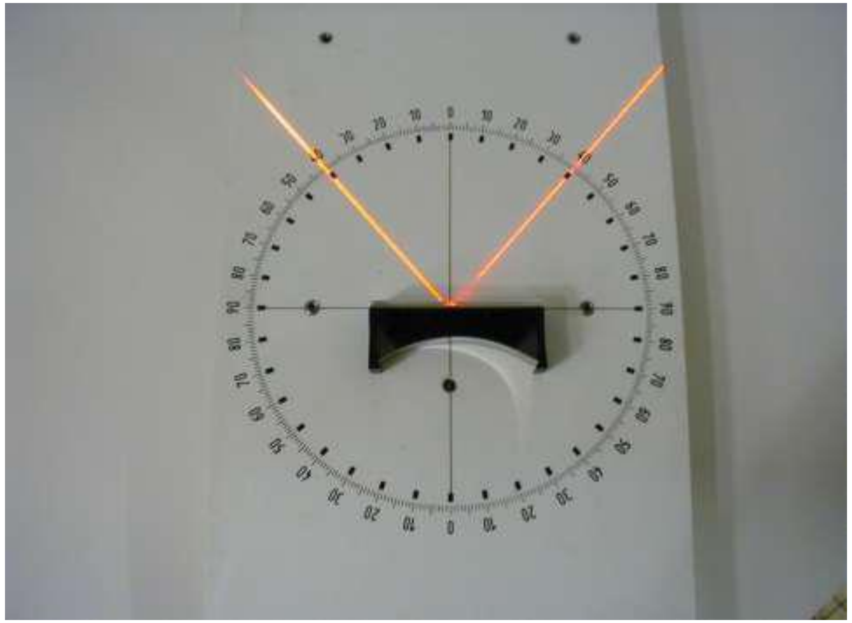
Laws of reflection and refraction:

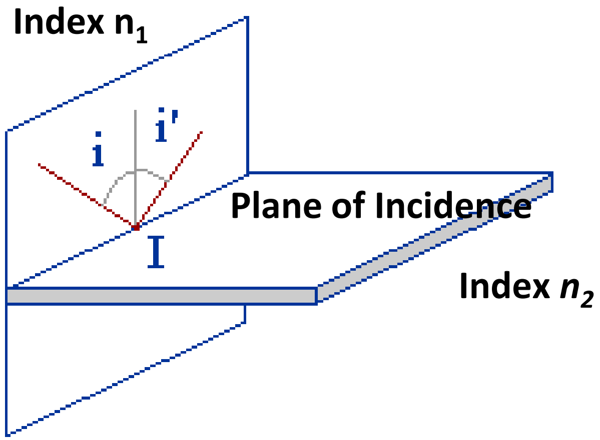
1. Reflection: We speak of reflection when a light ray suddenly changes direction while remaining in the same propagation medium.

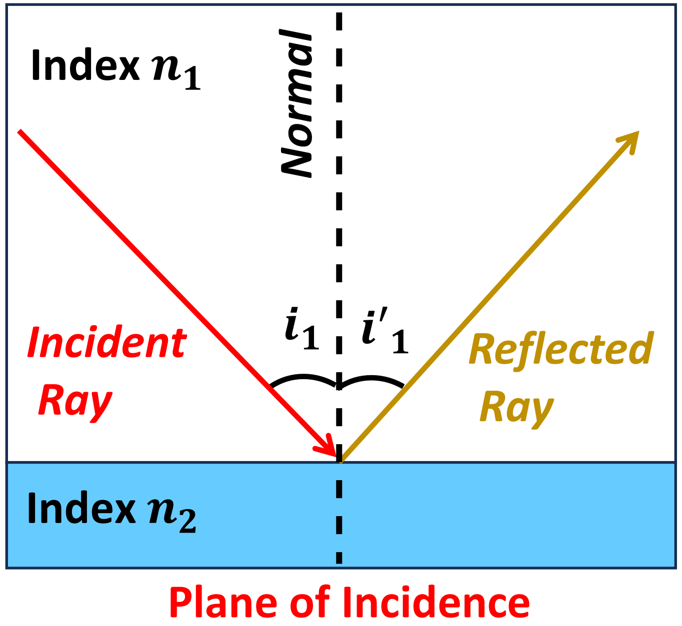
- The experiment shows the following laws of reflection:

1. The incident ray, the reflected ray and the normal to the surface are in the same plane called ***the plane of incidence***.

2. The angles of incidence and reflection are equal: $i_1 = i_1'$

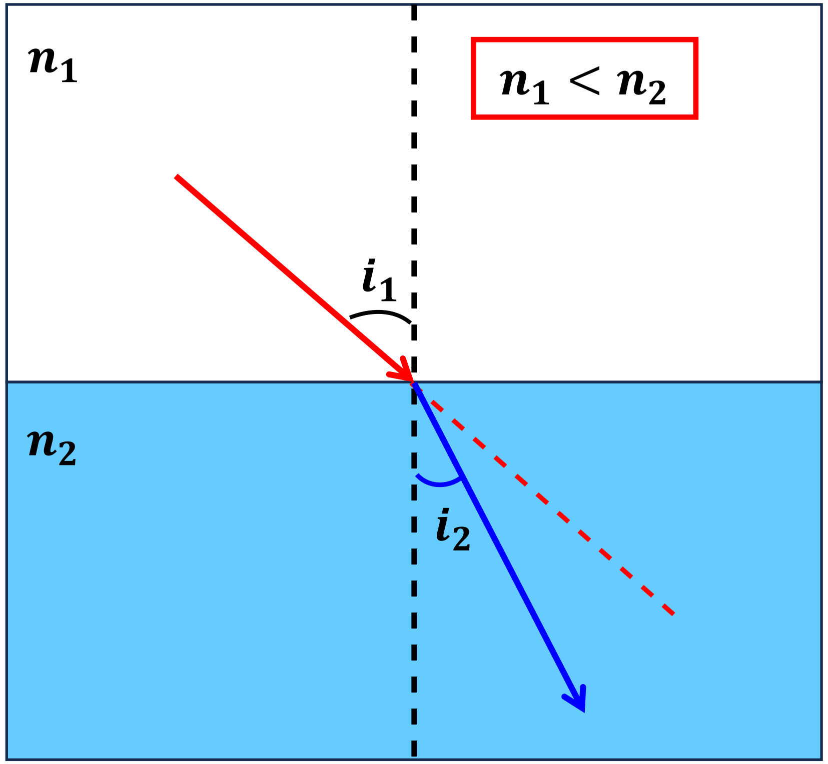


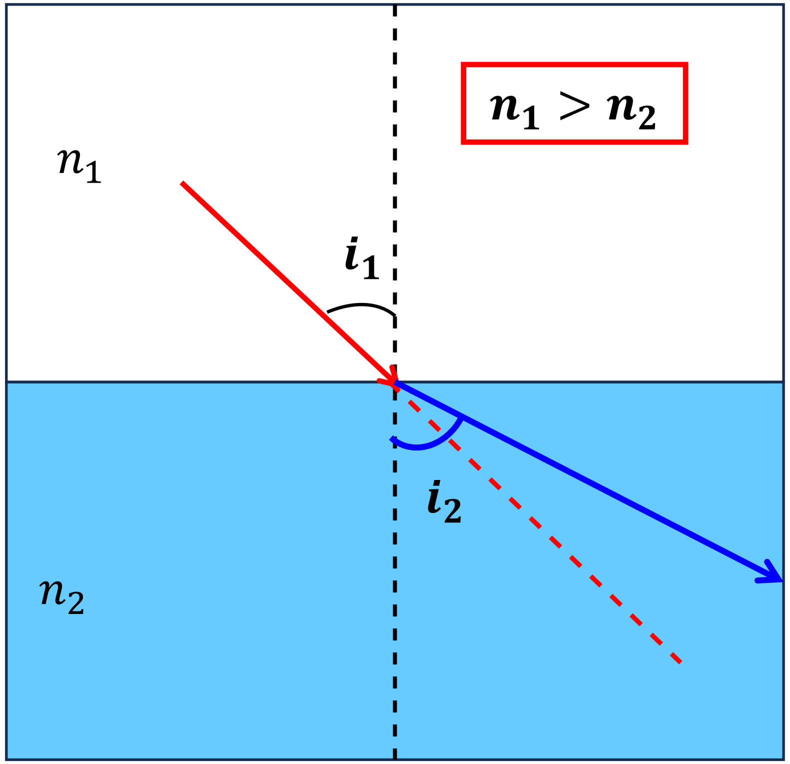




2. Refraction: We speak of refraction when there is a change in the direction of propagation of light when it crosses a diopter ***(separation surface between two transparent media)*** and therefore changes transparent media.

- **In other words**: Refraction is the deviation of light when it passes through the interface between two transparent media of different optical indices.

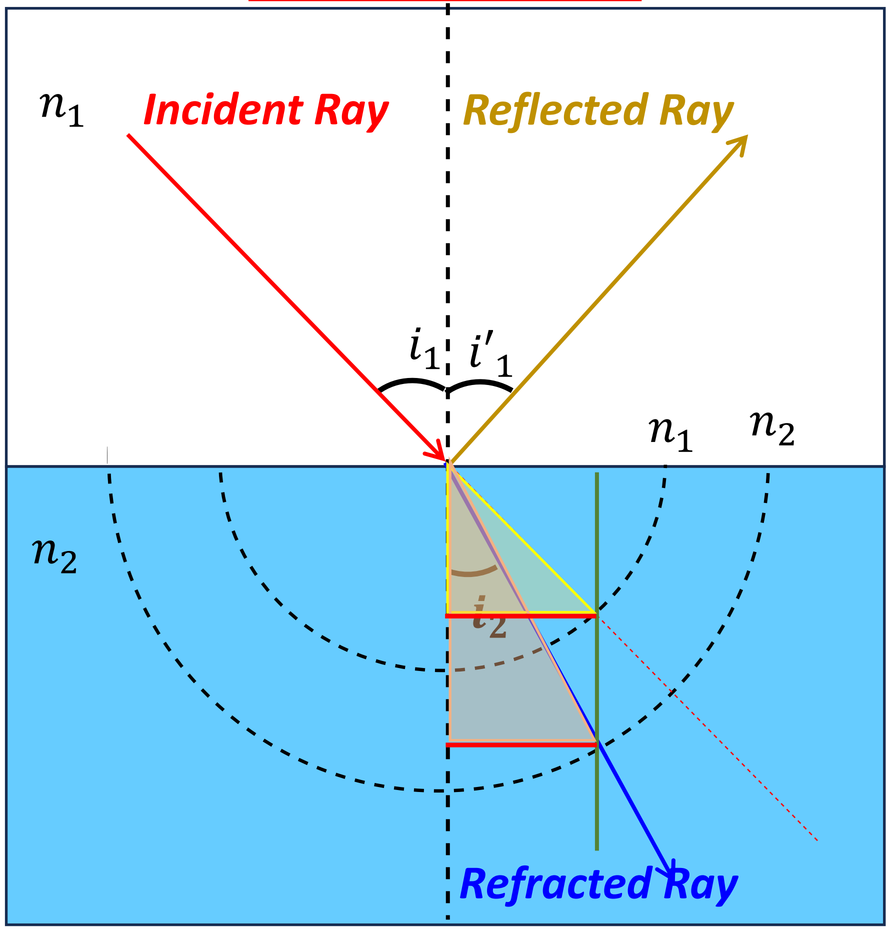




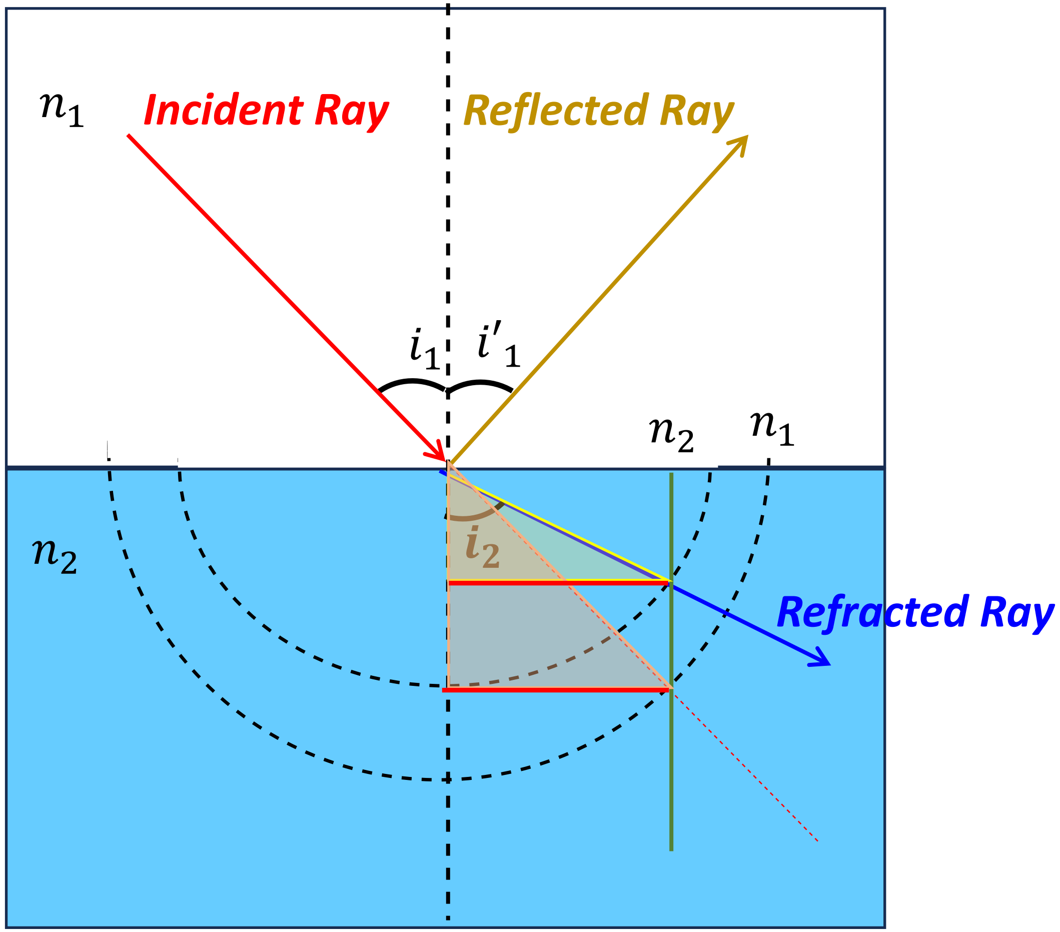
Please note: **If n2 > n1, we say that the medium (2) is more refractive;**
By passing from medium (1) to medium (2), the light then approaches to the normal.

I.3. Snell Descartes laws:

$n_1 < n_2$



$n_1 > n_2$

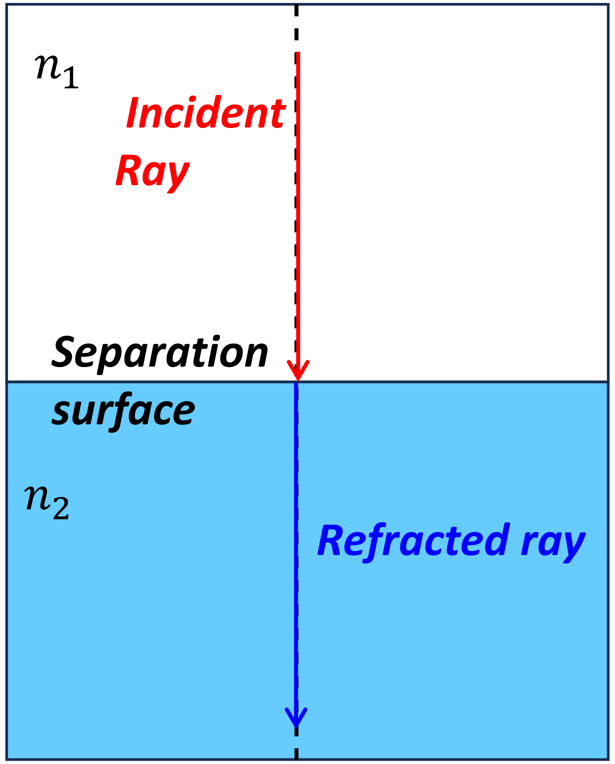


$n_1 \sin(i_1) = n_2 \sin(i_2)$ $\Leftarrow$ **Snell Descartes law** $\Rightarrow$ $n_1 \sin(i_1) = n_2 \sin(i_2)$

Total reflection phenomenon:

1er cas : Normal Incidence :

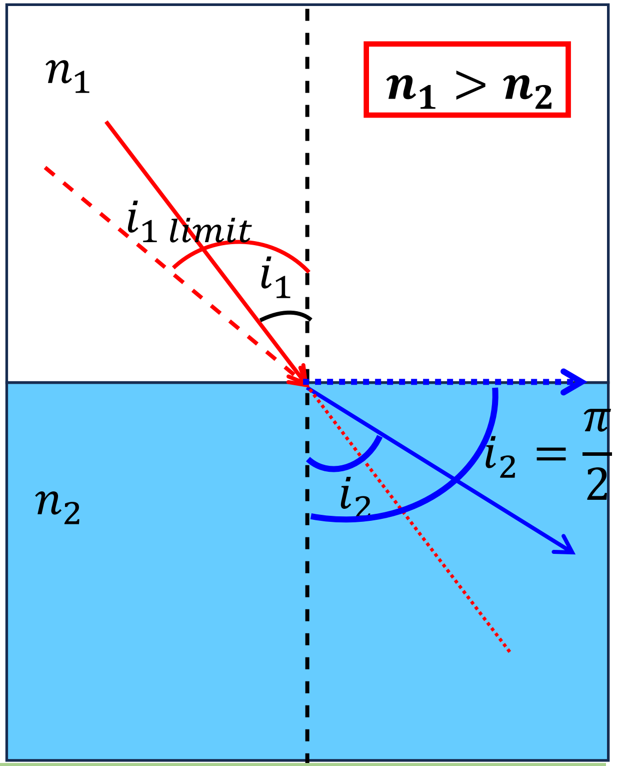
$$i_1 = 0 \Rightarrow i_2 = 0$$



All light passes through the 2nd medium

2nd case: Limited incidence (Critical): Limit refraction

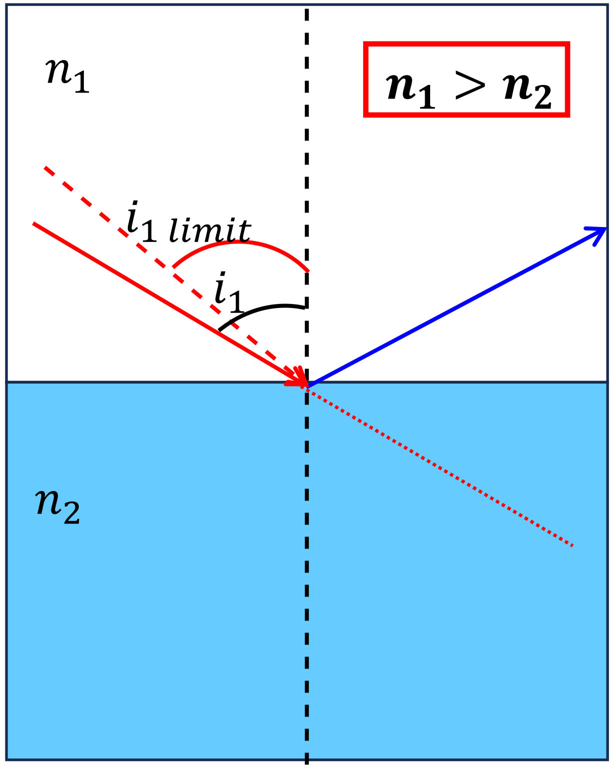
$$i_1 = i_{1\,limit} \Rightarrow i_2 = \frac{\pi}{2}$$



$$n_1 \sin(i_{1\,limit}) = n_2 \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow i_{1\,limit} = arc\,sin\left(\frac{n_2}{n_1}\right)$$

3rd case: Total reflection



Si $i_1 > i_{1\,limit} \Rightarrow$ Total reflection

Exercise 1:

Determine the limit angle of light in the glass ($n_G = 1.50$), whether it is immersed in air ($n_a = 1.00$) and whether it is immersed in water ($n_W = 1{,}33$).

Solution :

1. If the glass is immersed in air, the limit angle is given by the relation:

$$\sin(i_{limit})_G = \frac{n_a}{n_G} = \frac{1{,}00}{1{,}50} = 0{,}667 \quad \Longrightarrow (i_{limit})_G = 41{,}8°$$

2. If the glass is immersed in water, the limit angle is:

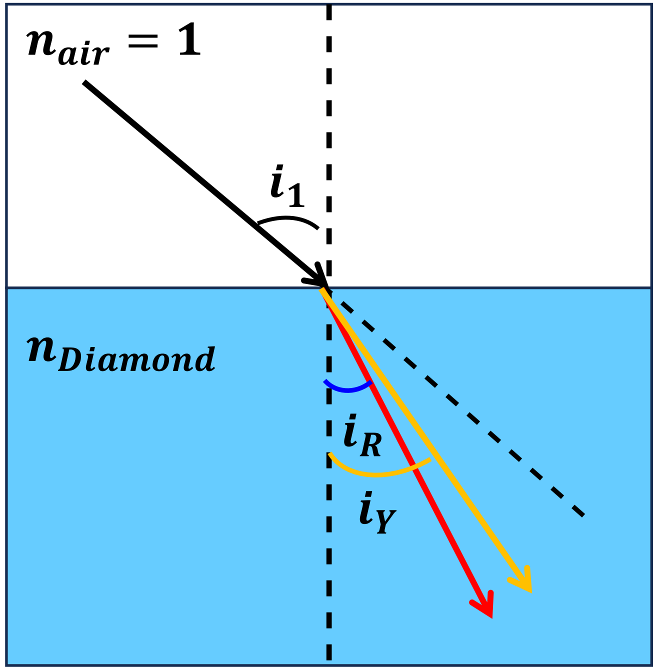
$$\sin(i_{limit})_{water} = \frac{n_W}{n_G} = \frac{1{,}33}{1{,}50} = 0{,}887 \quad \Longrightarrow (i_{limit})_{water} = 62{,}5°$$

Exercise 2:

The diamond has an index $n_R = 2{,}435$ for red wavelength of $486\ nm$ and an index $n_Y = 2.417$ for the $589nm$ wavelength yellow. A beam of white light falls on the diamond with an angle of incidence $i = 45°$.

Determine the angle that the red ray and the yellow ray form inside the diamond.

Solution :



The refractive angles of the red ray is given by:

$$n_{air} \sin 45° = n_R \sin i_R \implies \sin i_R = \frac{\sin 45°}{n_R} = 0.2904 \implies i_R = 16{,}88$$

The refractive angles of the yellow ray is also given by:

$$n_{air} \sin 45° = n_Y \sin i_Y \implies \sin i_Y = \frac{\sin 45°}{n_Y} = 0.2926 \implies i_Y = 17.01$$

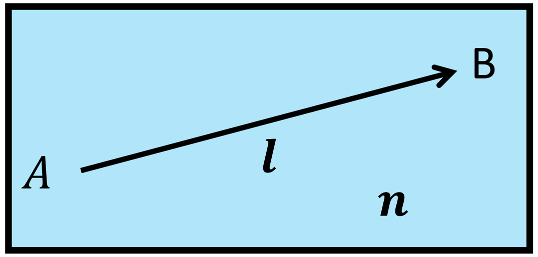
The angle formed by these two rays is therefore: $\mathbf{17,01 - 16,88 = 0,13}$

Prince of Fermat (1601-1665) (The principle of least time):

Light propagates from one point to another along the path that takes the minimum time.

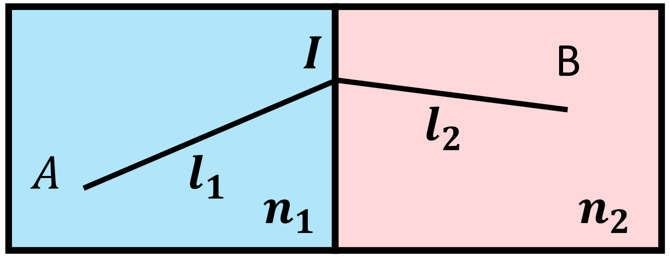
- Imagine a ray of light starting at point A and heading to another point B.
- Fermat proposes that, among all possible paths between A and B, light takes only the fastest path.

❑ ***In a transparent, homogeneous and isotropic medium (n=Cts):***



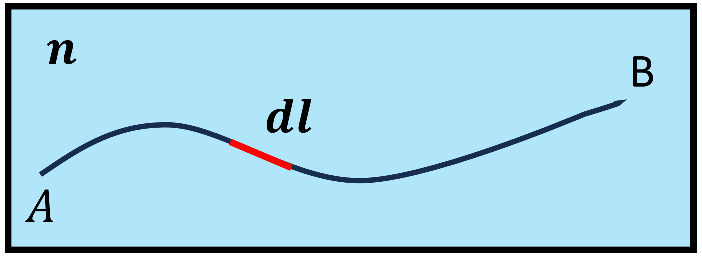
- The path that minimizes the time between A and B corresponds to the segment [AB]
- The time taken by light to go from A to B is equal to the length l of the path divided by the velocity v of light: $\boldsymbol{\tau = \frac{l}{v}}$ we have: $v = \frac{C}{n}$ $\boxed{\Rightarrow \boldsymbol{\tau = \frac{nl}{C}}}$

❑ ***In two transparent, homogeneous and isotropic medium:***



$$\boldsymbol{\tau = \tau_1 + \tau_2} \quad \Rightarrow \boldsymbol{\tau = \frac{l_1}{v_1} + \frac{l_2}{v_2}} \quad \boxed{\Rightarrow \boldsymbol{\tau = \frac{n_1 l_1}{C} + \frac{n_2 l_2}{C}}} \quad \boxed{= \boldsymbol{\frac{n_1 l_1 + n_2 l_2}{C}}}$$

❑ **Generalization:** For a medium of variable index n with light path AB.



For a small light path dl, The duration is written:

$$d\tau = \frac{dl}{v} = \frac{ndl}{C} \quad \Rightarrow \boldsymbol{\tau = \int d\tau = \frac{1}{C}\int_{AB} ndl}$$

- Let's imagine a lifeguard on a beach, and a swimmer in the water, drowning.
- The edge of the beach (passage from the beach to the water) is the diopter.

Which path the lifeguard will take to save the swimmer, knowing that this path must be the fastest????

- Run along the beach until he is opposite the swimmer, then swim straight toward him. It is the blue path
- Run straight ahead to get into the water as quickly as possible. This is the black path.
- Go straight toward the swimmer, along the green path

In fact, the fastest path will be the red one, which optimizes travel time.

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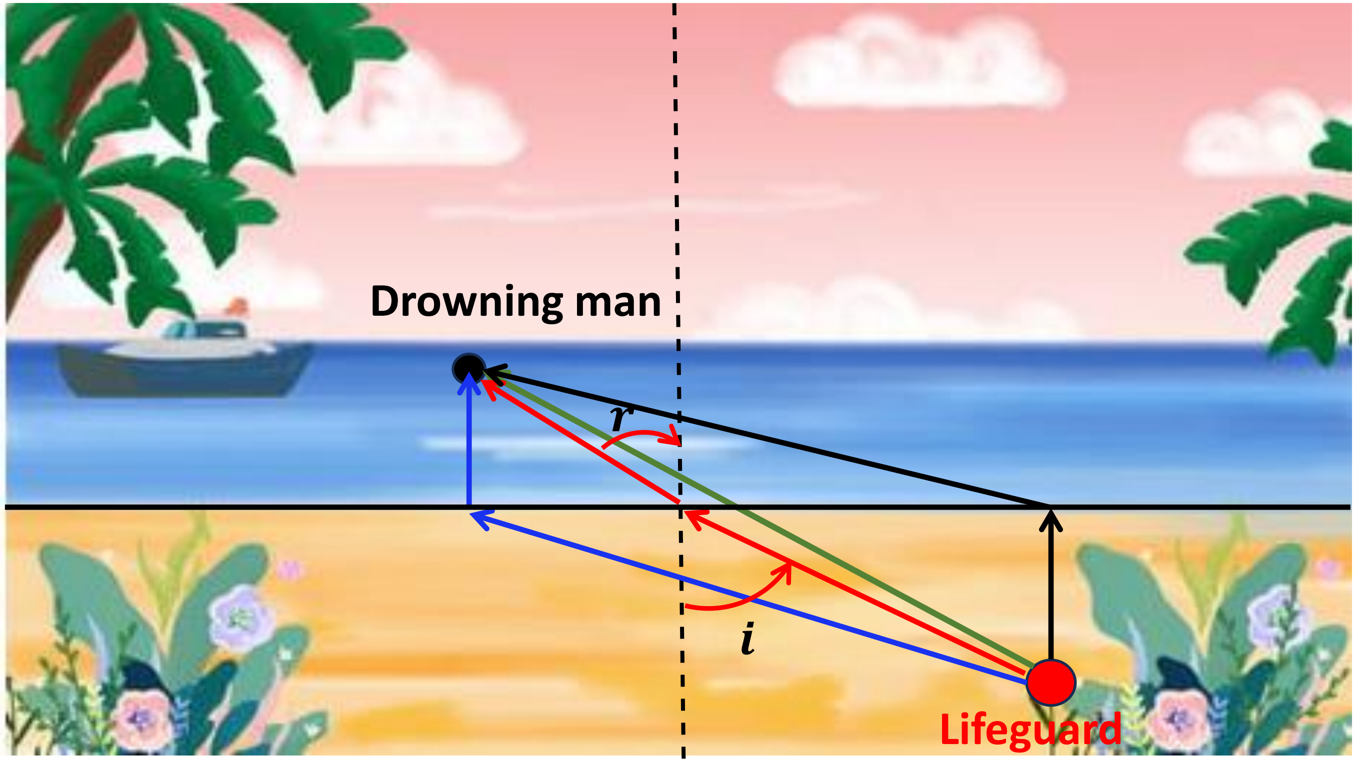
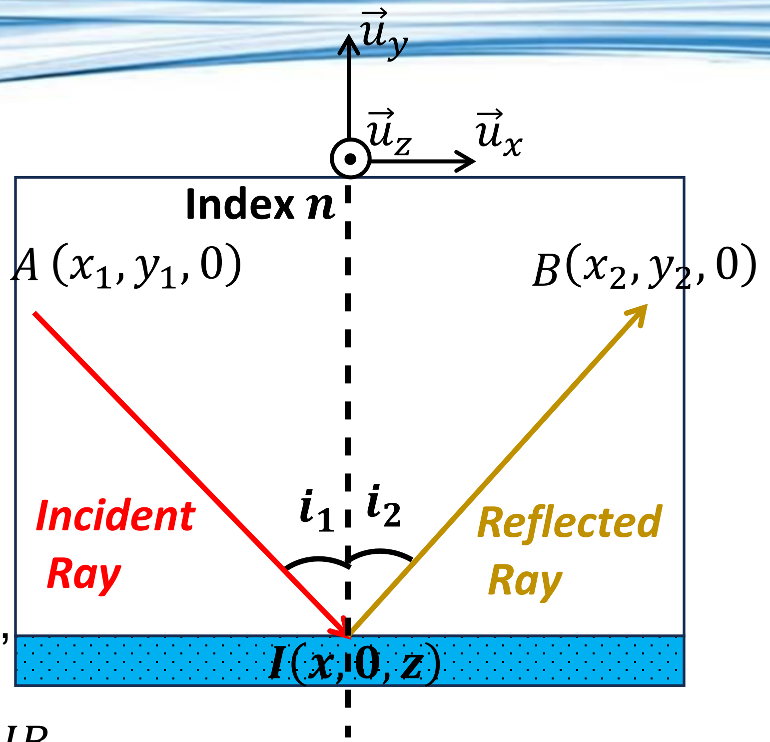


❑ Reflection on a plane mirror:

- Consider a light ray from source A propagating in a homogeneous medium towards a plane mirror.
- After reflection on it, the ray reaches point B.



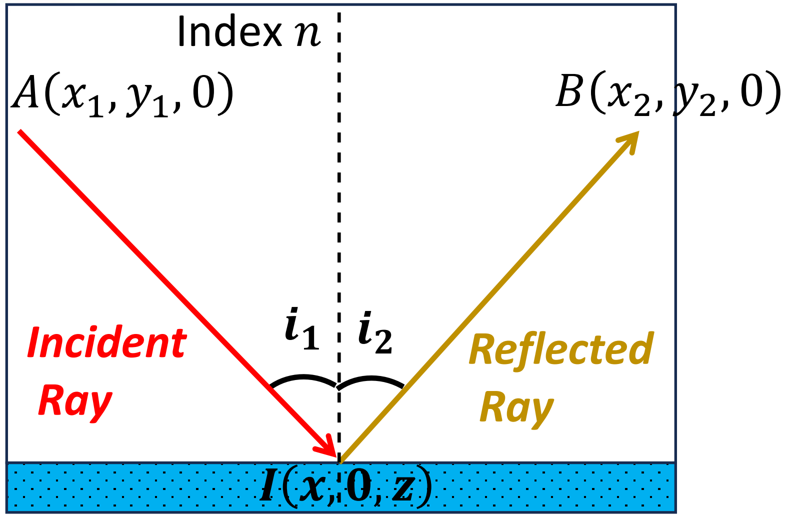
Let us find out what path the light should follow????

- For this, the plane mirror is represented by the xOz plane.
- Let A and B be placed in the xOy plane, with coordinates $(x_1, y_1, 0)$and $(x_2, y_2, 0)$, where $y_1 > 0$and $y_2 > 0$
- Calculate the duration $T_{AB}$of the path of a ray reflected at $I(x, 0, z)$: $T_{AB} = \dfrac{AI + IB}{v_1}$ (v_1 is the speed of light).

$$T_{AB}(x,z) = \frac{1}{v_1}\left(\sqrt{(x-x_1)^2 + {y_1}^2 + z^2} + \sqrt{(x-x_2)^2 + {y_2}^2 + z^2}\right)$$

- $T_{AB}(x, z)$ has a minimum if : $$\begin{cases} \dfrac{\partial T_{AB}(x,z)}{\partial x} = 0 \\ \dfrac{\partial T_{AB}(x,z)}{\partial z} = 0 \end{cases}$$

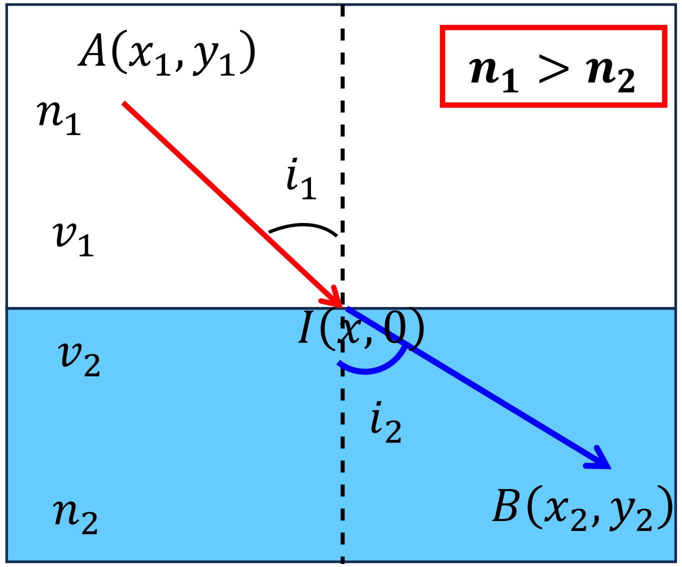
$$\Rightarrow \begin{cases} \dfrac{x-x_1}{\sqrt{(x-x_1)^2+{y_1}^2+z^2}} + \dfrac{x-x_2}{\sqrt{(x-x_2)^2+{y_2}^2+z^2}} = 0 \ldots\ldots(1) \\ \dfrac{z}{\sqrt{(x-x_1)^2+{y_1}^2+z^2}} + \dfrac{z}{\sqrt{(x-x_2)^2+{y_2}^2+z^2}} = 0 \ldots\ldots(2) \end{cases}$$



$(2) = 0 \Rightarrow z = 0 \;\Rightarrow\; A, I$ and B belong to the incidence plan.

$$(1) \Leftrightarrow \frac{x-x_1}{AI} = \frac{x_2-x}{IB} \quad \Leftrightarrow sin(i_1) = sin(i_2) \quad \boxed{\Leftrightarrow i_1 = i_2}$$

❑ Refraction



$$T_{AB}(x) = \frac{AI}{v_1} + \frac{IB}{v_2} \quad = \frac{\sqrt{(x-x_1)^2+{y_1}^2}}{v_1} + \frac{\sqrt{(x-x_2)^2+{y_2}^2}}{v_2}$$

$$\frac{\partial T_{AB}(x)}{\partial x} = 0 \qquad \Rightarrow \frac{1}{v_1}\frac{x-x_1}{\sqrt{(x-x_1)^2+{y_1}^2}} - \frac{1}{v_2}\frac{x_2-x}{\sqrt{(x-x_2)^2+{y_2}^2}} = 0$$

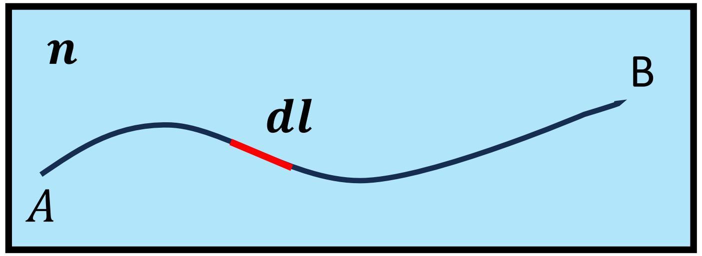
$$\Rightarrow \frac{1}{v_1}\frac{x-x_1}{AI} = \frac{1}{v_2}\frac{x_2-x}{IB} \qquad \Rightarrow \frac{1}{v_1} sin(i_1) = \frac{1}{v_2} sin(i_1)$$

With $v = \frac{c}{n}$, We get $\boxed{n_1 sin(i_1) = n_2 sin(i_2)}$

❑ The notion of the optical path:

The optical path corresponds to the distance that light would travel in a vacuum during the same time it takes to pass through a given medium.

- For a given path AB, the time taken by light to travel along it is: $\Delta t = \int_{t_A}^{t_B} dt$



We have: $dl = vdt$

and we know that: $v = \frac{c}{n}$

$$\Rightarrow dt = \frac{1}{c} ndl \qquad \Rightarrow \Delta t = \frac{1}{c}\int_{t_A}^{t_B} ndl$$

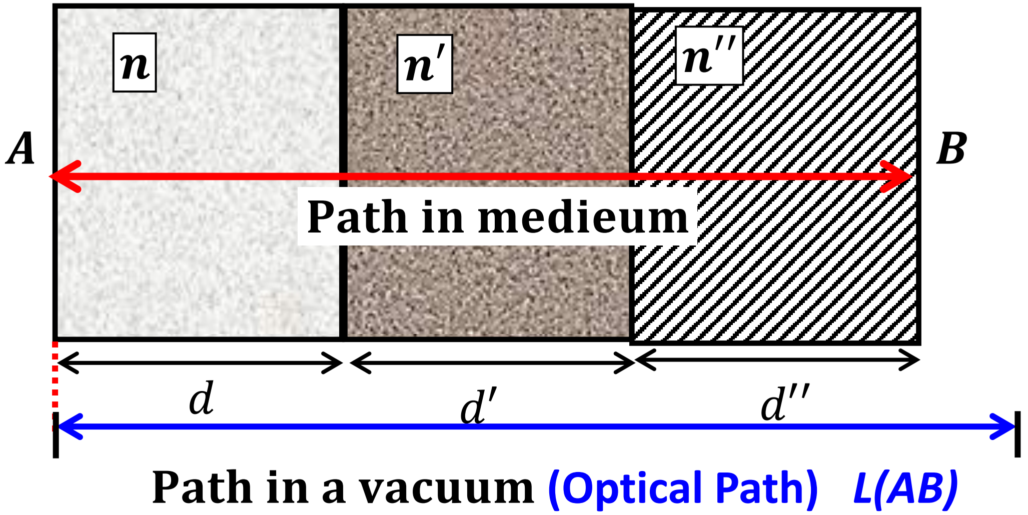
The optical path is therefore by definition(***distance that light would travel in a vacuum during the same time):***

$$L_{AB} = c\Delta t = \int_A^B ndl$$

Remark:

If a light ray travels through a series of optical media of constant thickness ***d, d', d", …*** and refractive indices ***n, n', n", … ,*** the total optical path is just the sum of the separate values:

$$L(AB) = nd + n'd' + n'd'' + \cdots$$



Law of rectilinear propagation of light:

- In the general case, the optical path is written: $\boldsymbol{L_{AB} = \int_{AB} n dl}$
- In a transparent, homogeneous and isotropic medium, the index n is uniform and constant, therefore:

$$\boldsymbol{L_{AB} = n\int_{AB} dl}$$

$\Longrightarrow$ So the optical path depends only on the length of the path

- Minimizing the optical path is the same as minimizing the length of the path, and we know that the smallest distance between two points is the straight line. $\Longrightarrow \boldsymbol{L_{AB} = nAB}$ $\Longrightarrow$ ***the law of rectilinear propagation of light***

Law of Reversibility of Light.

On a path AB, we have: $L_{AB} = \int_A^B ndl$

Let's calculate the optical path on the path BA: $L_{BA} = \int_B^A ndl = -\int_A^B ndl = -L_{AB}$

So the path followed by light between two points A and B is independent of the direction of propagation of light between these two points,

hence the law of Reversibility of Light : $\boldsymbol{L_{AB} = -L_{BA}}$

Construction of Huygens:

The aim is to find the Snell-Descartes law of refraction ($n_1 \sin i = n_2 \sin r$)

- We draw circles whose radii R_1 and R_2 are inversely proportional to the refractive indices n_1 and n_2 : $R_1 = \mathrm{k}/n_1$ et $R_2 = \mathrm{k}/n_2$
- The extension of the incident ray in the second medium intersects the circle of radius R_1 at a point A. ***The tangent of this circle intersects the interface at B***
- The line passing through B is tangent to the circle of radius R_2 at a point C,

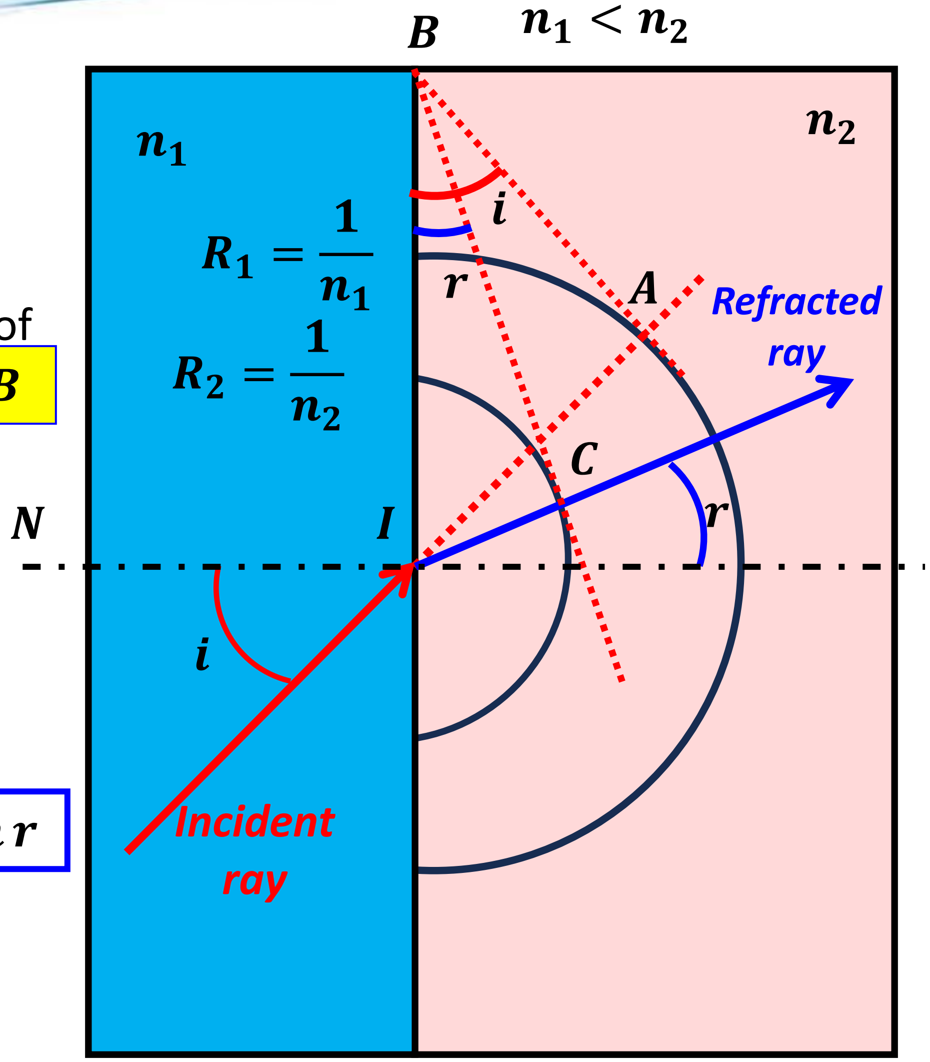
  $\Longrightarrow$The refracted ray then points to the point C.
- From the triangles BCI and BAI we can easily find the Snell-Descartes law:

$$\sin i = \frac{AI}{BI} \Longrightarrow BI = \frac{AI}{\sin i} = \frac{R1}{\sin i}$$

$$\sin r = \frac{CI}{BI} \Longrightarrow BI = \frac{CI}{\sin r} = \frac{R2}{\sin r}$$

$$\Longrightarrow \frac{R1}{\sin i} = \frac{R2}{\sin r}$$

$$\boxed{\Longrightarrow n_1 \sin i = n_2 \sin r}$$



Remark: Study the case where $n_1 > n_2$?

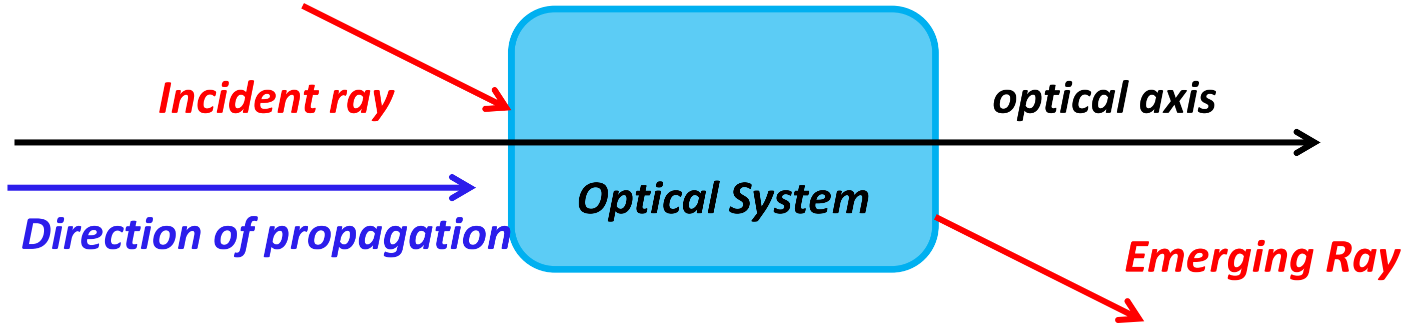
I.5.Mirrors

This part is devoted to the study of plane and spherical mirrors. We show how these systems allow the formation of images.

1.5.1. Generalities about optical systems:

Définition: An optical system is generally made up of diopters (separation surfaces of 2 media of different indices such as lenses, prism.....) and mirrors.

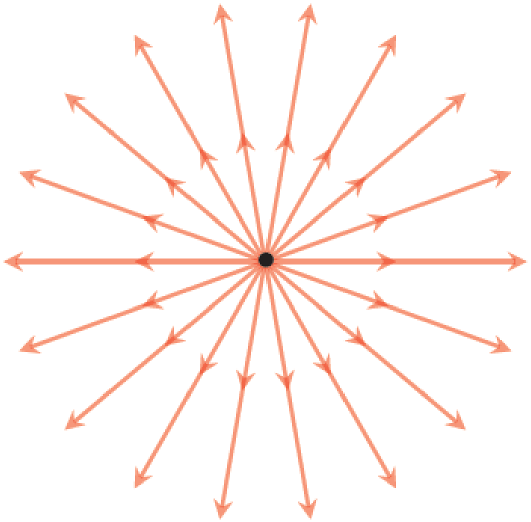
- An optical system is said to be ***dioptric*** if it consists only of diopters (lentilles, lunettes. Microscope) (e.g. Eye)
- An optical system is said to be a ***catadioptric*** if it contains at least one mirror. (e.g. Telescope)
- An optical system is said to be ***centered*** *(called Centered Optical System)* if it admits an axis of symmetry of revolution (microscope, camera lens, ...). This axis is called the ***optical axis***
- This system transforms an incident light ray into a ray emerging in a different direction from the incident direction.



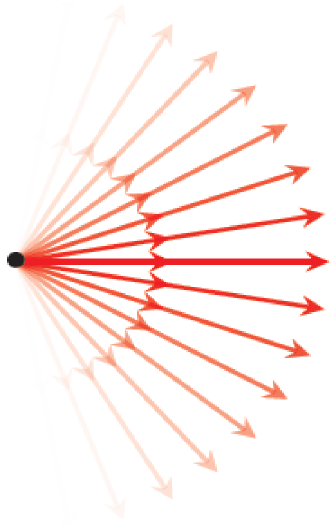
Types of Light sources

- We usually distinguish between ***primary sources*** (autonomous source of light such as the sun, a lamp, a flame, etc.) and ***secondary sources*** which return light by reflection, diffraction or diffusion (the moon, mirrors, etc.)

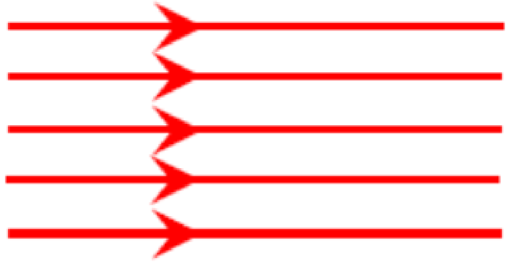
- A light source can be decomposed into an infinite number of point sources emitting light rays in all directions in space..



Isotropic point source



Anisotropic point source

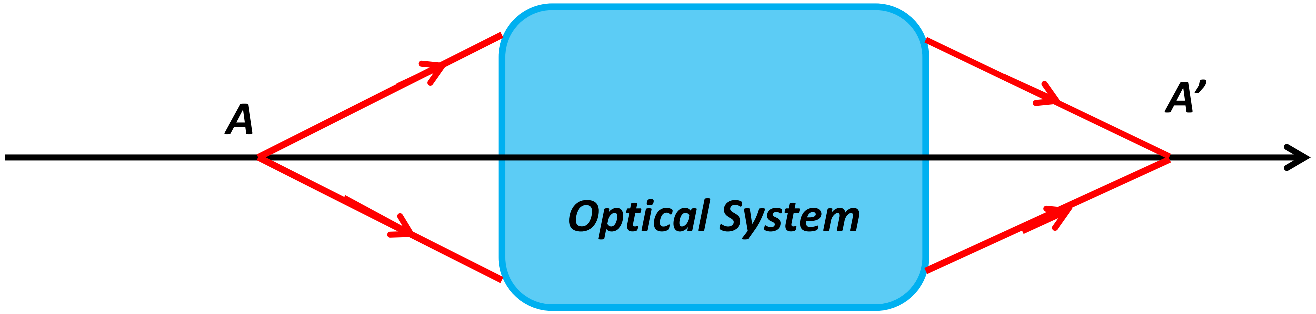


Infinite point source

❑ Stigmatism :

❖ Definitions:

- We call ***"object"*** the source of light rays whose propagation is studied through a given optical system.
- ***A'*** is said to be the image of a luminous point ***A*** through an optical system, if any incident ray of ***A*** converges to ***A'***.



- We say that the points ***A*** and ***A'*** are conjugated with respect to the optical system.

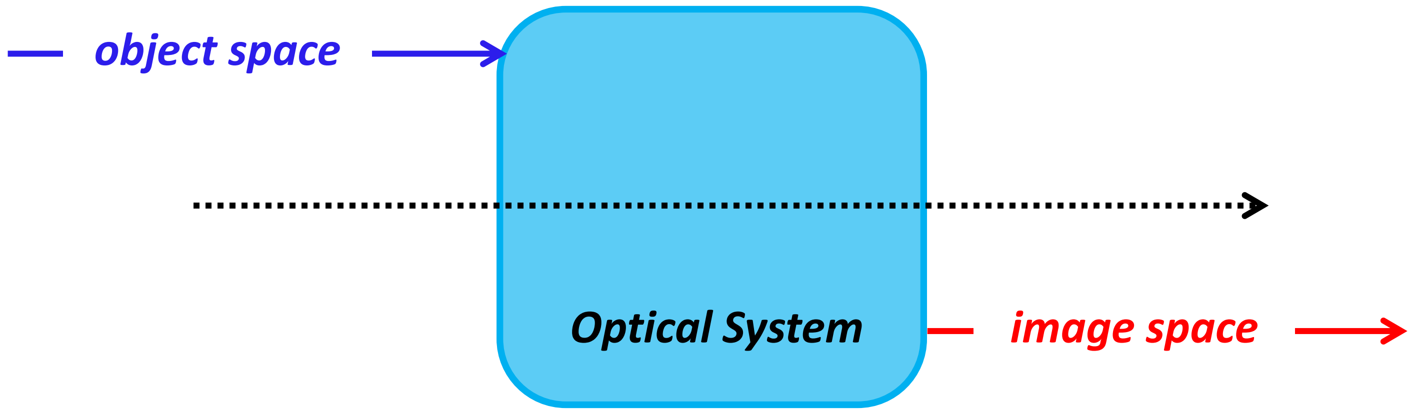
We say that An optical system is strictly stigmatic if all the rays emitted by (object point) A converge at a single point A' (image point), after having passed through the optical system.

➢ Object and image spaces:

Around an optical system, two spaces are organized: object space and image space.

1- Object space is the region of space located before the input face of the system

*Any object located in this space is **a real object**.*



2- Image space is the region of space located after the system output face

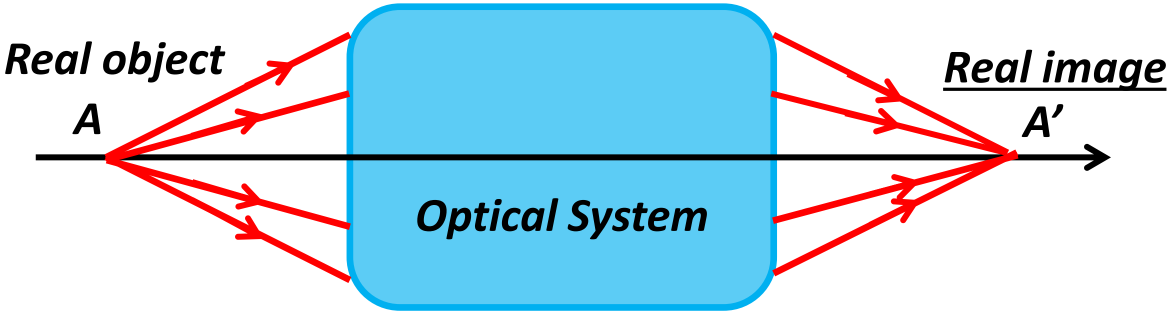
*Any object located in this space is a **virtual object**.*

❑ Nature of the object and the image

❖ ***Consider a source point A sending light rays onto an optical system:***

✓ We will say that A is ***a real point object***(**Real Object)**(**located in the Object space)**.

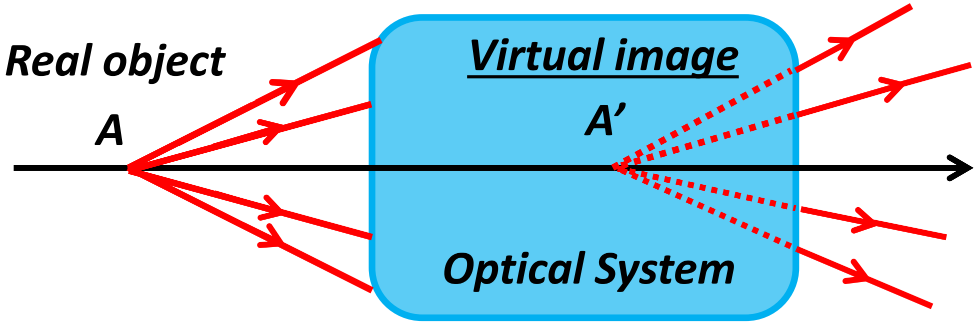
➢ ***Two cases are possible for the image:***



1. The emerging rays converge at a point A' located in the image space
   ✓ We say that A' is ***a real image*** (**A' est une image réelle)**.

> ➢ ***The real image is formed by the meeting of the emerging rays.***

2. If the rays coming from A do not converge at ***A'.***



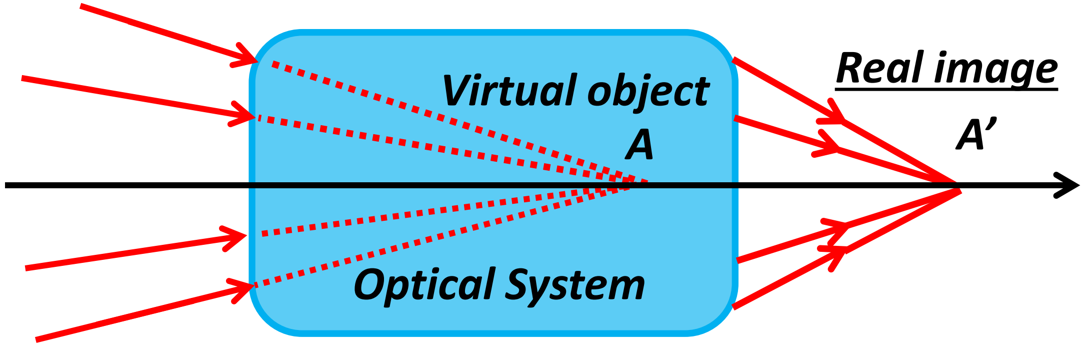
➢ The emerging rays seem to come from a point ***A'*** (their extensions intersect at **A'**).

✓ We say that A' ***is a virtual image*** **(image virtuelle).**

> ➢ ***The virtual image is formed by the extension of the emerging rays.***

❖ ***Consider that the incident rays do not come from a source point A to the optical system***

- It is also possible to create ***a virtual object point A*** by converging the incident ray extensions to the optical system

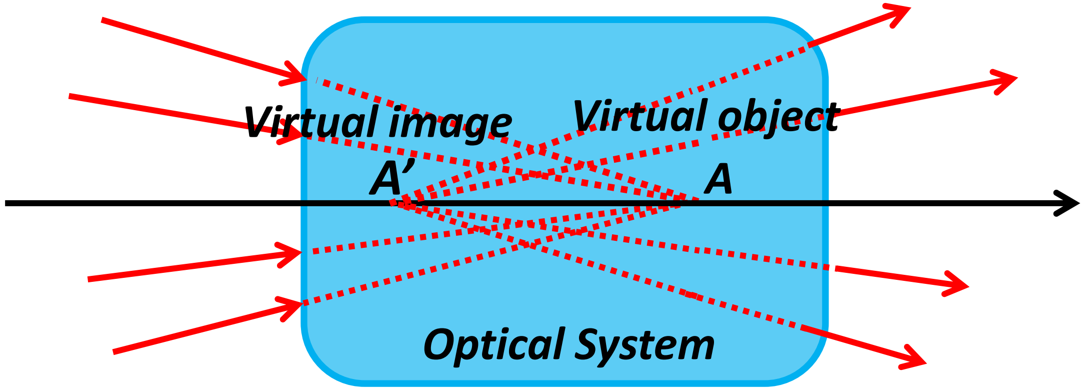


- The image of this virtual object point A can be, according to the same principles :

A real image point

Or

A virtual image point

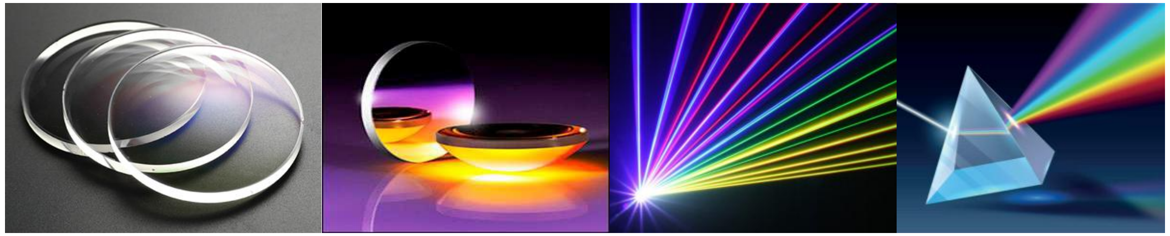


I.4.Mirrors

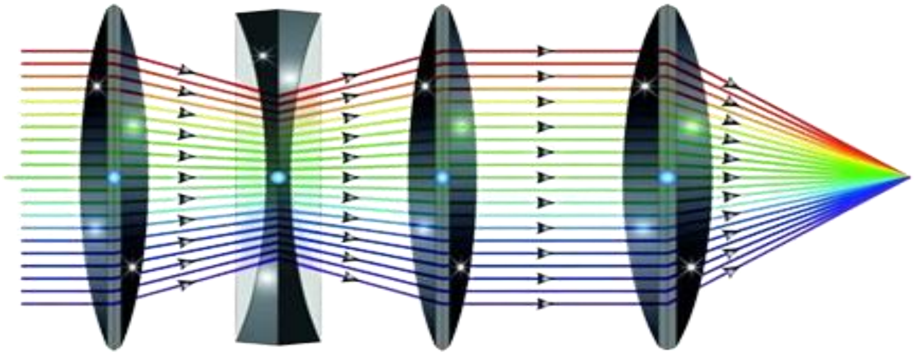
This part is devoted to the study of plane and spherical mirrors. We show how these systems allow the formation of images.

1.5.1. Generalities about optical systems:

Définition: An **optical system** consists of a succession of elements, which may include: lenses, mirrors, light sources, detectors, reflecting prisms, ……



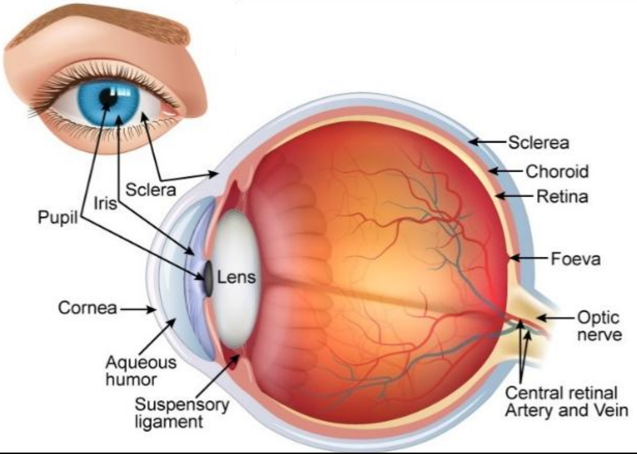
…



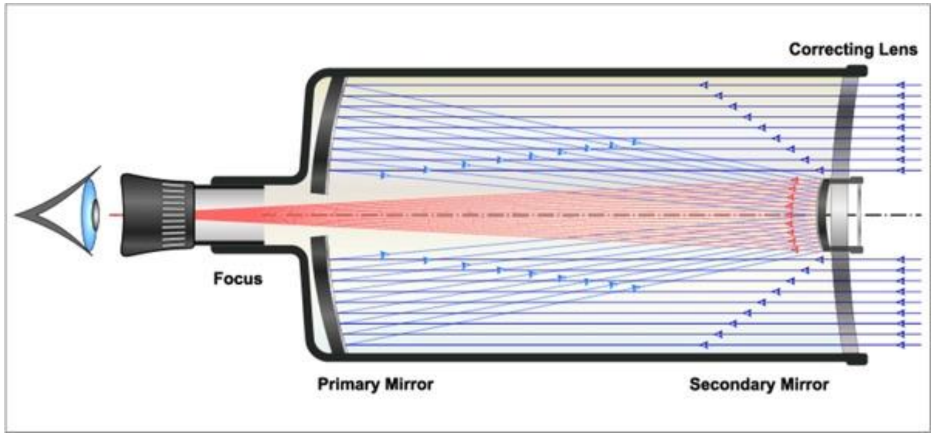
Optical system

In other words, an optical system is any element capable of modifying the path of light.

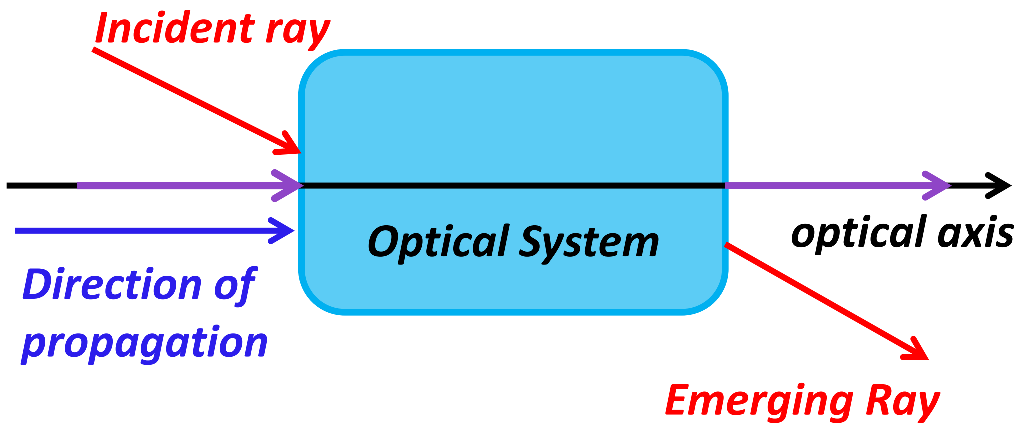
- An optical system is said to be ***dioptric*** if it consists only of diopters (lenses, glasses. Microscope) (e.g. Eye)



- An optical system is said to be a ***catadioptric*** if it contains at least one mirror. (e.g. Telescope)

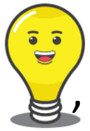


- An optical system is said to be ***centered*** ***<u>(called Centered Optical System)</u>*** if it admits an axis of symmetry of revolution (microscope, camera lens, ...). This axis is called the *optical axis*

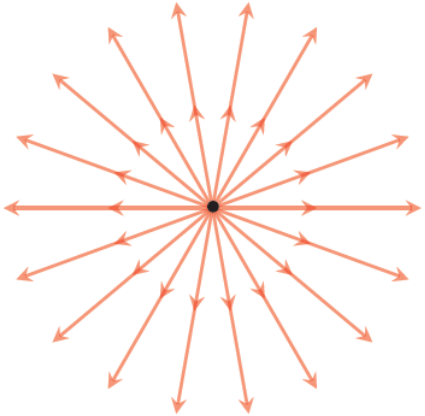


- This system transforms an incident light ray into a ray emerging in a different direction from the incident direction.
- The rays that arrive along the optical axis are not deflected by the optical system

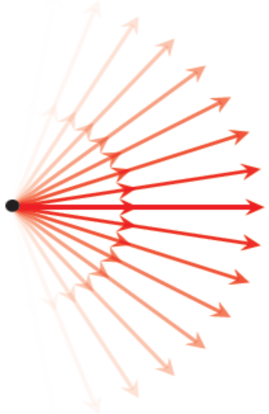
Types of Light sources

- We usually distinguish between:
  - ✓ **Primary sources** (autonomous source of light such as the sun , a lamp , a flame , etc.)
  - ✓ **secondary sources** which return light by reflection, diffraction or diffusion (the moon , mirrors , etc.)
- A light source can be decomposed into an infinite number of point sources emitting light rays in all directions in space……

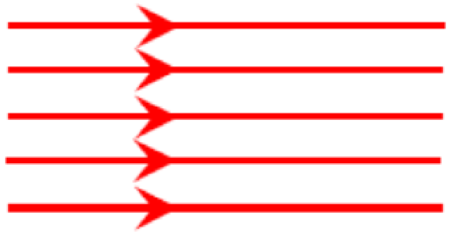
We can distinguish:



Isotropic point source



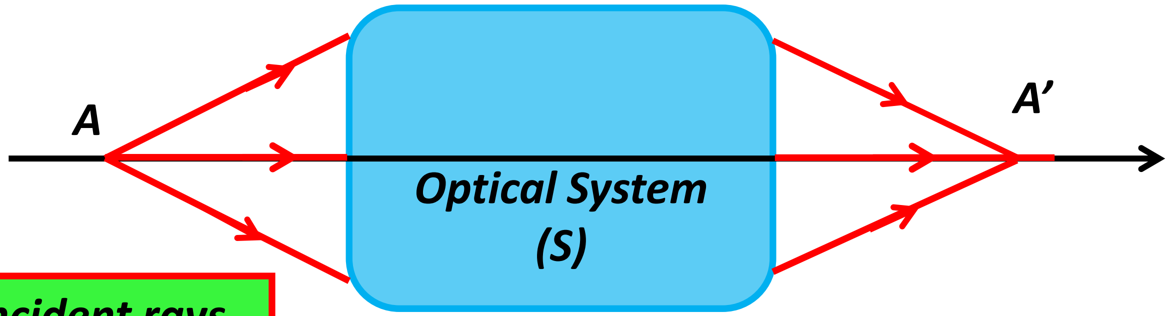
Anisotropic point source



Infinite point source

❑ Stigmatism :

❖ **Definitions:** Let be an optical system (S)



➢ We call ***"object point"*** the source of light rays whose propagation is studied through (S)

An object point is the point of intersection of the incident rays.

➢ ***A'*** is said to be ***the image point*** of an object point ***A*** through an optical system, if any incident ray of ***A*** converges to ***A'***.

An image point is the point of intersection of the emerging rays

➢ We say that the points ***A*** and ***A'*** are conjugated with respect to the optical system.

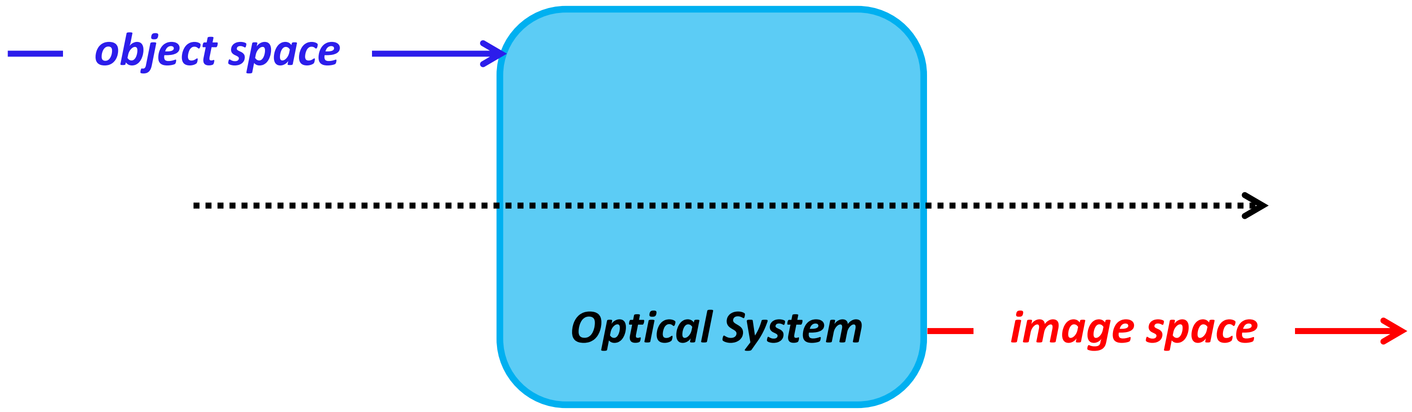
We say that An optical system is <u>rigorously stigmatic</u> if all the rays emitted by (object point) A converge at a single point A' (image point), after having passed through the optical system.

Object and image spaces:

Around an optical system, two spaces are organized: **object space** and **image space**.

1- Object space is the region of space located before the input face of the system

*Any object located in this space is **a real object**.*



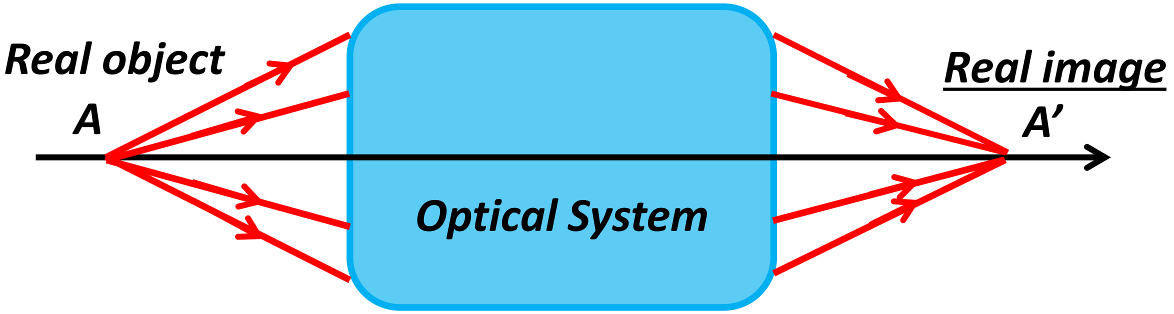
2- Image space is the region of space located after the system output face

*Any object located in this space is a **virtual object**.*

❑ Real, virtual object. Real, virtual image:

❖ ***Consider a source point A sending light rays onto an optical system:***

✓ We will say that A is ***a real point object***(**Real Object)(located in the Object space)**.



➢ ***Two cases are possible for the image:***

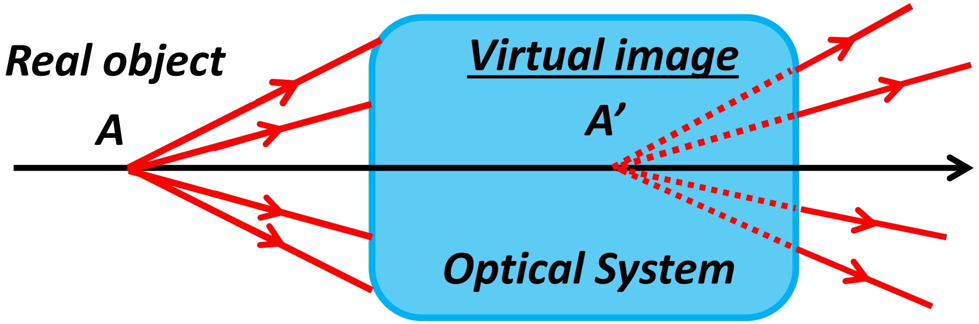
1. The emerging rays converge at a point A’ located in the image space

✓ We say that A' is ***a real image*** (**A’ est une image réelle)**.

> ➢ ***The real image is formed by the intersection of the emerging rays.***

2. If the rays coming from A do not converge at ***A'.***

➢ The emerging rays seem to come from a point ***A'*** (their extensions intersect at **A’**).

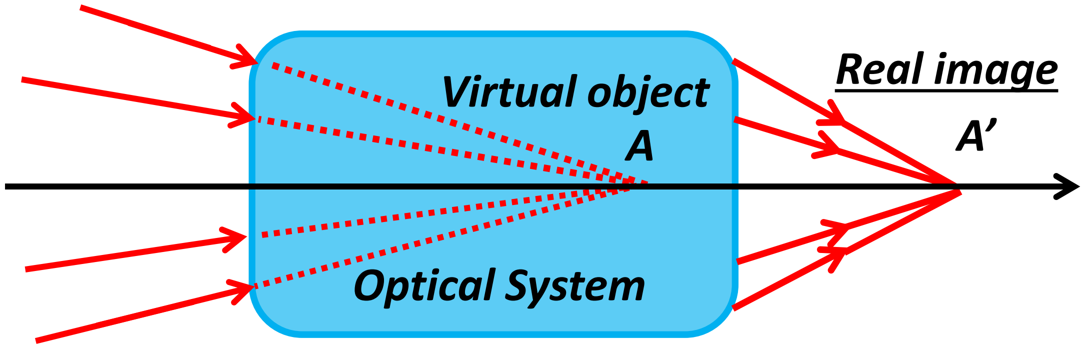


✓ We say that A’ ***is a virtual image*** **(image virtuelle).**

> ➢ ***The virtual image is formed by the extension of the emerging rays.***

❖ ***Consider that the incident rays do not come from a source point A to the optical system***

➢ It is also possible to create ***a virtual object point A*** by converging the incident ray extensions to the optical system

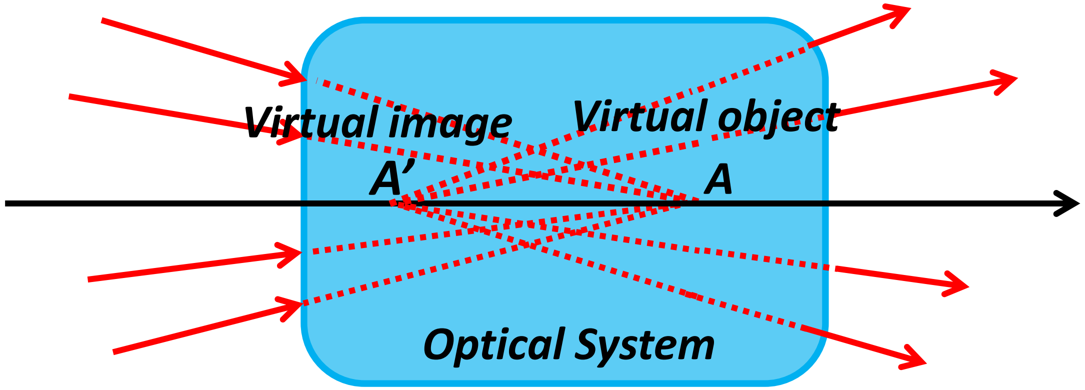


➢ The image of this virtual object point A can be, according to the same principles :

A real image point

Or

A virtual image point

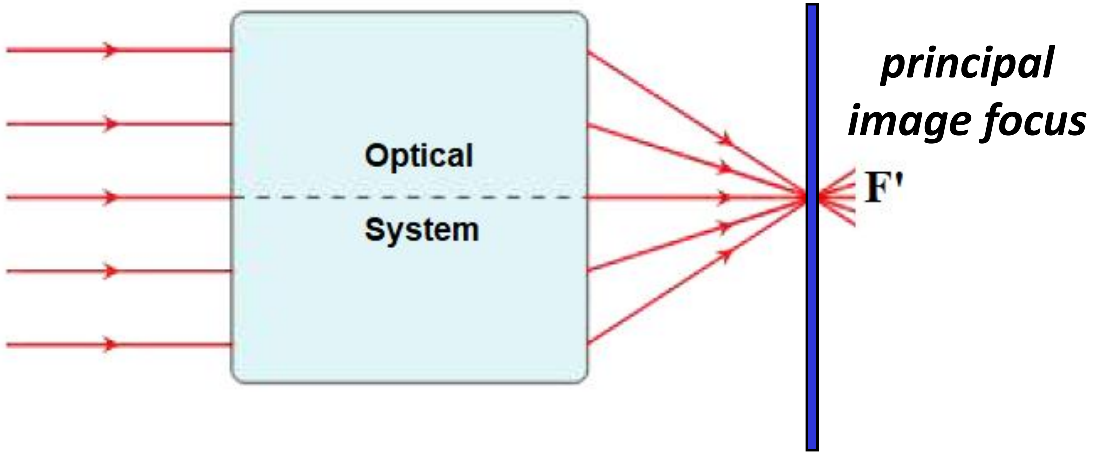


❑ The Foci of an Optical System

The foci of an optical system are special points:

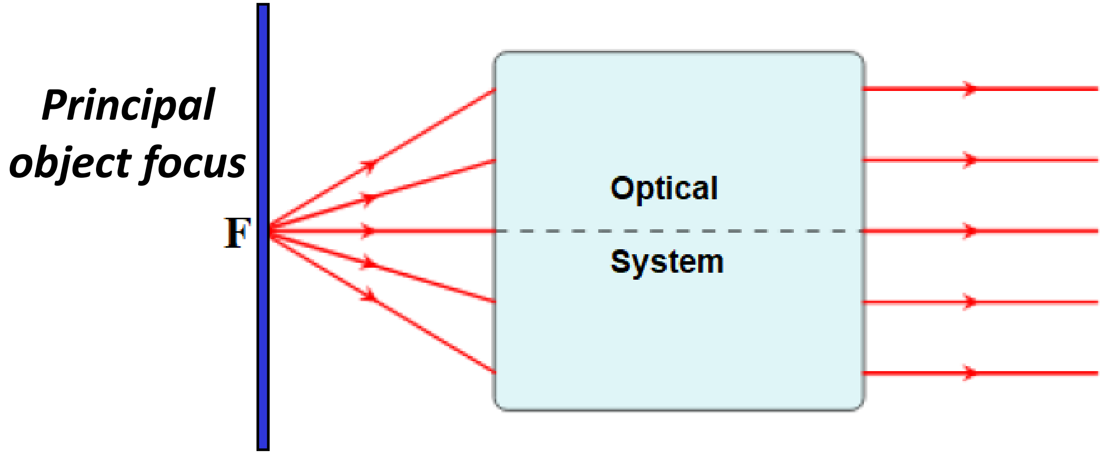
1. ***The principal image focus F'*** is the image point of an object located at infinity, whose rays arrive parallel to the optical system and parallel to its optical axis.

> ➢ *The plane passing through F' and perpendicular to the optical axis of the system is called the* ***image focal plane.***



2. ***The principal object focus F*** is the object point of an image located at infinity, the rays emerge from the optical system parallel to each other and parallel to the optical axis.

> ➢ *The plane passing through F and perpendicular to the optical axis of the system is called the* ***object focal plane.***



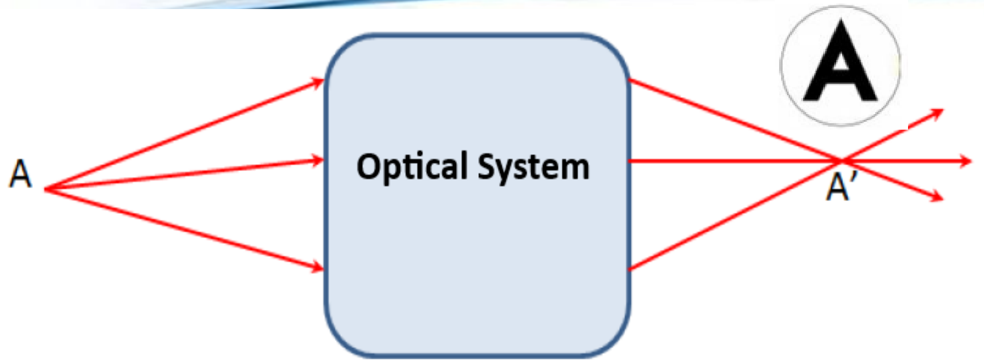
❑ Quality criterion for the formation of an image (stigmatism)

Stigmatism: is a fundamental concept in geometrical optics, as it characterizes the **sharp formation of an image** from a single object point

Rigorous Stigmatism:

An optical system is stigmatic if all the rays emitted by an object point A cross at a single image point A' after their emergence from this system.

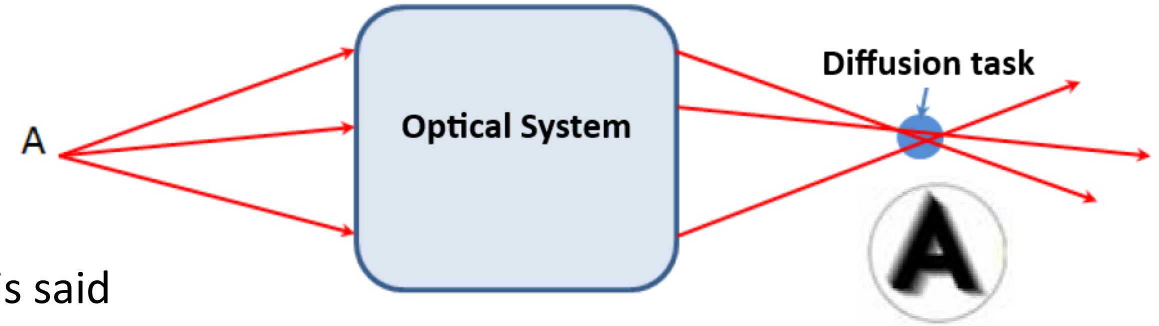
- ✓ The image of an object point is a point.
- ✓ we say that A and A' are conjugated with respect to the optical system



Note: There exists only one optical system that is rigorously stigmatic: ⟹ ***The plane mirror.***

Approximate Stigmatism:

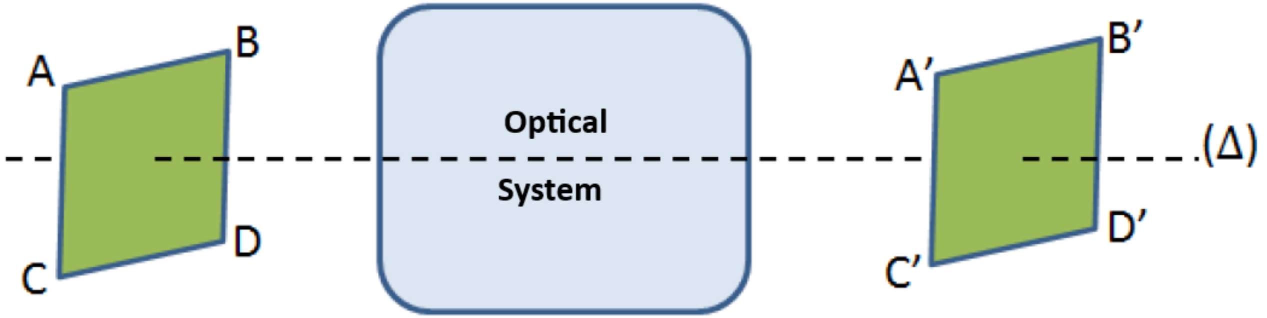
In general, optical systems are not perfectly stigmatic. The image of a point object is not a single point but rather a small spot, called the **diffusion spot** (or **blur spot**).



If this spot is small enough to be considered as a point, the system is said to exhibit **approximate stigmatism**

Aplanatism of Centered Optical Systems

For a stigmatic, centered, optical system with an optical axis (Δ), there is rigorous aplanetism if the image A'B'C'D' of an object ABCD, plane and perpendicular to (Δ), is also plane and perpendicular to (Δ).



Conséquences :

- L'aplanétisme garantit que les images restent **nettes et non déformées** sur tout le plan de l'image.
- Dans la pratique, aucun système n'est parfaitement aplanétique : on parle alors d'**aplanétisme approché**.
- Les **lentilles simples** ne sont généralement **pas aplanétiques**, tandis que certains **systèmes composés** (comme les objectifs photographiques) sont **corrigés pour le rendre quasi aplanétique**.

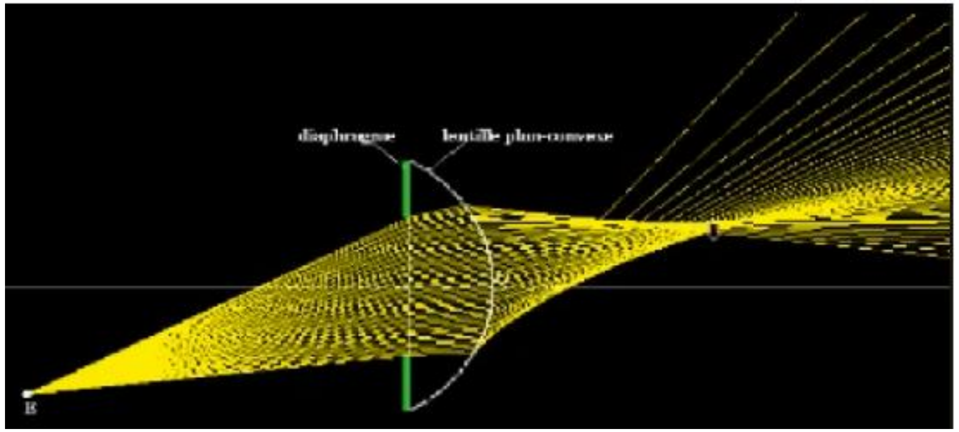
- **Gaussian approximation:**

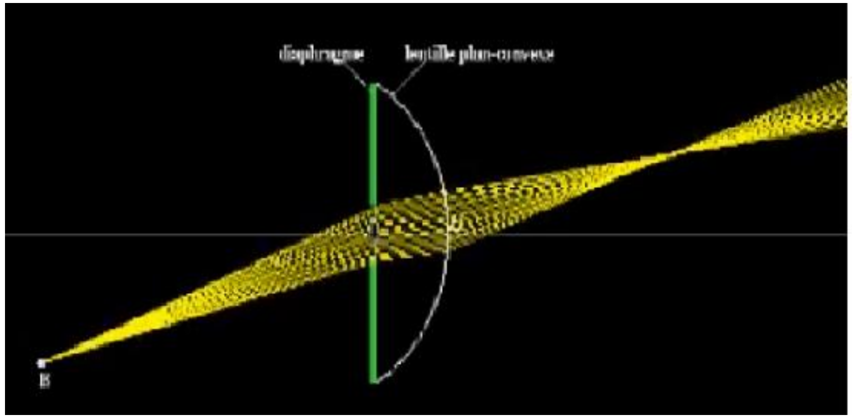
A centered optical system will produce **a high-quality image** of an object if the following two conditions, known as the Gauss conditions, are satisfied:

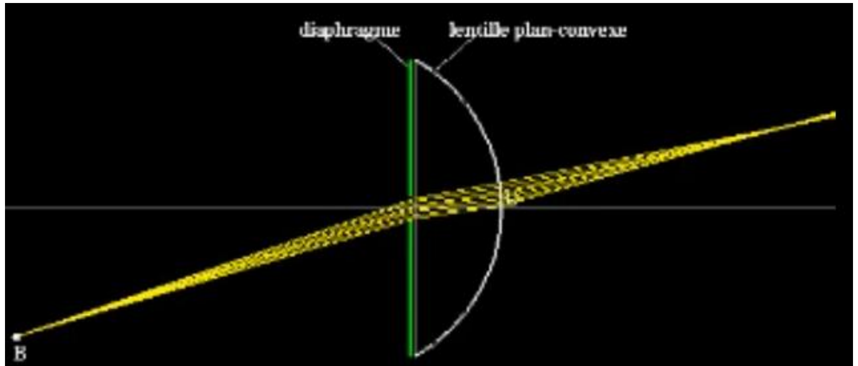
1- The incident rays are slightly inclined with respect to the optical axis:

The incident rays make a small angle with the optical axis ($\sin i \approx i$) $\boldsymbol{n_1 \sin i = n_2 \sin r \;\Longrightarrow n_1 i = n_2 r}$

2- the incident rays are very close to the optical axis



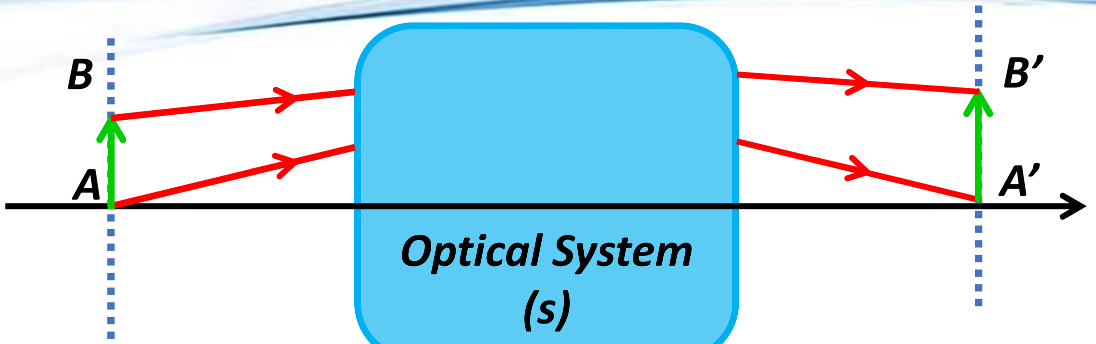




❑ Magnification (Grandissement) :

We distinguish tow type of magnification:

1- Tranverse magnification:



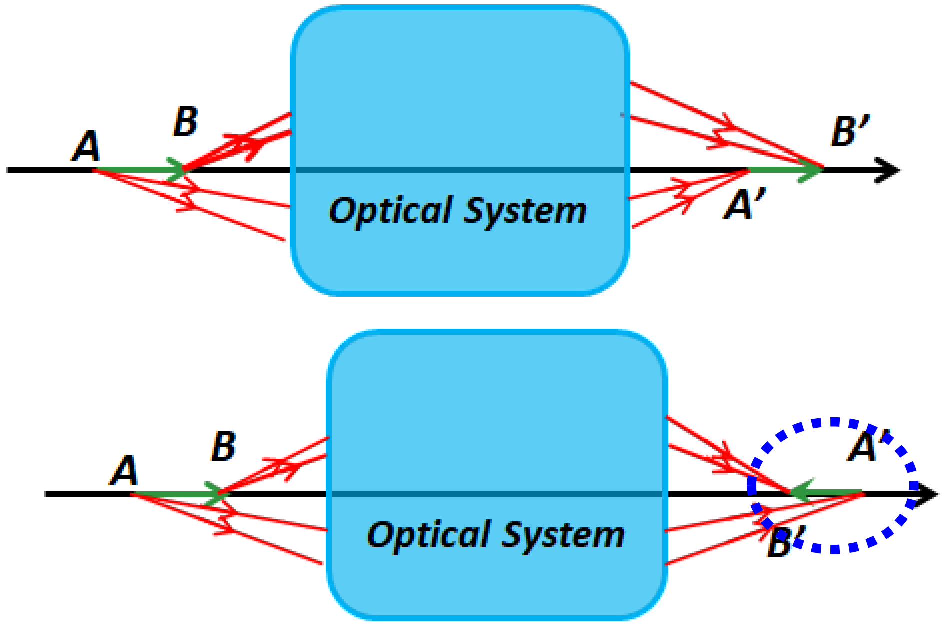
Let A'B' be the image of an object AB through an optical system (S).

The ***Tranverse magnification*** is the ratio of the algebraic values of the transverse dimensions of the image A'B' and the object AB:

$$\gamma_T = \frac{\overline{A'B'}}{\overline{AB}}$$

- If $\gamma<1$ then the image is smaller than the object;
- If $\gamma>1$ then the image is larger than the object;
- If $\gamma<0$ then the image is reversed.

2- Longitudinal magnification:



➢ The object points A and C located on the optical axis and A' and C' the corresponding images.

A longitudinal magnification is defined as: $\gamma_L = \frac{\overline{A'B'}}{\overline{AB}}$

➢ If $\gamma_L < 0$, We say that there is an inversion of the image.

Plane and spherical mirrors

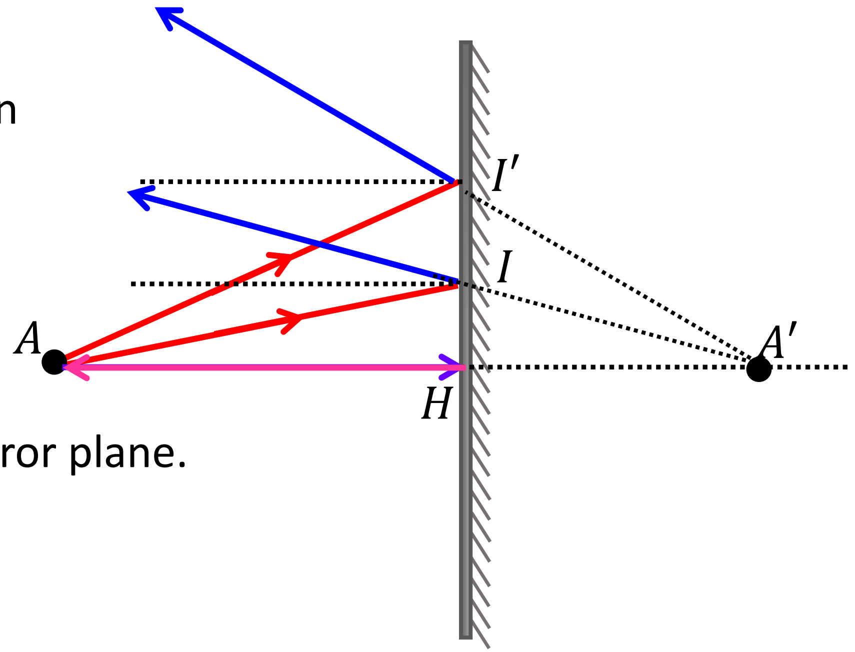
A mirrors is a glass surface covered with a thin metal deposit that make it perfectly reflective

I- Plane mirrors A plane mirror is a rigorously stigmatic optical system: all the rays emitted by the object point A converge at a single point A', called the image point.

Constructing the Image of an Object Point

- Let be a real object A. The rays coming from point Astrike the mirror M.
- The rays are reflected at points I and H by respecting the conditions of reflection
- Their extensions intersect to give a virtual image point A'.
- ***A'*** is a virtual image because it is the point of intersection of the extension of emerging rays.
- the image point A' is the symmetry of the object point A with respect to the mirror plane.
- The conjugation relation of the plane mirror is written: $\overrightarrow{HA} = -\overrightarrow{HA'}$

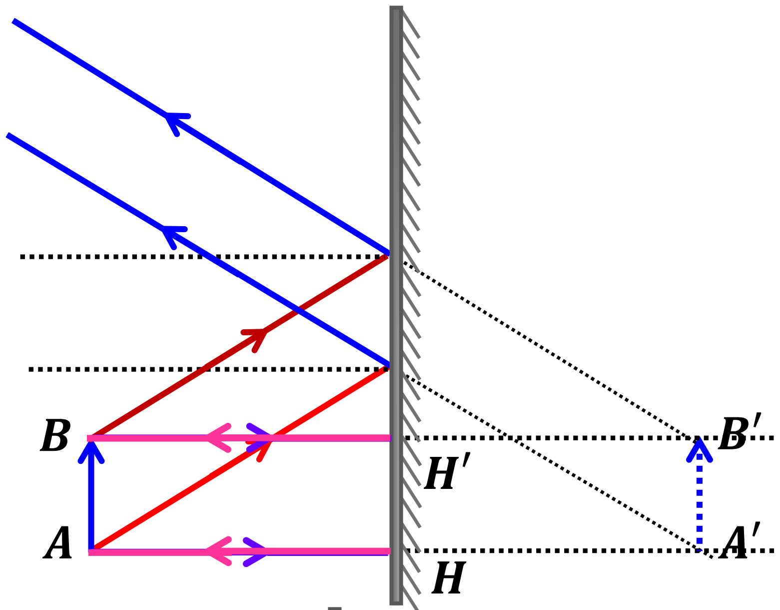
where H is the orthogonal projection of A on the plane mirror.



Applications: **Transverse and longitudinal magnification**

1. ***Construct the image of a transverse segment AB*** (parallel to the mirror).

- H is the orthogonal projection of A on the mirror: Image A' is the orthogonal symmetry of A.
- H' is the orthogonal projection of B on the mirror: Image B' is the orthogonal symmetry of B.



- Calculate the transverse magnification defined by $\gamma_t = A'B'/AB$

$$\gamma_t = \frac{A'B'}{AB} = 1$$

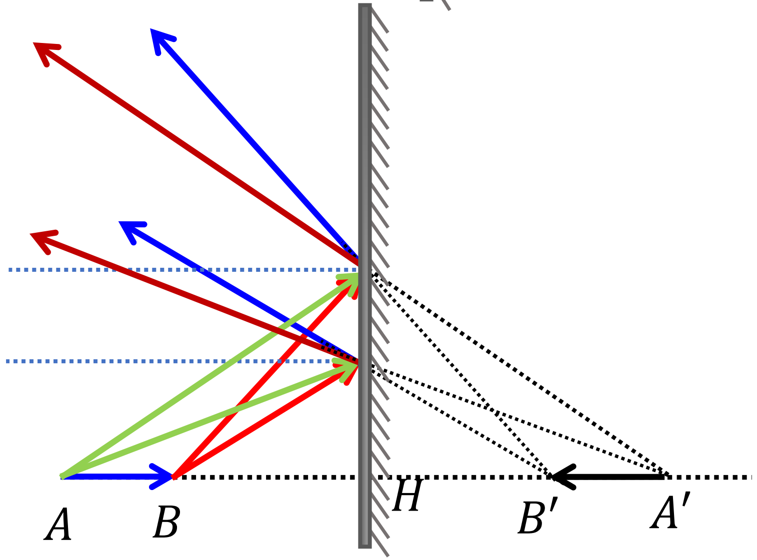
There is a left-right inversion between the object and the image.

2. ***Construct the image of an axial segment AB (perpendicular to the mirror),***

- Calculate the longitudinal magnification defined by $\gamma_L = A'B'/AB$

The image of AB is reversed and no distortion:

$$\overline{AB} = -\overline{A'B'} \quad \Rightarrow \gamma_L = \frac{\overline{A'B'}}{\overline{AB}} = -1$$



Association of two mirrors:

Let A be a real object, its image through the mirror M1 is A1.

We can show that if we rotate the mirror by an angle α (position in M2), the final image rotates by an angle β=2 α.

- We have: $\theta + \alpha = \frac{\pi}{2} \quad \Longrightarrow \theta = \frac{\pi}{2} - \alpha$

- In the yellow triangle, we have:

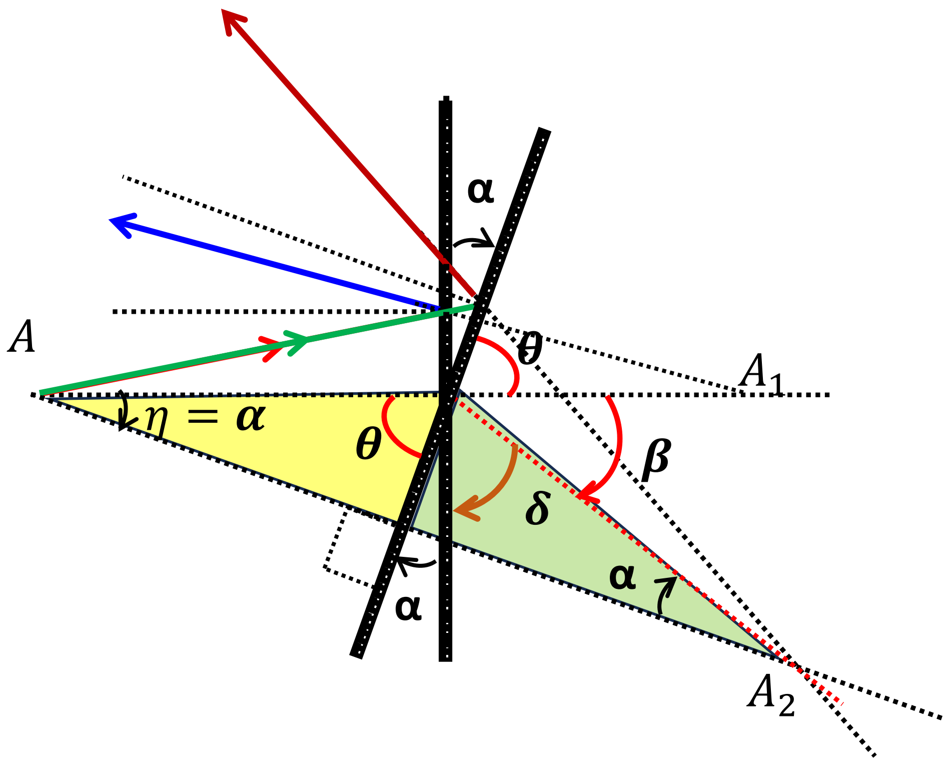
$$\theta + \eta = \frac{\pi}{2} \quad \Longrightarrow \frac{\pi}{2} - \alpha + \eta = \frac{\pi}{2} \quad \Longrightarrow \boldsymbol{\eta = \alpha}$$

- We also have, according to the diagram:

$$\beta + \delta = \frac{\pi}{2}$$

From the green, we have

$$\delta + \alpha + \alpha = \frac{\pi}{2} \quad \boxed{\Longrightarrow \boldsymbol{\beta = 2\alpha}}$$

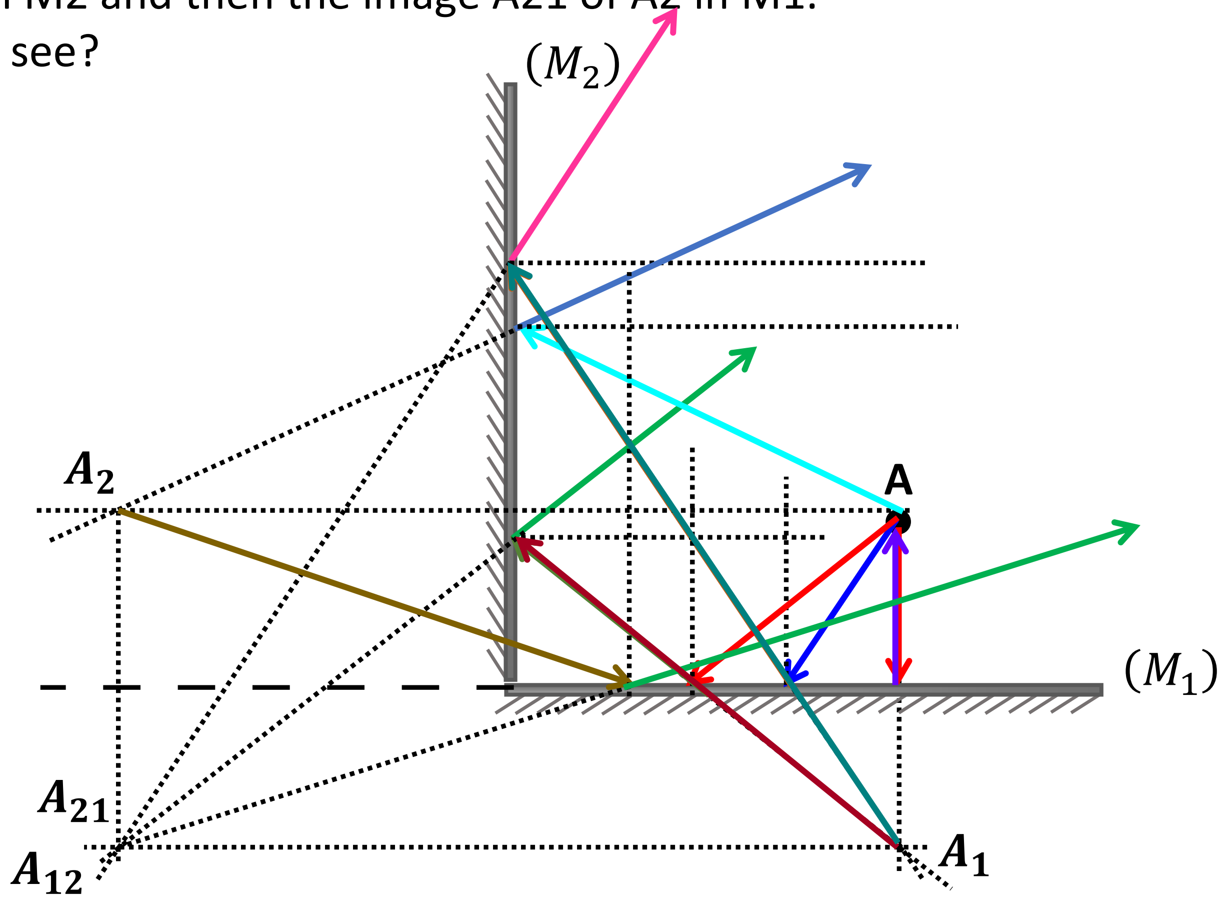


Exercise1:

Two mirrors M1 and M2 are arranged perpendicular to each other, and a point object A is located so that it can be seen simultaneously in these 2 mirrors.

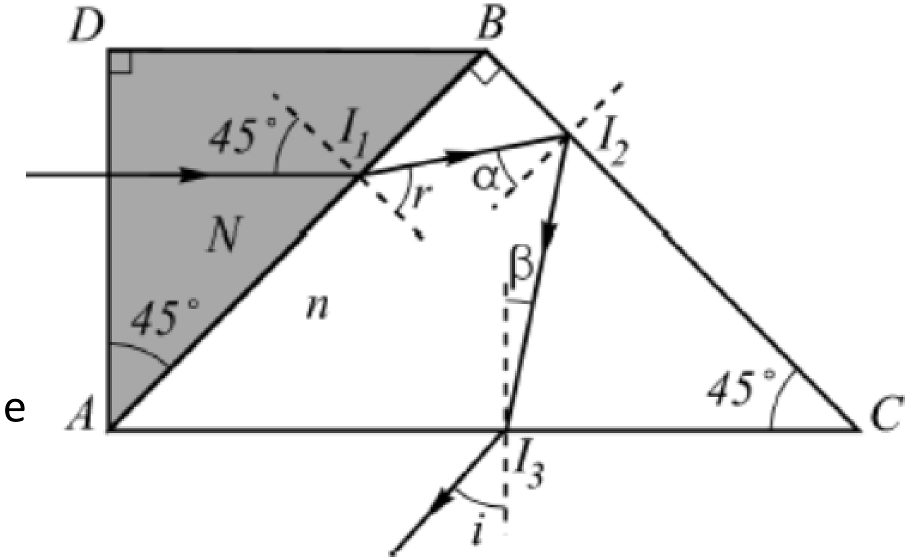
1) Construct the image A1 of A in the mirror M1 and trace a beam of rays from A and reflected by M1.
   - ✓ Can A1 play the role of object in relation to the mirror M2? If so, build his image A12 in M2 and the corresponding rays.
   - ✓ Can the process continue with a new reflection on M1?

2) in the same way, construct the image A2 of A in M2 and then the image A21 of A2 in M1.
   - ✓ Finally, how many images of A can the observer see?



Exercise2:

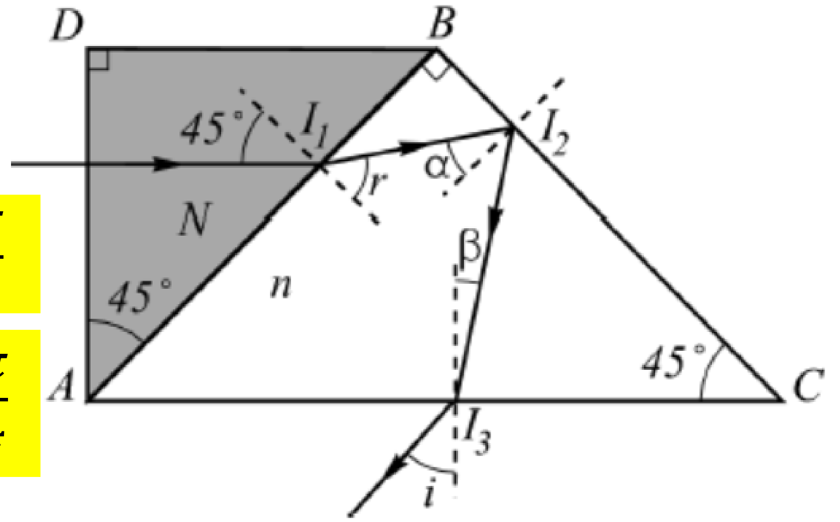
Two pieces of glass cut in the form of right and isosceles triangles with indices N and n respectively have their common AB face. An incident ray hits AD under a normal incidence, refracts at I_1, is reflected at I_2 and then exits at I_3 under the incidence i. The values of N and n are such that the reflection is total in I_2.



1. Write the Snell-Descartes relation at points I_1 and I_3?
2. What relations verify the angles r and α; α and β?
3. What relation do N and n verify for the reflection to be limited in I_2?
   - Calculate N, r, α, β and i for $n = 3/2$ when this limited condition is me
   - This limit value of N is called N0. For the reflection to be total in I_2, does N have to be larger or smaller than N_0?
1. Write the relation verified by N and n so that the angle i is zero. Calculate N?

Solution :

1. At point I_1: $N\frac{\sqrt{2}}{2} = n\sin r$ At point I_3: $n\sin\beta = \sin i$



2. The normal at BC and the normal at AB are perpendicular to each other. In the triangle formed by these normals and I_1I_2, we have : $\boldsymbol{r + \alpha = \frac{\pi}{2}}$

Also, in the triangleI2CI3, we have : $\frac{\pi}{2} - \alpha + \frac{\pi}{2} - \beta + \frac{\pi}{4} = \pi$ $\boldsymbol{\Rightarrow \alpha + \beta = \frac{\pi}{4}}$

3. **for the reflection to be limited in I_2:** The angle of refraction $r' = \frac{\pi}{2}$

$$\Rightarrow n\sin\alpha = 1 \;\Rightarrow n\sin\left(\frac{\pi}{2} - r\right) = 1 \;\Rightarrow n\cos r = 1 \;\Rightarrow n^2cos^2r = 1 \;\Rightarrow n^2(1 - sin^2r) = 1$$

$$\Rightarrow n^2 - n^2sin^2r = 1 \;\Rightarrow n^2 - \frac{N^2}{2} = 1 \;\Rightarrow 2n^2 - N^2 = 2 \;\boldsymbol{\Rightarrow N^2 = 2(n^2 - 1)}$$

for $n = 3/2$: $N = 1.58$; $r = 48.189°$; $\alpha = 41.81°$; $\beta = 3.189°$; $i = 4.787°$

For the reflection to be total in I_2, the angle α must be greater than the angle of incidence for the limit refraction α_0 that we have just calculated. We then have:

$$n\sin\alpha > 1 \;\Rightarrow r < r_0 \;\Rightarrow N < N_0 \;\boldsymbol{\Rightarrow N^2 < 2(n^2 - 1)}$$

4. Writing the relation verified by N and n so that the angle i is zero:

$$n \sin\beta = \sin i \qquad i = 0 \Longrightarrow \beta = 0 \qquad \Longrightarrow \alpha = \frac{\pi}{4} \Longrightarrow r = \frac{\pi}{4}$$

$$\Longrightarrow N\,\frac{\sqrt{2}}{2} = n \sin\frac{\pi}{4} \qquad \Longrightarrow \boldsymbol{N = n = 3/2}$$

II. spherical mirror: A spherical mirror is a type of curved mirror that has a sphere-shaped reflection surface.

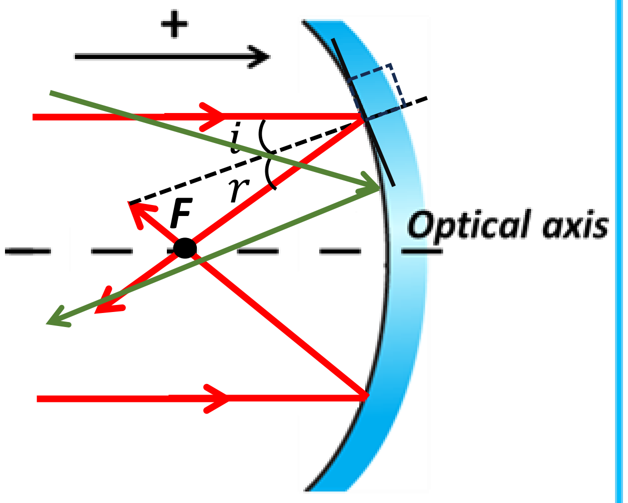
We distinguish tow type of spherical mirrors:

Concave mirrors.

Convex mirrors

1. ***A concave mirror:*** has a reflection surface that curves inwards.

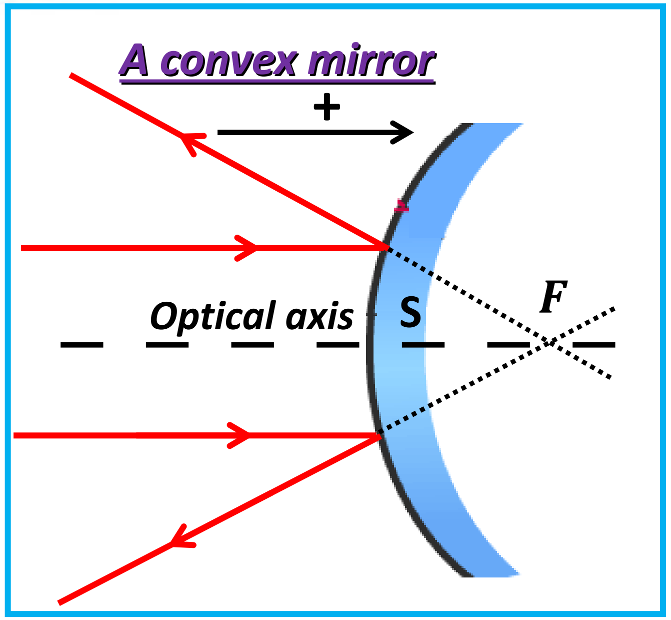
- **Schematization:** A concave mirror is metallized towards the inside of the sphere:



- It is convergent.
- Any ray arriving at the mirror approaches its axis of symmetry.
- When parallel light ray hits this mirror, it is reflected inwards, converging at a point called ***the focus point F***.
- Concave mirrors are often used in telescopes, movie projectors, and car headlights.

2. ***A convex mirror:*** has a reflecting surface that curves outwards.

- **Schematization:** A convex mirror is metallized towards the outside of the sphere:



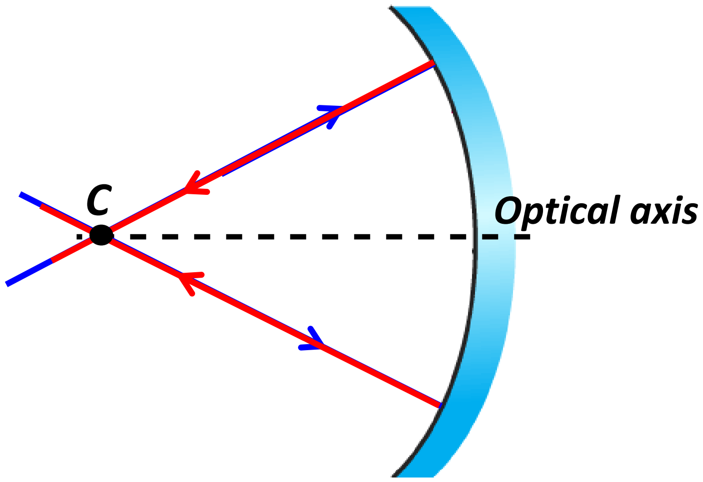
- It is divergent.
- When parallel light ray hits this mirror, it is reflected outward, diverging from a point called ***a virtual focus F***.
- Convex mirrors are often used in car mirrors, retail stores, and intersections to help prevent collisions.

The special points of a spherical mirror:

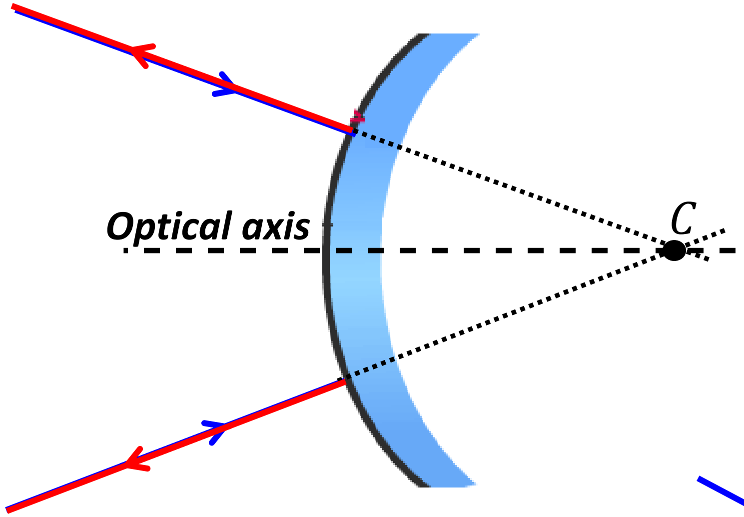
1. The center of the mirror "C":

Any ray passing through C arrives under normal incidence on the mirror and returns on itself after reflection.

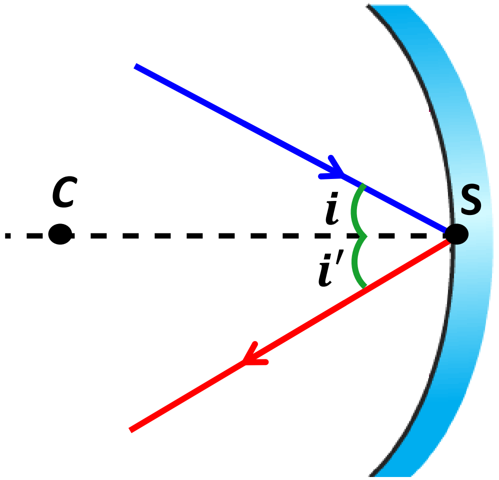
A concave mirror



A convex mirror



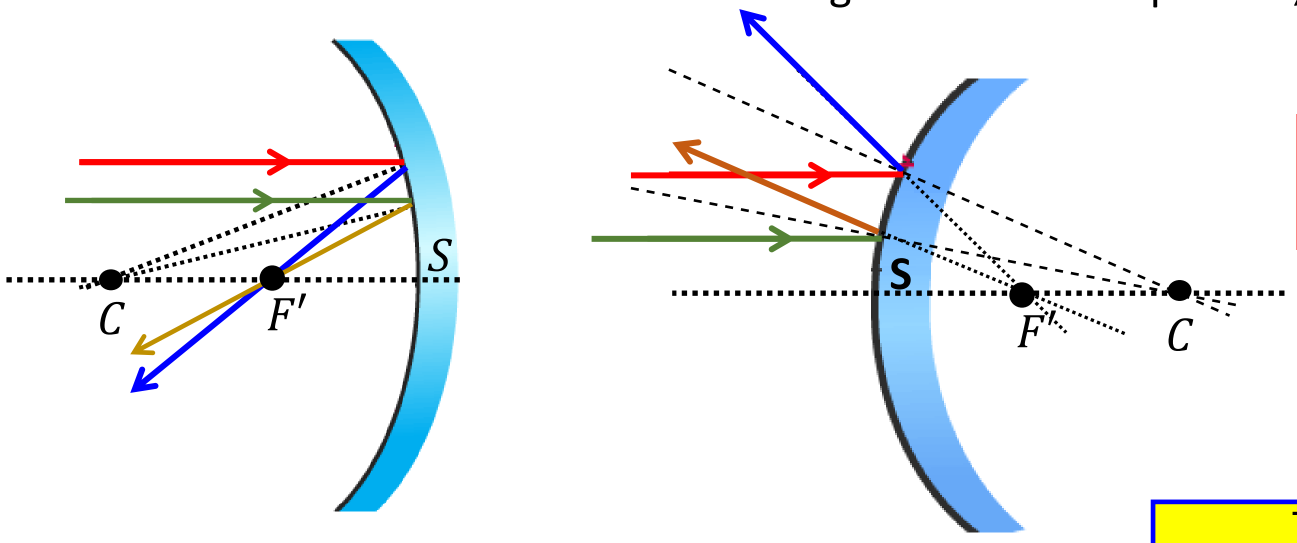
2. The vertex of the mirror "S":



Any ray arriving at S is reflected according to Descartes' laws.

3. *Focal Center System:*

3.1. Principal image focal point: If a beam is sent to the mirror parallel to the optical axis, it is reflected into a beam that converges towards the point F’, the principal image focal point of the mirror.



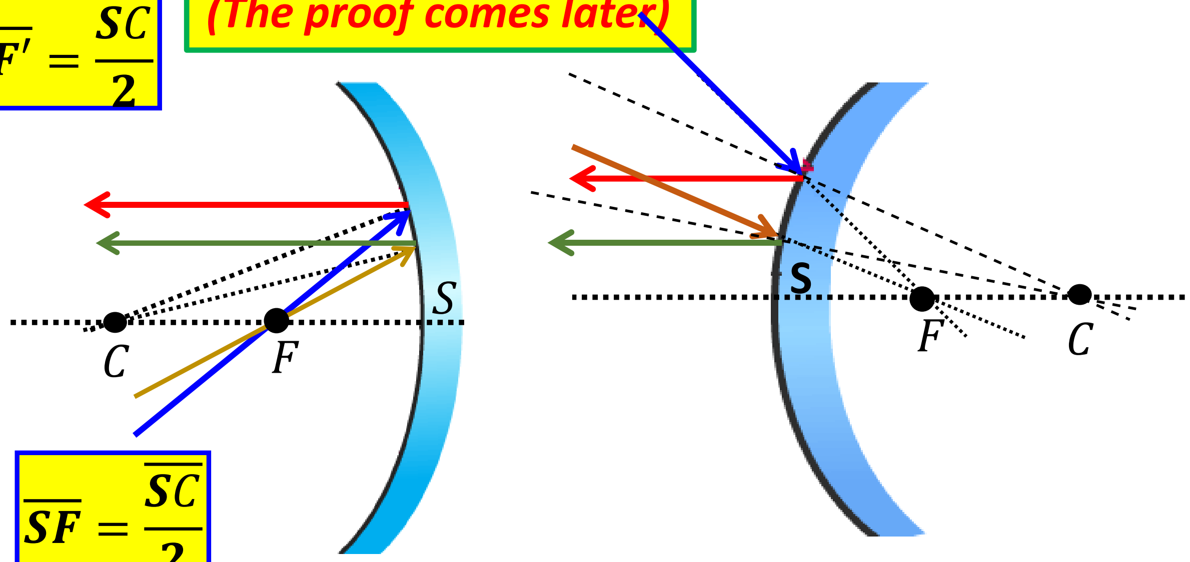
The image focal point is the image of an object point located at infinity

We can show that F’ is the midpoint of the segment CS: $\overline{SF'} = \frac{\overline{SC}}{2}$

(The proof comes later)

3.2. Principal object focal point:

- The object focal point is the point **F** located on the optical axis whose image is located at infinity,
- In this case a beam of light rays, coming from **F**, will emerge from the system in a beam of rays parallel to each other and to the optical axis.



$\overline{SF} = \frac{\overline{SC}}{2}$

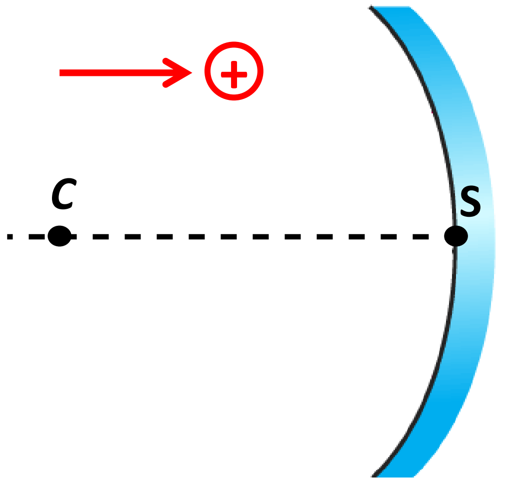
<u>The vergence</u> of a spherical mirror is defined as the inverse of its focal length, called diopter and denoted δ :

$$\delta = \frac{1}{\overline{SF}} = \frac{2}{\overline{SC}}$$

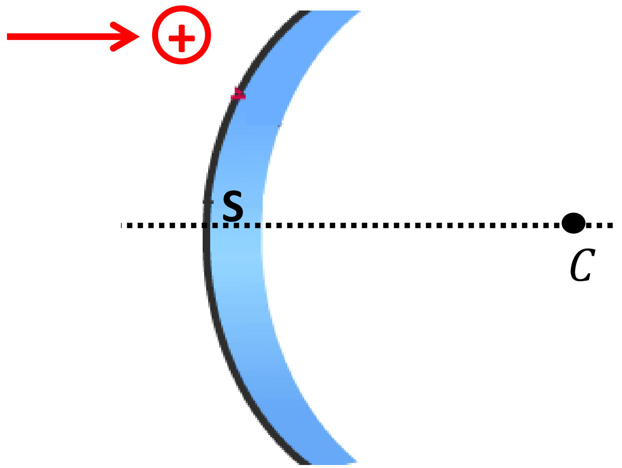
The unit of vergence is therefore the $metre^{-1}$ (m^{-1}) ,

4. *<u>The radius of the sphere mirror (R = SC)</u>:*

The radius of the spherical mirror (called the radius of curvature) is defined by the algebraic measure: $R = \overline{SC}$



$R = \overline{SC} < 0$



$R = \overline{SC} > 0$

This radius of curvature is: <u>negative</u> for a concave mirror and <u>positive</u> for a convex mirror

Approximate stigmatism, Gaussian conditions:

The conditions of approximate stigmatism:

The approximate stigmatism for a spherical mirror depends on two conditions:

- The object observed must be located on the optical axis of the spherical mirror;

  This means that the light rays coming from the object must be parallel to the optical axis before hitting the mirror.

- The light rays must hit the spherical mirror at very small angles;

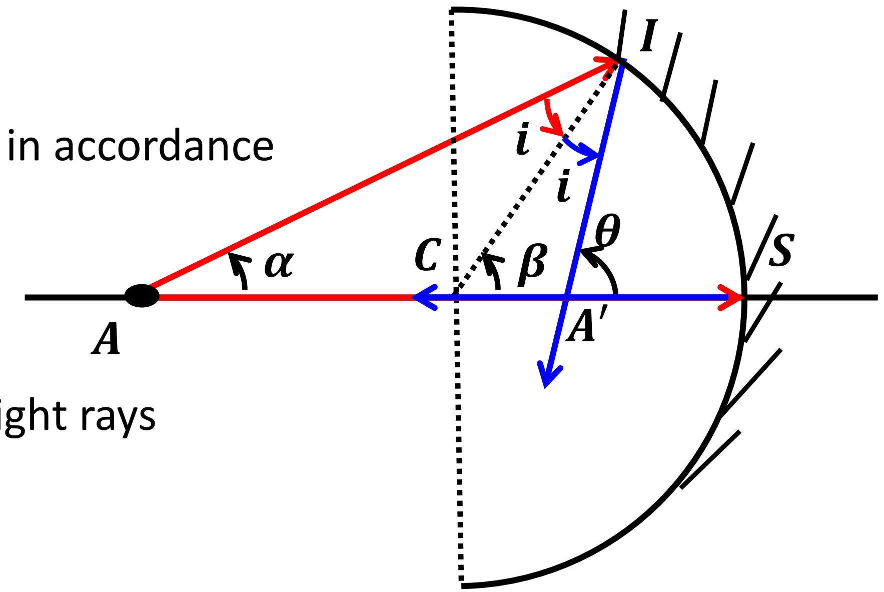
- When these two conditions are met, the light rays cross at a single point on the optical axis of the mirror,
- This makes it possible to obtain a clear and undistorted image of the object observed.
- This is what is called the approximate stigmatism for a spherical mirror.

Note: In geometric optics, the measurement of distances is algebraic. Along the optical axis, the direction of propagation of light (usually from left to right) is chosen as the positive direction

Conjugation relations - Magnification

Conjugation relations:

There is a relationship between the positions of an object A and its image A' called the conjugation relation.



- Consider a real object point A located on the optical axis of a concave mirror.
- We consider that the ray emitted from A and which is reflected at the point I in accordance with the laws of reflection.
- Let a ray be confused with the optical axis, it reflects on itself:
- The image A' of A is located at the point of intersection of any reflected two light rays from A
- A' is always situated on the optical axis.

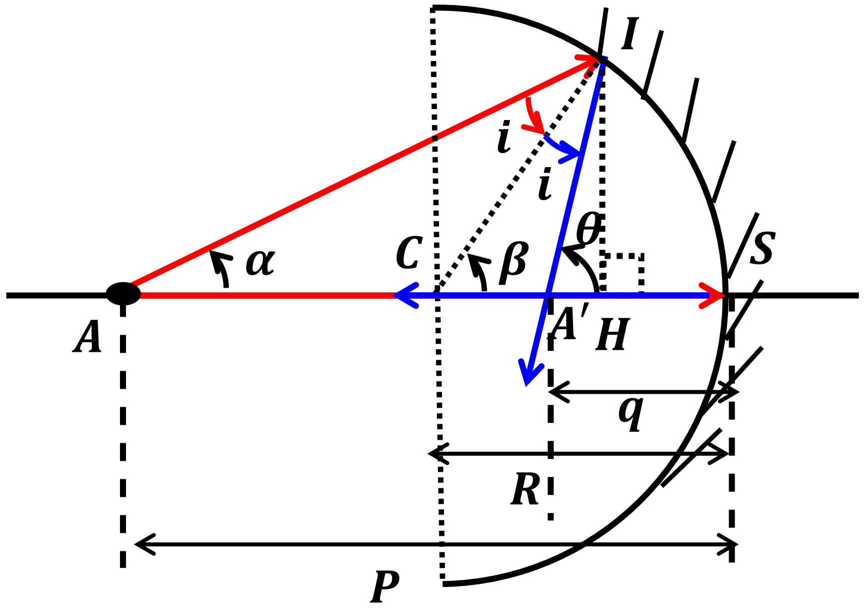
Let's establish the conjugation relationships in the case of a concave mirror for example:

In the triangles AIC and $A'IC$ the sum of the interior angles must be equal to π, i.e.:

$$\alpha + i + (\pi - \beta) = \pi \quad \Rightarrow i = \beta - \alpha$$
$$\beta + i + (\pi - \theta) = \pi \quad \Rightarrow i = \theta - \beta$$
$$\Rightarrow 2\beta = \theta + \alpha$$

$$2\beta = \theta + \alpha$$

- Let consider H the normal project of I on the optical axis.
- Under Gaussian conditions, (very low angle of incidence), and the angles α, β and θ are very small and can be assimilated to their tangents.
- the points H and S are practically confused

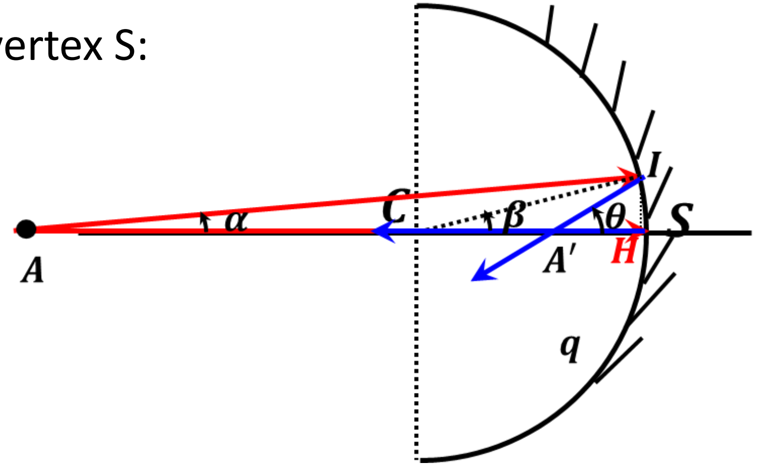


$$\operatorname{tg}\alpha \cong \alpha = \frac{\overline{IS}}{\overline{SA}} \qquad \operatorname{tg}\beta \cong \beta = \frac{\overline{IS}}{\overline{SC}} \qquad \operatorname{tg}\theta \cong \theta = \frac{\overline{IS}}{\overline{SA'}},$$

We have:

$$2\beta = \theta + \alpha \quad \Rightarrow 2\frac{\overline{IS}}{\overline{SC}} = \frac{\overline{IS}}{\overline{SA'}} + \frac{\overline{IS}}{\overline{SA}}$$

We finally obtain the conjugation relation of the spherical mirror with origin at the vertex S:

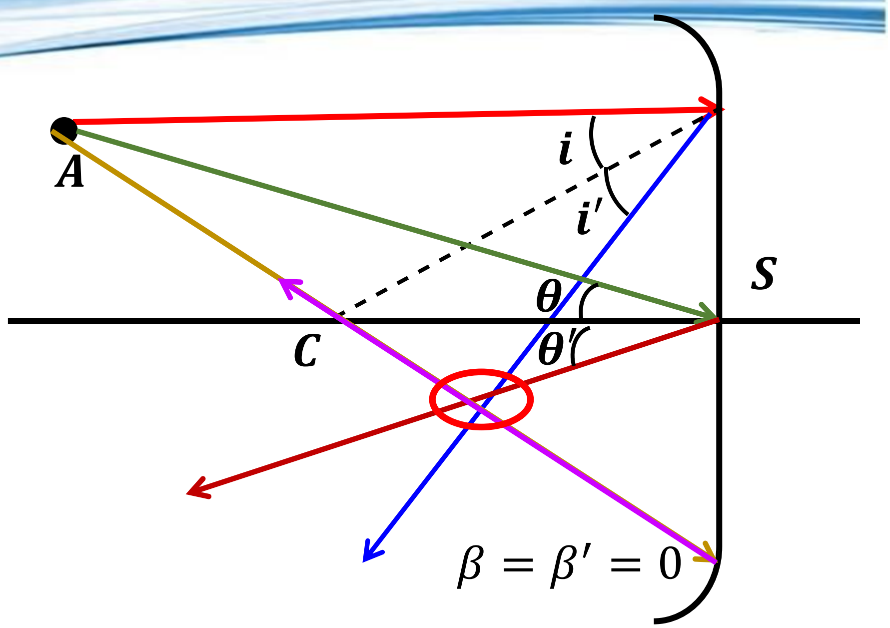


$$\Rightarrow \frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

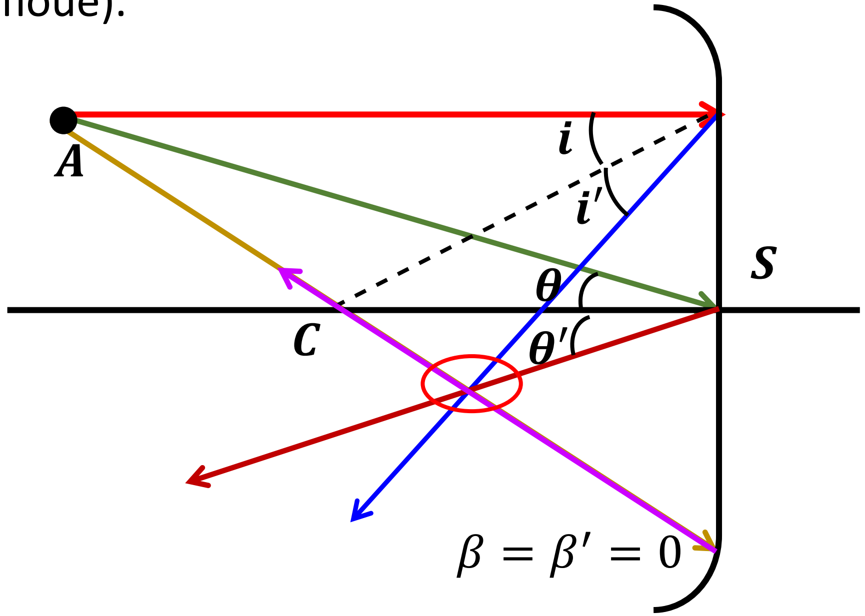
$$\Rightarrow \frac{1}{q} + \frac{1}{P} = \frac{2}{R}$$

Spherical mirror equation

Formation of a clear image under an approximation



- To form a clear image, all the rays from an object must be directed towards the same point after reflection.
- Let's follow the path of three rays from a real object that reflects off a concave mirror
- The three rays respecting the law of reflection are: $i = i'$, $\theta' = \theta$, $\beta = \beta' = 0$
- Since the three rays do not intersect in the same point, the final image is blurry (floue).



In order to solve this situation, we will introduce the approximation of paraxial rays (Gaussian approximation):

- considers that all the rays reflected by the mirror forming the image are relatively parallel to the optical axis.
- The angles in play are therefore much smaller than 1 radian

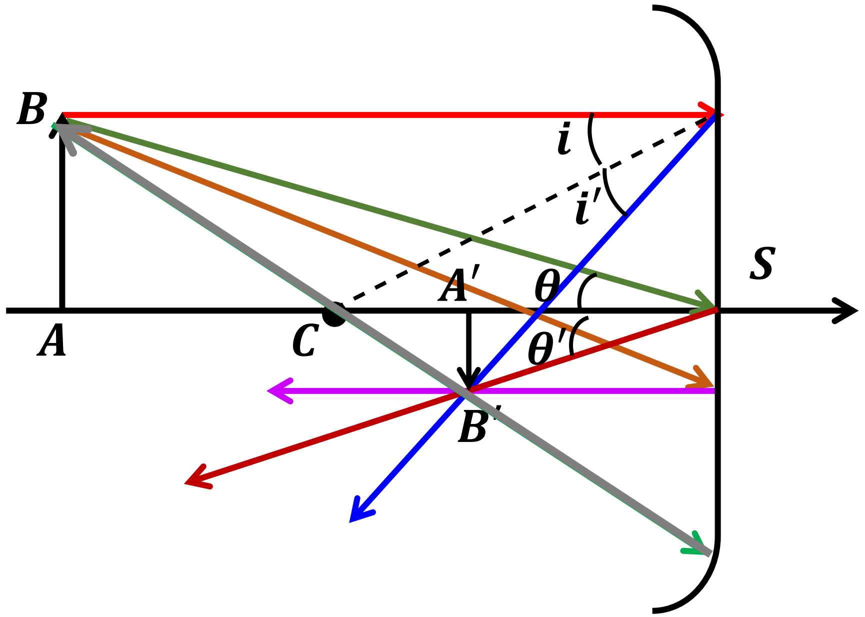
Magnification:

$$\Rightarrow \frac{\overline{AB}}{\overline{SA}} = \frac{\overline{A'B'}}{\overline{SA'}} \quad \operatorname{tg}\theta = \operatorname{tg}\theta'$$

If $\boldsymbol{AB}$ has for image $\boldsymbol{A'B'}$,

By definition, the magnification $\boldsymbol{\gamma}$ is the algebraic ratio of the size of the image to the size of the object:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}}$$



Consider a real object point AB located on the optical axis **of a concave mirror**.

The image $\boldsymbol{A'B'}$ is obtained by the phenomenon of reflection.

We have:

$$tg\theta \cong \theta = \frac{\overline{AB}}{\overline{SA}} \quad \Rightarrow \overline{AB} = \theta.\overline{SA}$$

$$tg\theta' \cong \theta' = \frac{\overline{A'B'}}{\overline{SA}} \quad \Rightarrow \overline{A'B'} = \theta'.\overline{SA'}$$

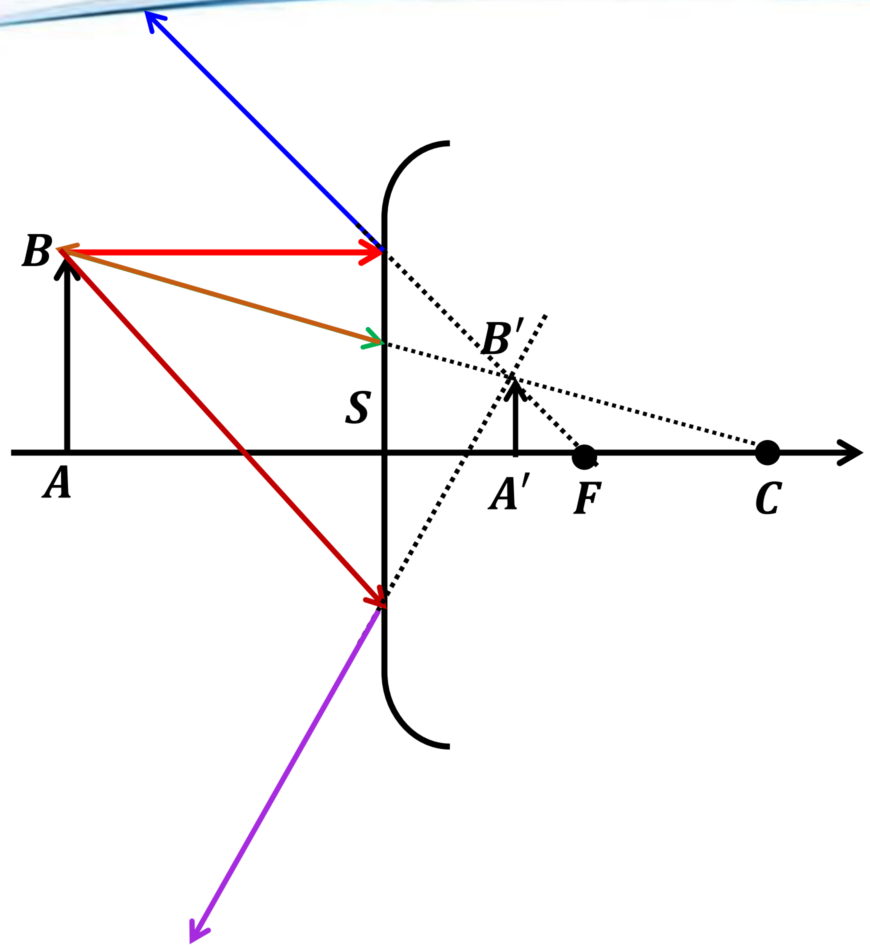
$$\Rightarrow \gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\theta.\overline{SA'}}{\theta'.\overline{SA}}$$

To respect the sign convention $(\theta = -\theta')$, the last equation must be written as follows:

$$\boldsymbol{\frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}}$$

Exercise:

Finding the relation of Magnification in the case of a Convex Mirror



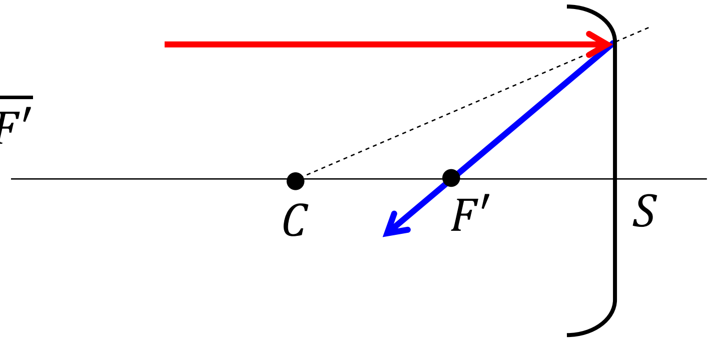
Focal points of the spherical miror: Focus and Newton's formulas

Image focal point F': is the image point of an object located at infinity

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}};$$

Object located at infinity: $\Longrightarrow \overline{SA} \rightarrow \infty \; et \; \overline{SA'} = \overline{SF'}$

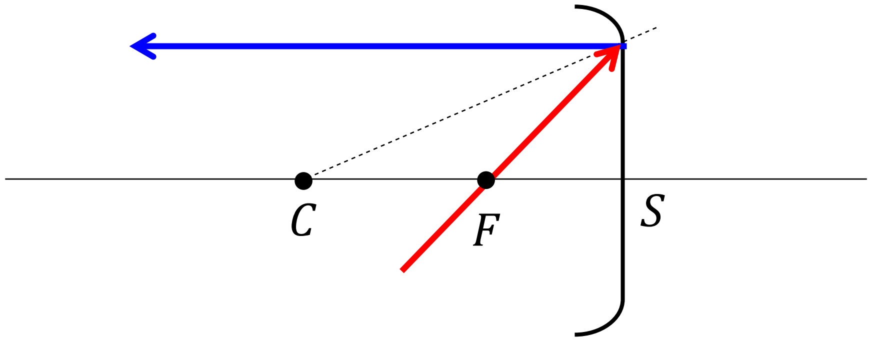
$$\Longrightarrow \overline{SF'} = \frac{\overline{SC}}{2}$$



object focal point F: is the object point of an image formed to infinity.

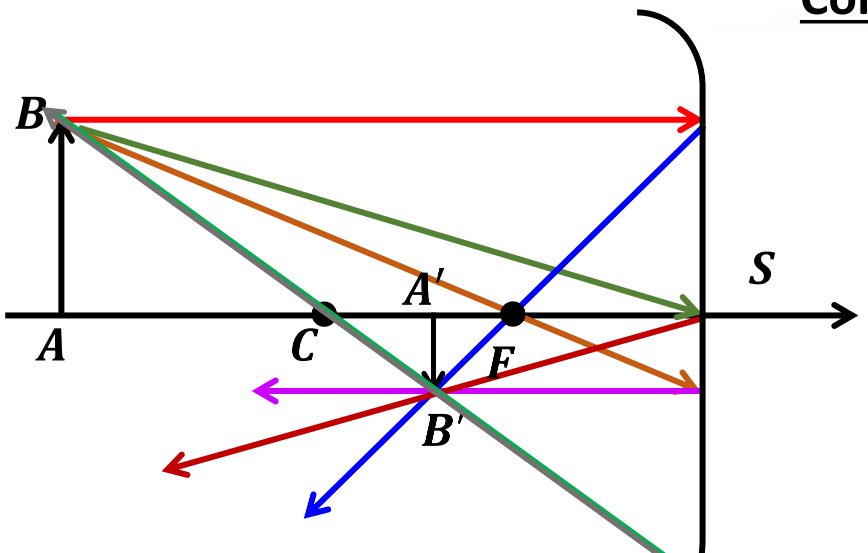
Image Formed to Infinity: $\Longrightarrow \overline{SA} \rightarrow \infty \quad et \; \overline{SS} = \overline{SF}$

$$\Longrightarrow \overline{SF} = \frac{\overline{SC}}{2}$$

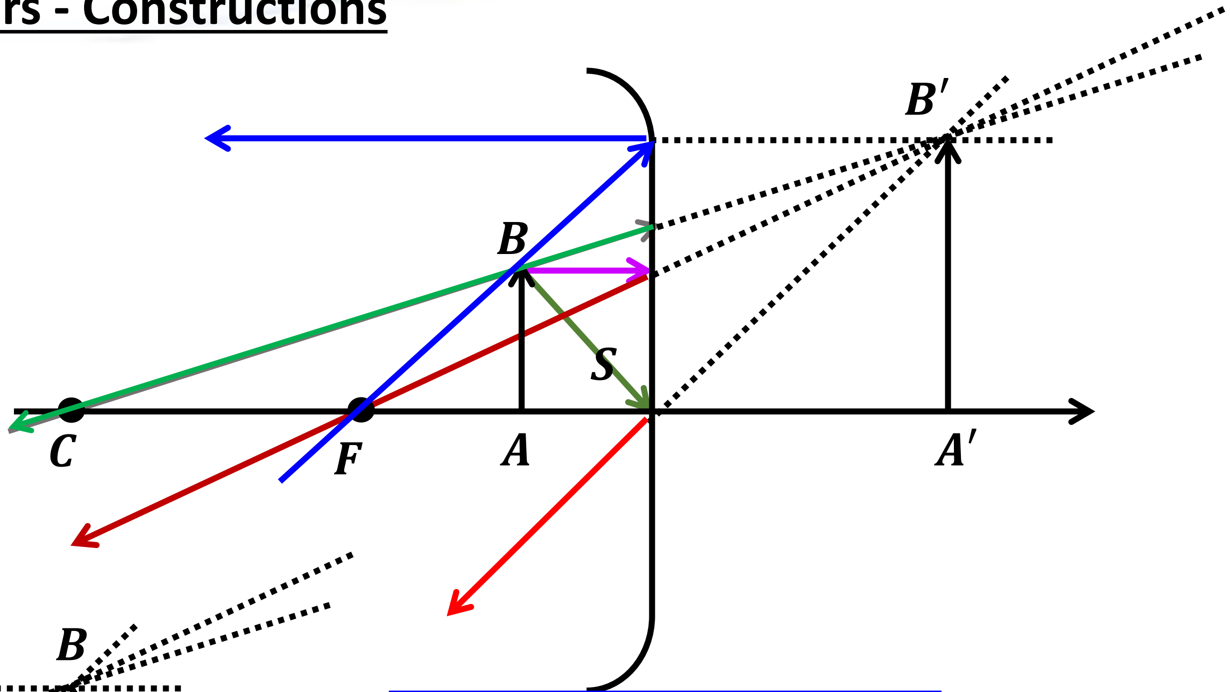


Remark: $\overline{SF} = \overline{SF'}$ ***Image focal point F' and the object focal point F are confused with the midpoint of the segment CS of the spherical mirror.***

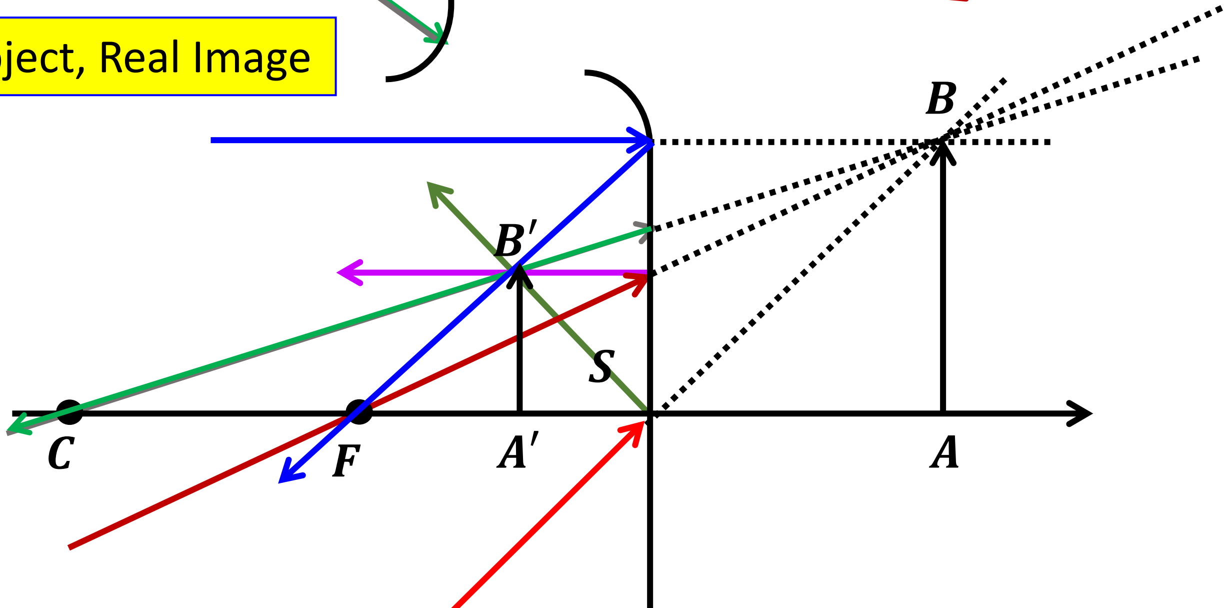
Concave Mirrors - Constructions



Real Object, Real Image

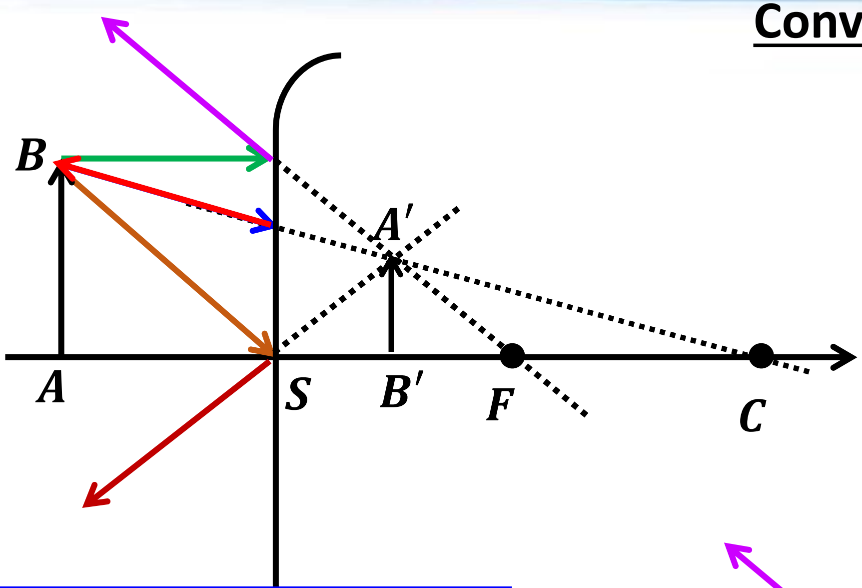


Real Object, Virtual Image

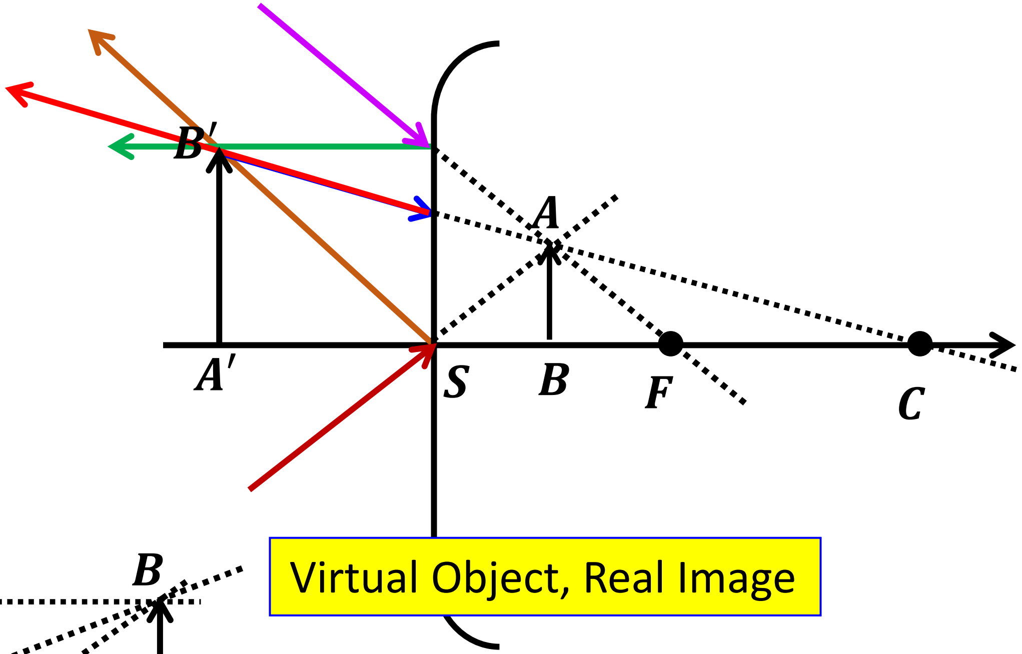


Virtual Object, Real Image

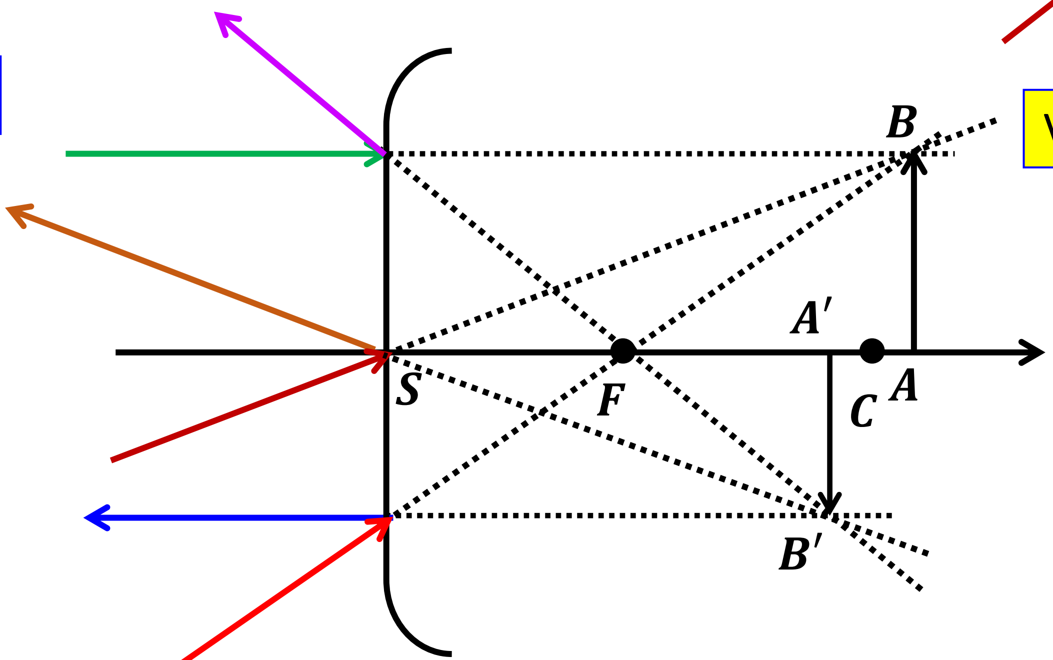
Convex Mirrors - Constructions



Real Object, Virtual Image



Virtual Object, Real Image



Virtual Object, Virtual Image

Exercise 1:

A concave spherical mirror has a radius of curvature of **8 cm**. A real object is located at $\boldsymbol{y_0} = \boldsymbol{4mm}$ from the optical axis, with respect to a point located at $\boldsymbol{P} = \boldsymbol{12\ cm}$ cm from the vertex of the mirror **S**.

We want to trace the principal rays and determine the position of the image by the equations.

Solution:

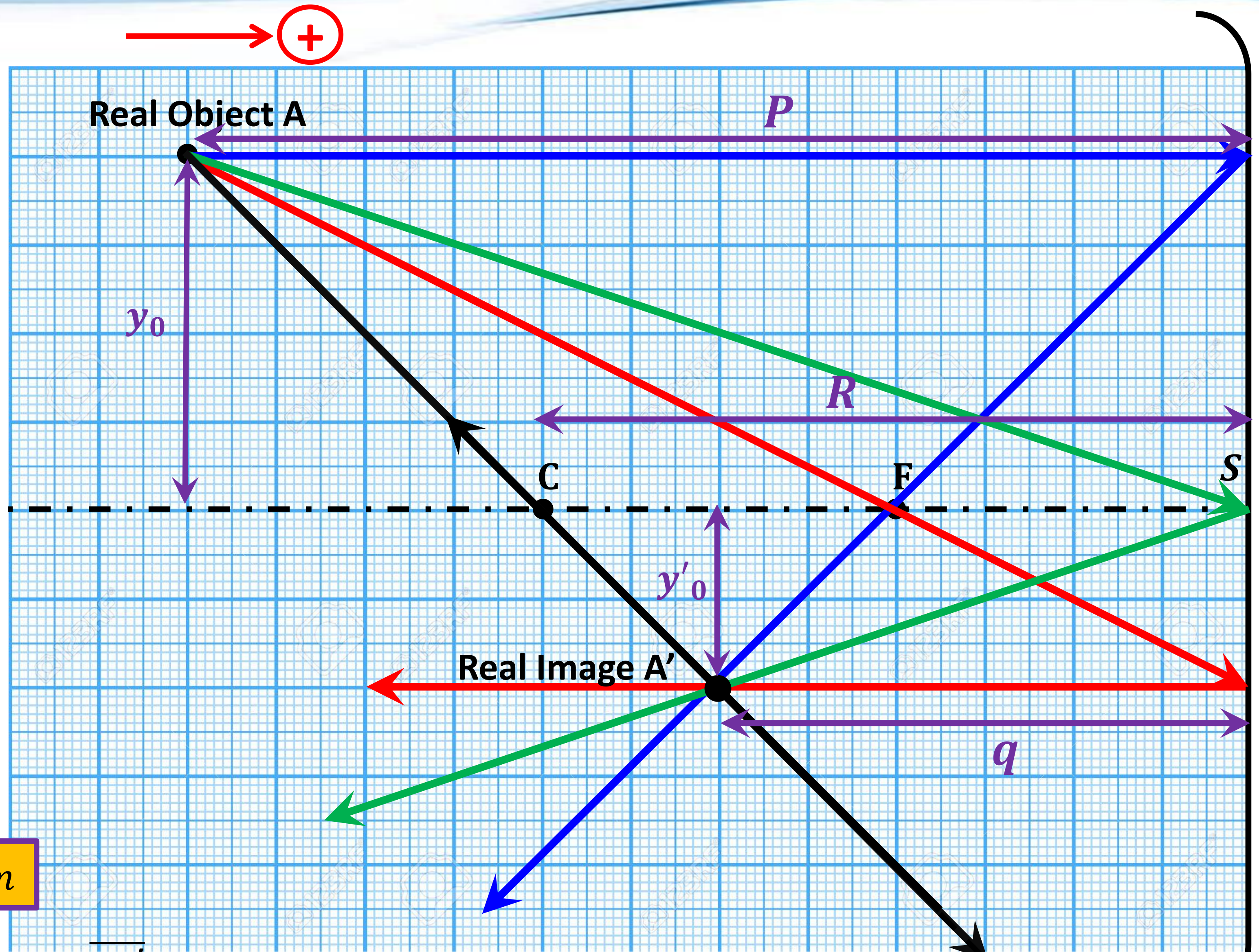
We have: $\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$ (Conjugation relation)

- ✓ Concave mirror: $\Rightarrow R = \overline{SC} < 0$
- ✓ Real object $\Rightarrow P = \overline{SA} < 0$

$$\Rightarrow \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}} - \frac{1}{\overline{SA}} \Rightarrow \frac{1}{\overline{SA'}} = \frac{2}{-8} - \frac{1}{-12}$$

$$\Rightarrow \frac{1}{\overline{SA'}} = -\frac{1}{4} + \frac{1}{12} = -\frac{1}{6}$$

$$\Rightarrow \overline{SA'} = q = -6\ cm$$



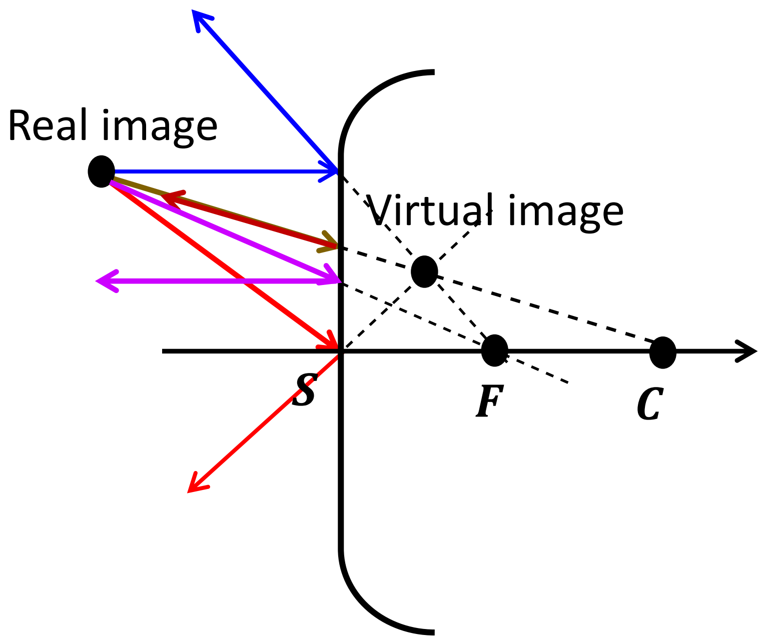
$$\boldsymbol{\gamma} = \frac{\boldsymbol{A'B'}}{\boldsymbol{AB}} = -\frac{\overline{\boldsymbol{SA'}}}{\overline{\boldsymbol{SA}}} \Rightarrow \frac{y'_0}{y_0} = -\frac{\overline{SA'}}{\overline{SA}} \Rightarrow y'_0 = -y_0\frac{\overline{SA'}}{\overline{SA}} = -4\frac{-6}{-12}$$

$$\Rightarrow \boldsymbol{y'_0} = \boldsymbol{-2mm}$$

Inverted image

Exercise 2: The trace of the Main Rays for a Convex Mirror

A convex spherical mirror has a radius of curvature of 8 cm. A real object is located at yo = 4 mm from the optical axis, with respect to a point located at p = 4 cm from the mirror. We want to trace the principal rays and determine the position of the image by the equations.



$$\frac{1}{q}+\frac{1}{P}=\frac{2}{R} \quad \Rightarrow \frac{1}{q}=\frac{2}{R}-\frac{1}{P}$$

(Real Object: $P < 0$, Convex mirror: $R > 0$)

$$\Rightarrow \frac{1}{q}=\frac{2}{8}-\frac{1}{-4}=\frac{1}{2} \quad \Rightarrow q = 2\ cm > 0$$

(Virtual image)

- Let's evaluate the distance y_0' between the image and the optical axis (image size):

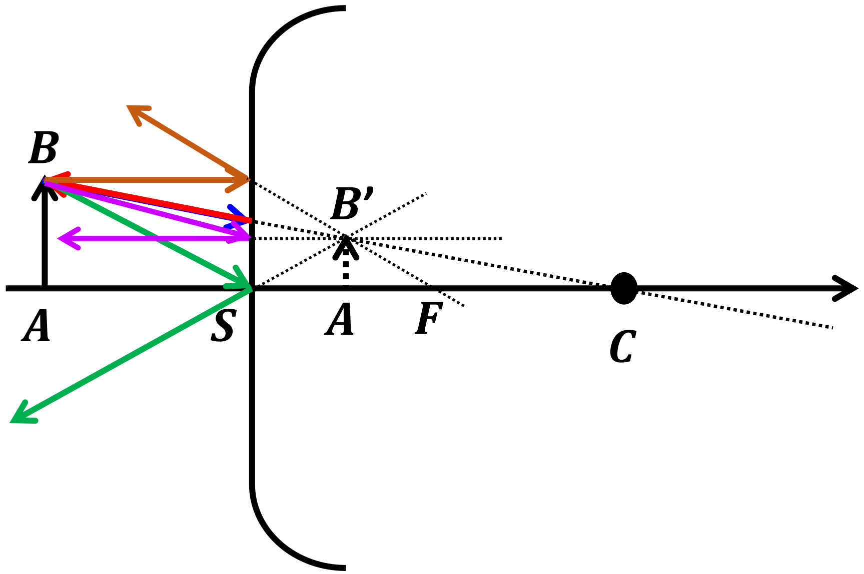
$$\frac{\overline{A'B'}}{\overline{AB}}=-\frac{\overline{SA'}}{\overline{SA}} \quad \Rightarrow \frac{y_0'}{y_0}=-\frac{q}{P} \quad \Rightarrow y_0'=-y_0\frac{q}{P} \quad = -4\frac{2}{-4}$$

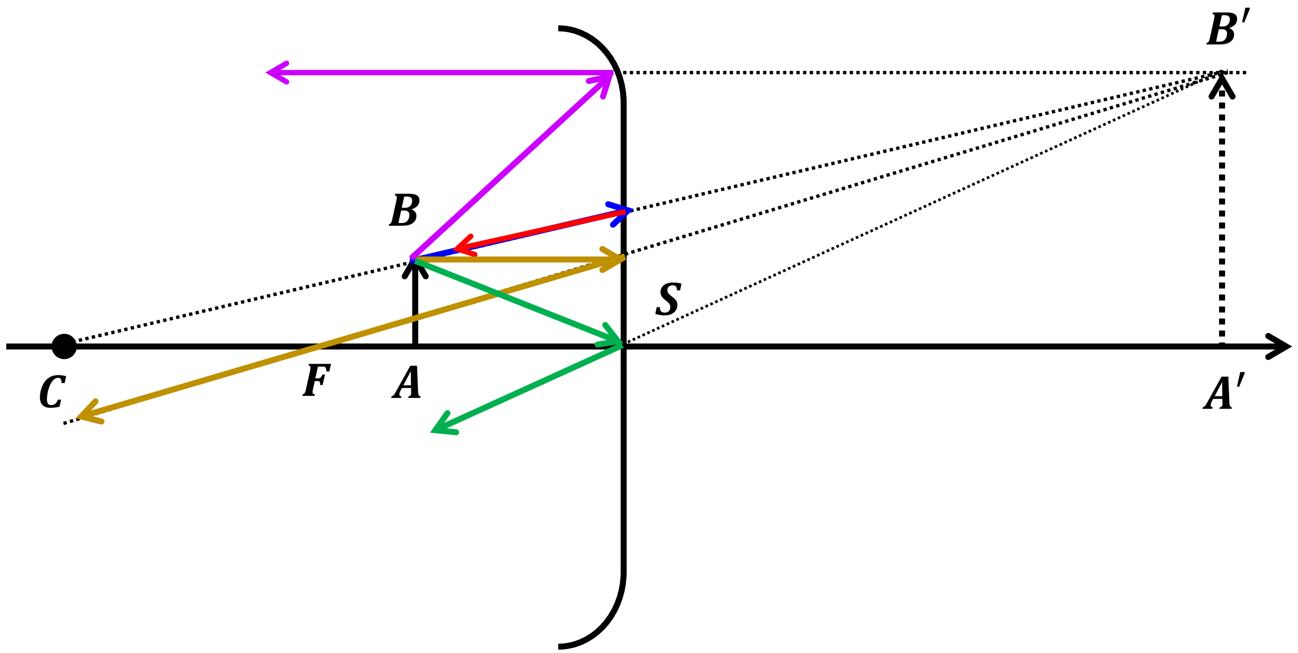
$$\Rightarrow y_0' = 2\ cm$$

image not inverted because $y_0' > 0$

Exercise 3:

Trace the image A'B' of the object AB in the following two cases (Concave mirror and convex mirror):

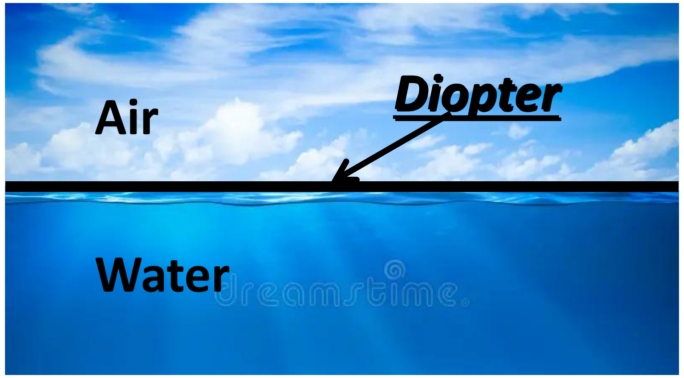




Plane diopter and spherical diopter

Definition:

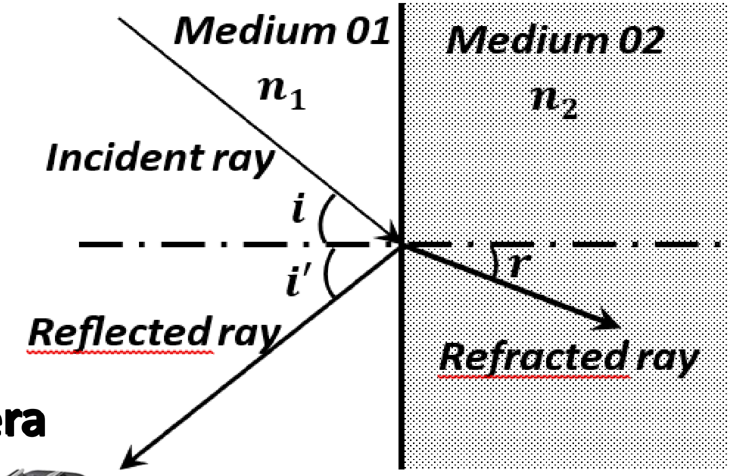
- ❑ A dioper is a surface separating two transparent media of different refractive index
- ❑ This surface can be plane (the diopter is then called plane), spherical (spherical diopter) or of any shape.



1. Plane diopter :

- ➢ A plane diopter is a plane surface that separates two optical media: Transparent, homogeneous and isotropic.
- ➢ When an object is placed in front of a plane diopter, the light reflected and refracted by the plane diopter produces a virtual image of the object.
- ➢ Plane diopters are used in many optical devices, such as:

Plane diopter



glasses

magnifying glasses

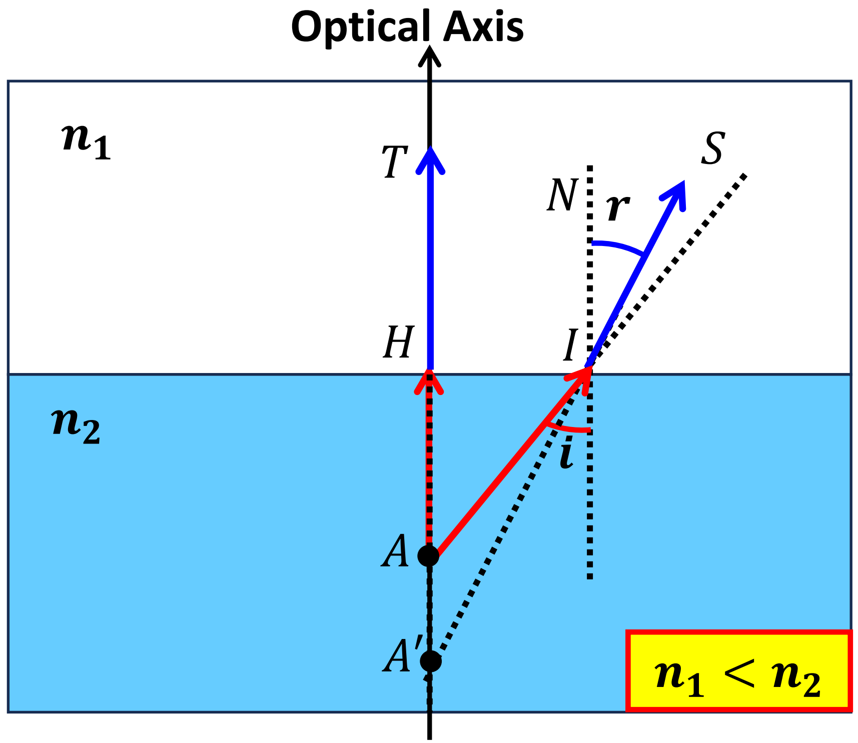
Telescope

Microscope

Camera

Construction of image of a point by a plane diopter:

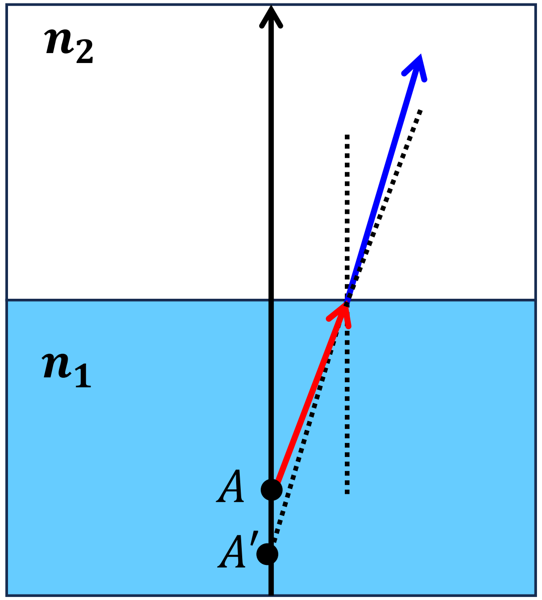
- Let a diopter separate two media with refractive indices n_1 and n_2, with $n_1 < n_2$
- An object point A is located on the optical axis, in the medium n_2
- Let's find the image A' of the object point A
- The image A' is, by definition, the point of intersection of the refracted rays ***(or their extensions).***
- To do, we must trace all the light rays coming from the object point A
- Then construct their refracted rays by applying the laws of refraction,
- And finally find the point of intersection of these refracted rays.



- Let AH be the ray that falls in H perpendicularly on the diopter, it refracts forming *HT* without changing direction
- Now, draw an arbitrary ray AI, It refracts to form the ray IS, which lies in the medium of refractive index n_2.
- The image point A' is the point of intersection of the extensions of the refracted rays *HT* and *IS*.

The image point A' is therefore a virtual image.

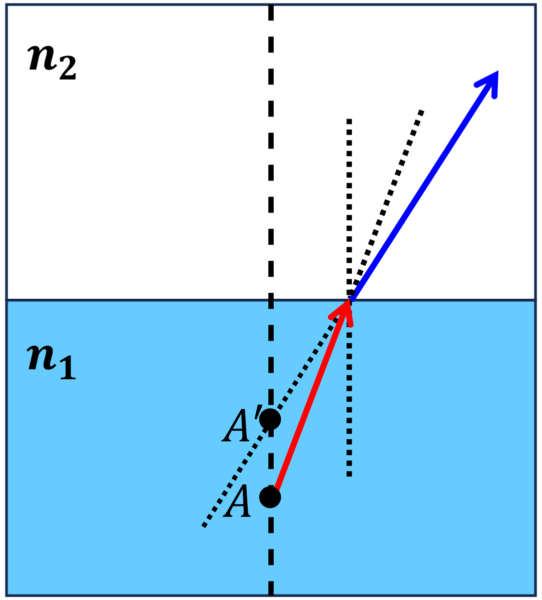
$n_1 < n_2$



A: real objet

A' : virtual image

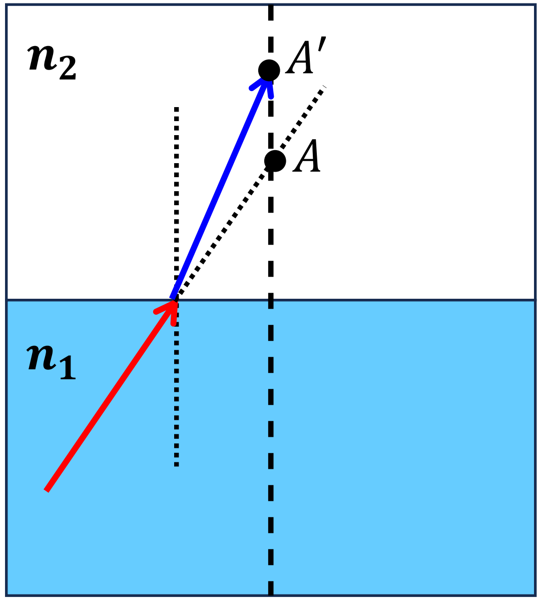
$n_1 > n_2$



A: real objet

A' : virtual image

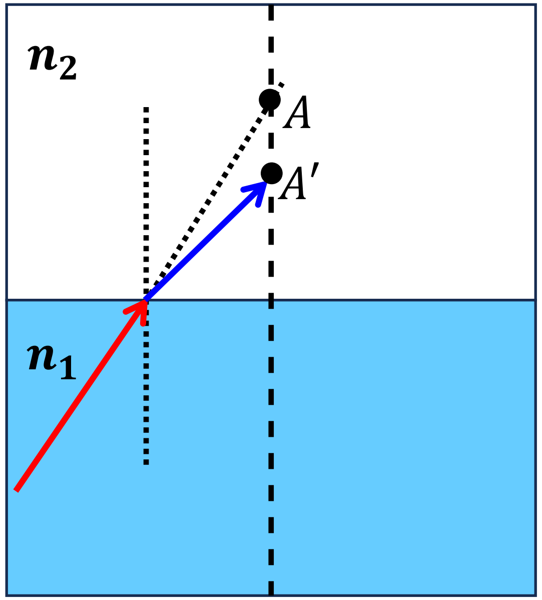
$n_1 < n_2$



A: virtual objet

A' : real image

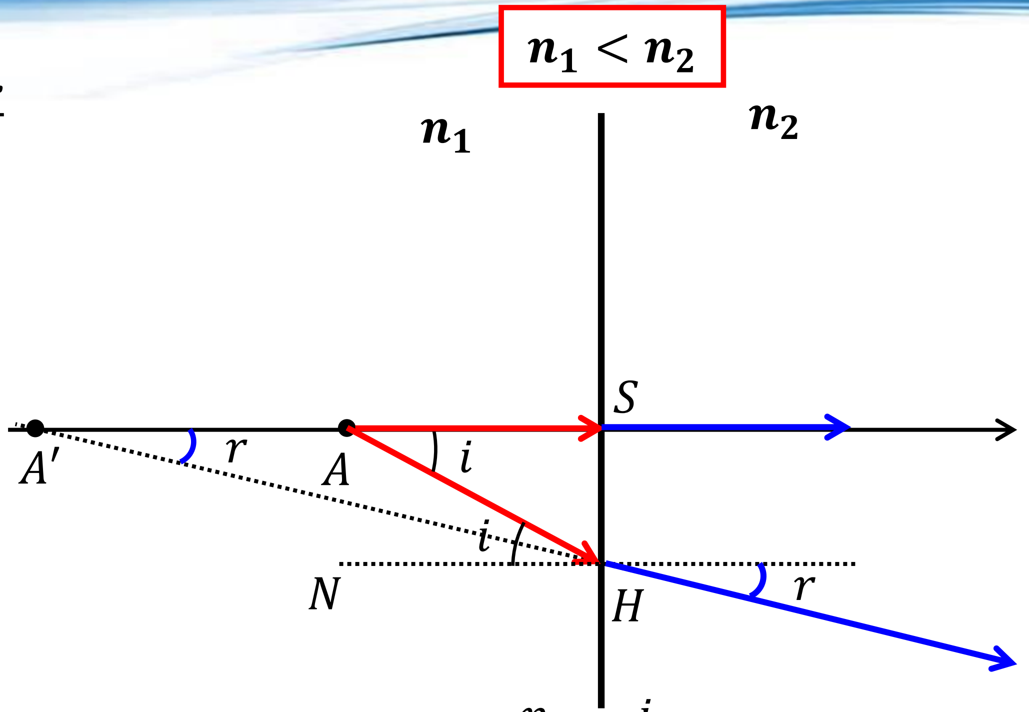
$n_1 > n_2$



A: virtual objet

A' : real image

Conjugation formula of the plane diopter, Gaussian conditions:



The triangles AHS and $A'HS$ are similar, so we have:

$$\operatorname{tg} i = \frac{\overline{HS}}{\overline{AS}} \Longrightarrow \overline{HS} = \overline{AS}\operatorname{tg} i$$

$$\Longrightarrow \overline{AS}\operatorname{tg} i = \overline{A'S}\operatorname{tg} r$$

$$\operatorname{tg} r = \frac{\overline{HS}}{\overline{A'S}} \Longrightarrow \overline{HS} = \overline{A'S}\operatorname{tg} r$$

On the other hand, we have: $n_1 \sin i = n_2 \sin r$

Applying Gaussian conditions: $\sin i \cong \operatorname{tg} i \cong i$ and $\sin r \cong \operatorname{tg} r \cong r \Longrightarrow n_1 i = n_2 r \Longrightarrow \frac{n_2}{n_1} = \frac{i}{r}$

Also we have: $\operatorname{tg} i \cong i$ and $\operatorname{tg} r \cong r \Longrightarrow \overline{AS}\, i = \overline{A'S}\, r \Longrightarrow \frac{i}{r} = \frac{\overline{A'S}}{\overline{A'S}}$

There are: $\frac{\overline{A'S}}{\overline{A'S}} = \frac{n_2}{n_1}$ $\Longrightarrow \frac{\overline{A'S}}{n_2} = \frac{\overline{AS}}{n_1}$ (The angles i et r are small enough: Approximate stigmatism)

The position of the image A' is depends on the angle of incidence

Image of an extended object, linear magnification

1. *the object AB is real and parallel to the surface of the diopter.*

- The conjugation formula of the plane dioptre for points A and B are:

$$\frac{\overline{A'H}}{n_2} = \frac{\overline{AH}}{n_1} \quad \Longrightarrow \frac{\overline{A'H}}{n_2} = \frac{\overline{AA'} + \overline{A'H}}{n_1} \quad \Longrightarrow \overline{AA'} = \frac{n_1 - n_2}{n_2}\overline{A'H}$$

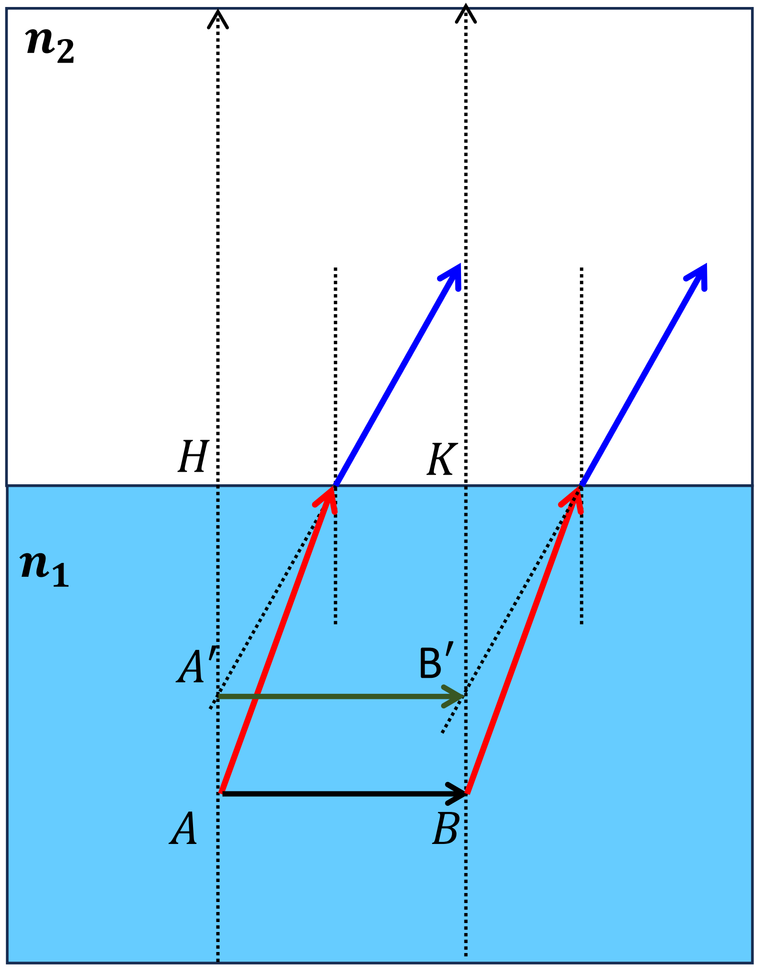
Of the same we find: $\overline{BB'} = \frac{n_1 - n_2}{n_2}\overline{B'K}$

$$\Longrightarrow \overline{AA'} = \overline{BB'}$$

- The image $A'B'$ is virtual and parallel to the surface of the diopter.
- The magnification γ is given by:

$$\boldsymbol{\gamma = \frac{\overline{A'B'}}{\overline{AB}} = 1}$$

$$\boldsymbol{n_1 > n_2}$$



2. *the object AB is real and perpendicular to the surface of the diopter.*

$n_1 > n_2$

- The virtual image A'B' of the object is formed on the normal AH of the diopter.

Conjugation formulas of the plane diopter are:

$$\frac{\overline{A'H}}{n_2} = \frac{\overline{AH}}{n_1} \Rightarrow \overline{A'H} = \frac{n_2}{n_1}\overline{AH} \ldots\ldots\ldots (1)$$

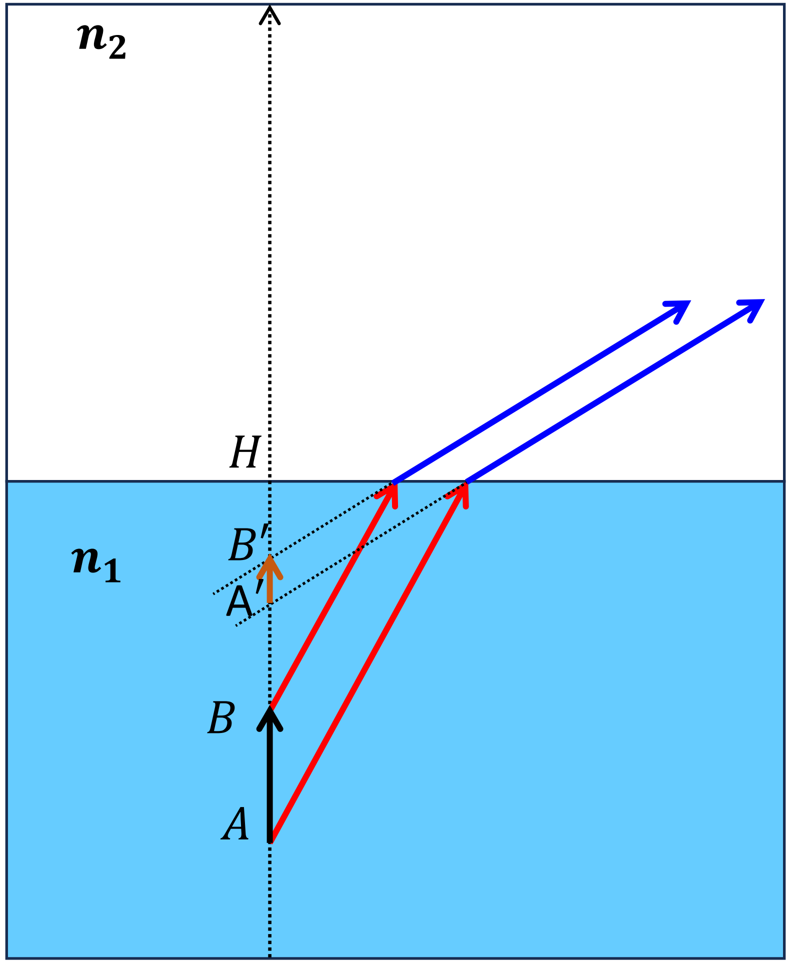
$$\frac{\overline{B'H}}{n_2} = \frac{\overline{BH}}{n_1} \Rightarrow \overline{B'H} = \frac{n_2}{n_1}\overline{BH} \ldots\ldots\ldots (2)$$

$$(1) \Rightarrow \overline{A'B'} + \overline{B'H} = \frac{n_2}{n_1}\overline{AB} + \frac{n_2}{n_1}\overline{BH} \ldots\ldots\ldots (3)$$

We replace (2) in (3) we find: $\overline{A'B'} = \frac{n_2}{n_1}\overline{AB}$

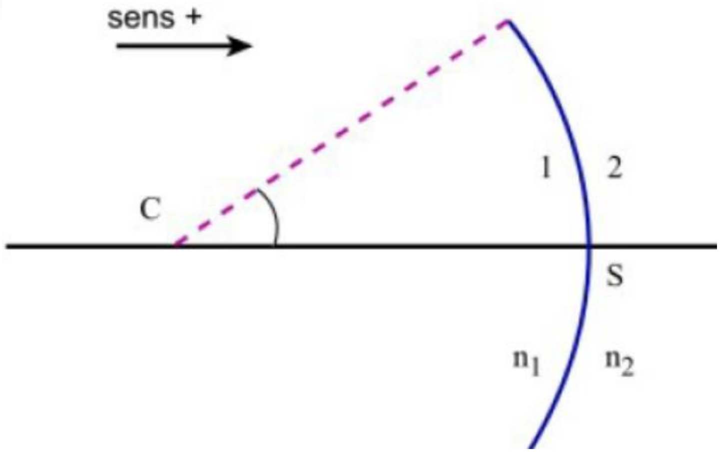
- The magnification γ is therefore given by:

$$\boldsymbol{\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_2}{n_1}}$$



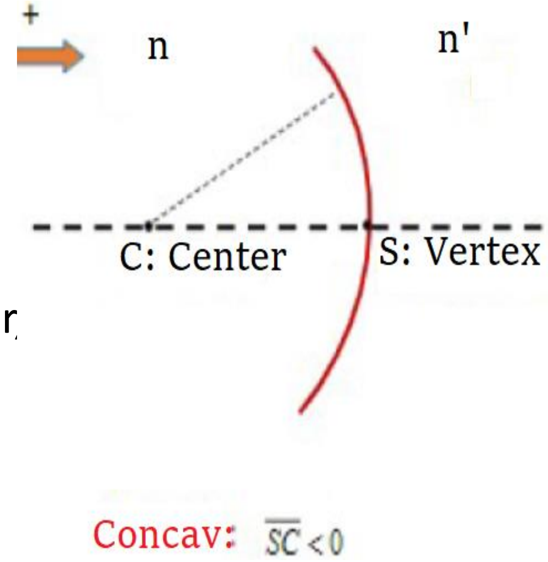
2. Spherical diopter:

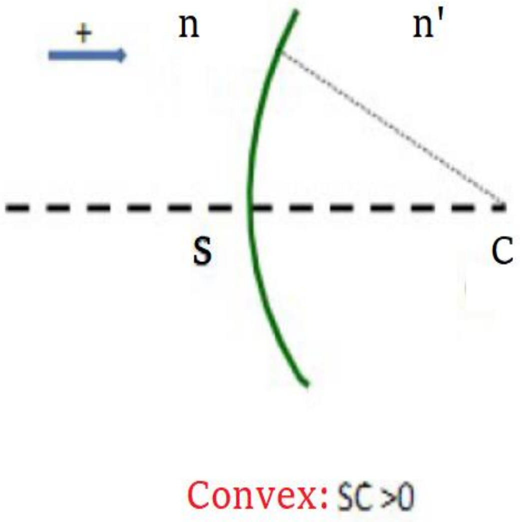
Spherical dioptre in the approximation of Gauss



Definition: A spherical diopter is made up of two transparent, homogeneous and isotropic media, with different indices $n1$ and $n2$ separated by a refractive spherical surface.

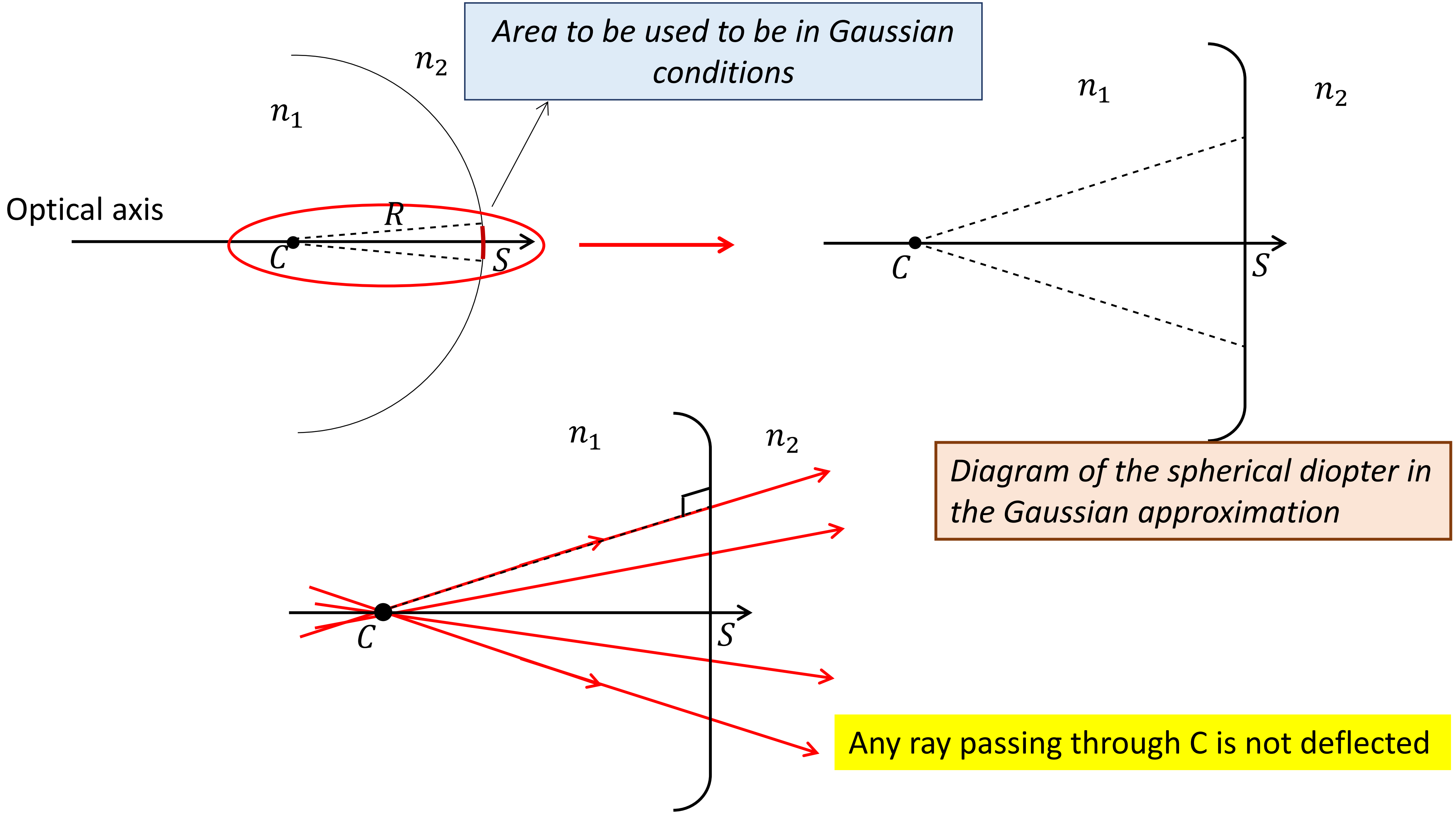
The spherical diopter is characterized by:





- ❖ The center C of the sphere called the diopter center
- ❖ The point S called the vertex of the diopter.
- ❖ The optical axis, the axis of symmetry of revolution of the diopter
- ❖ The radius of the sphere $\boldsymbol{R} = \overline{\boldsymbol{SC}}$, called the radius of curvature.
- ❖ for a concave spherical diopter $\overline{\boldsymbol{SC}} < 0$
- ❖ for a convex spherical diopter $\overline{\boldsymbol{SC}} > 0$

Representation of the spherical diopter in the Gaussian approximation

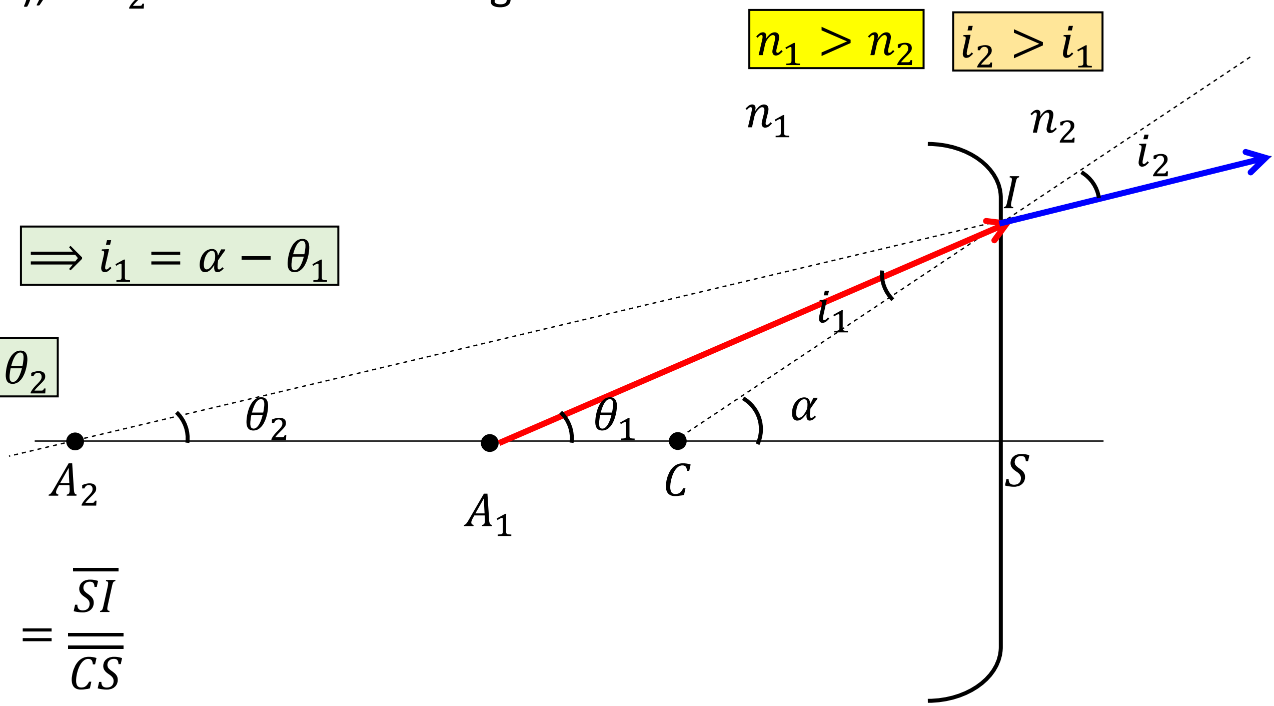


Conjugation relation, Gaussian conditions:

n_1: index of the object medium (on the side of the incident light),et n_2: index of the image medium

$\overline{A_1S}$: Position of the object; $\overline{A_2S}$: Position de l'image;

$\overline{CS}$:Radius of Curvature of thr diopter .



From th triangle A_1IC, we have: $\theta_1 + i_1 + (\pi - \alpha) = \pi$ $\Longrightarrow i_1 = \alpha - \theta_1$

$\theta_2 + i_2 + (\pi - \alpha) = \pi$ $\Longrightarrow i_2 = \alpha - \theta_2$

From th triangle A_2IC:

Applying the Gaussian conditions, we have:

$$\operatorname{tg}\theta_1 \cong \theta_1 = \frac{\overline{SI}}{\overline{A_1S}} \qquad \operatorname{tg}\theta_2 \cong \theta_2 = \frac{\overline{SI}}{\overline{A_2S}} \qquad \operatorname{tg}\alpha \cong \alpha = \frac{\overline{SI}}{\overline{CS}}$$

Let's apply Snell Descartes' law: $n_1 \sin i_1 = n_2 \sin i_2$

Gaussian conditions $\Longrightarrow \; sin\, i_1 \cong i_1 \; et \; sin\, i_2 \cong i_2 \;\; \Longrightarrow n_1\, i_1 = n_2 i_2 \;\; \Longrightarrow n_1\,(\alpha - \theta_1) = n_2(\alpha - \theta_2)$

$$\Longrightarrow n_1\left(\frac{\overline{SI}}{\overline{CS}} - \frac{\overline{SI}}{\overline{A_1S}}\right) = n_2\left(\frac{\overline{SI}}{\overline{CS}} - \frac{\overline{SI}}{\overline{A_2S}}\right) \quad \boxed{\Longrightarrow \frac{n_2}{\overline{A_2S}} - \frac{n_1}{\overline{A_1S}} = \frac{n_2 - n_1}{\overline{CS}}}$$

- We called the term: $\mathbf{V} = \dfrac{\boldsymbol{n_2 - n_1}}{\overline{\boldsymbol{CS}}}$ the vergence of the diopter,

$[V] = L^{-1}$; Unit = diopter($1\delta = 1m^{-1}$)

- $V > 0$: Convergent diopter, $V < 0$: Divergent dioptre.

Example: spherical glass diopter in air, convex, 1 cm radius

$$\mathrm{V} = \frac{n_2 - n_1}{\overline{CS}} = \frac{1{,}5 - 1}{+0{,}01} = 0{,}5 \times 100 = 50\delta$$

- ***Four possible cases, depending on the vergence, for the spherical diopter:***

- $\overline{SC} < 0$ et $n_1 < n_2$: Divergent concave spherical diopter.
- $\overline{SC} < 0$ et $n_1 > n_2$: Converging convex spherical diopter.
- $\overline{SC} > 0$ et $n_1 < n_2$: Converging convex spherical diopter.
- $\overline{SC} > 0$ et $n_1 > n_2$: Divergent concave spherical diopter.

Focal points of the spherical diopter:

Object focal point F **:** is the object point of an image formed at infinity.

The focal length object f is the algebraic measure $\overline{SF}$

$$\frac{n_2}{\overline{A_2S}} - \frac{n_1}{\overline{A_1S}} = \frac{n_2 - n_1}{\overline{CS}};$$

Image Formed at Infinity: $\Longrightarrow \overline{A_2S} \to \infty \; et \; \overline{A_1S} = \overline{FS}$

$$\Longrightarrow 0 - \frac{n_1}{\overline{FS}} = \frac{n_2 - n_1}{\overline{CS}}$$

$$\Longrightarrow f = \overline{SF} = -\frac{n_1\overline{CS}}{n_2 - n_1} = -\frac{n_1}{V}$$

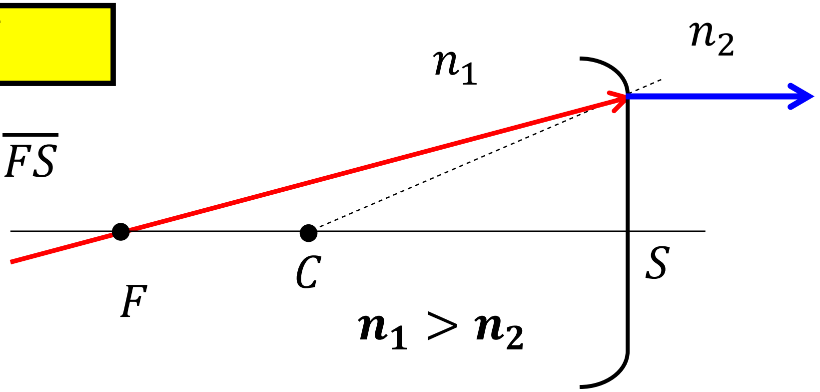


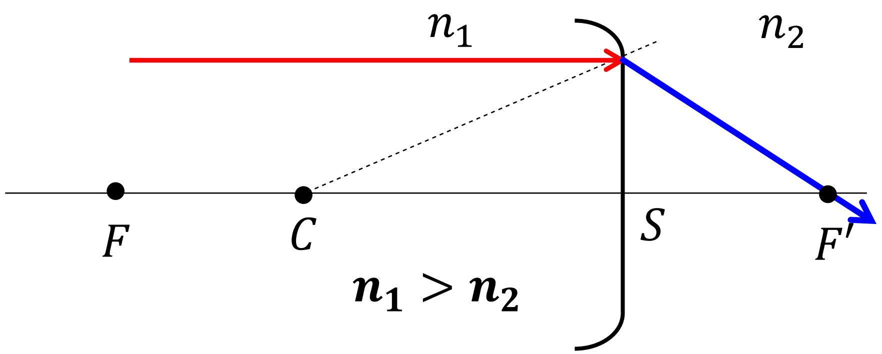
Image focal point F'**:** is the image point of an object located at infinity.

The focal length image f' is the algebraic measurement $\overline{SF'}$

Image point of an object located at infinity: $\Longrightarrow \overline{A_1S} \to \infty \quad et \; \overline{A_2S} = \overline{F'S}$

$$\frac{n_2}{\overline{F'S}} - 0 = \frac{n_2 - n_1}{\overline{CS}};$$

$$\Longrightarrow f' = \overline{SF'} = \frac{n_2\overline{CS}}{n_2 - n_1} = \frac{n_2}{V}$$

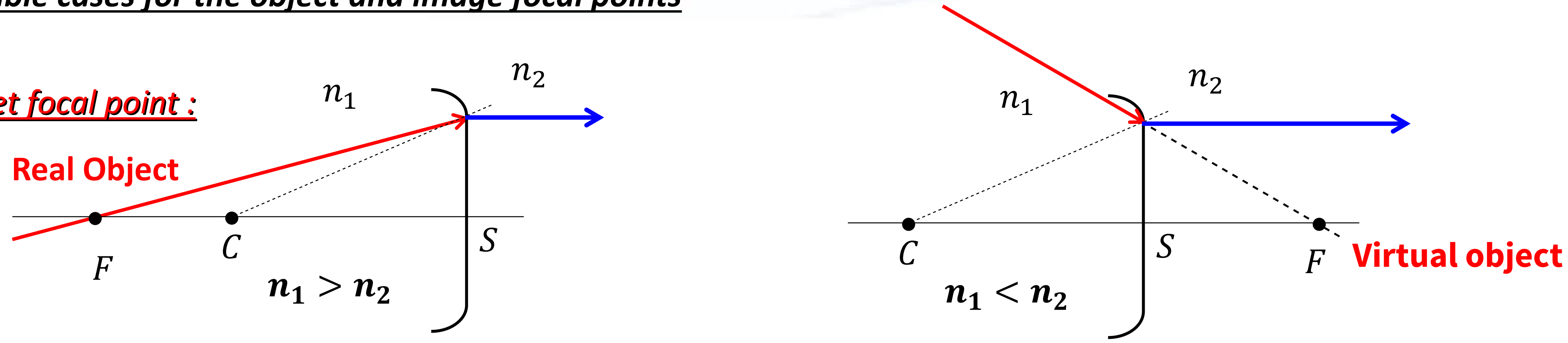


Remark: $\frac{\overline{SF'}}{\overline{SF}} = -\frac{n_2}{n_1} < 0 \quad \Longrightarrow \overline{SF}$ and $\overline{SF'}$ are of contrary signs,

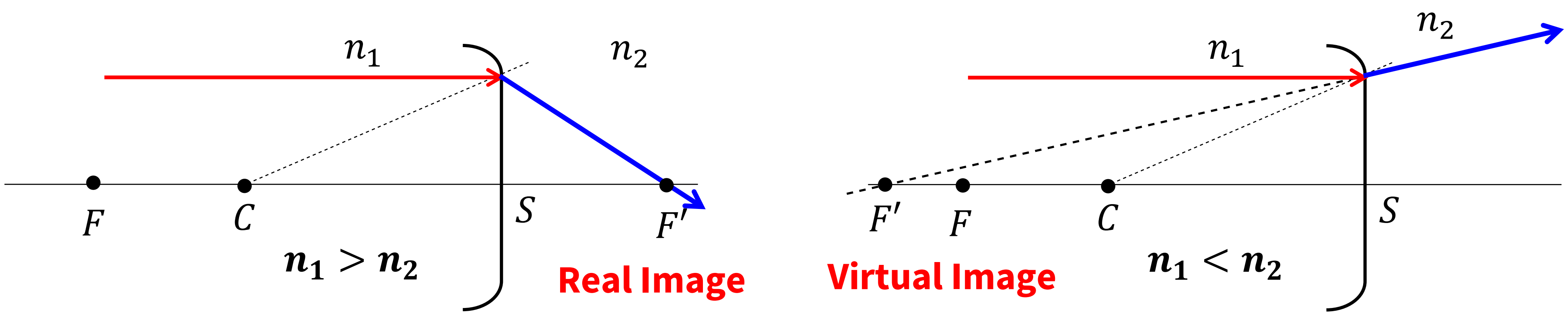
F and F' belong to two different media

Two possible cases for the object and image focal points

1. *Objet focal point :*



2. *Image focal point :*

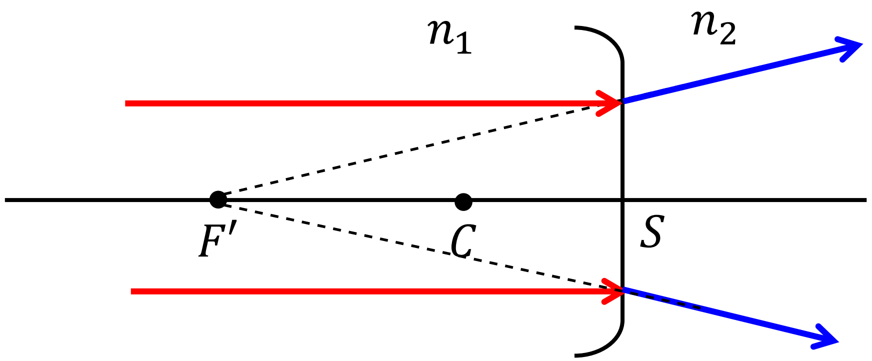


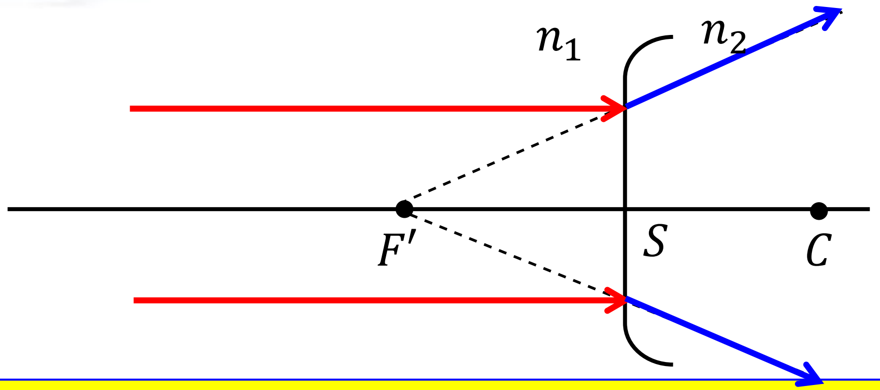
It results from this definition that any incident ray parallel to the optical axis is refracted by passing through the image focal F'.

Diopter Divergent:

$\overline{SC} < 0$ and $n_1 < n_2$

$\overline{SC} > 0$ and $n_1 > n_2$





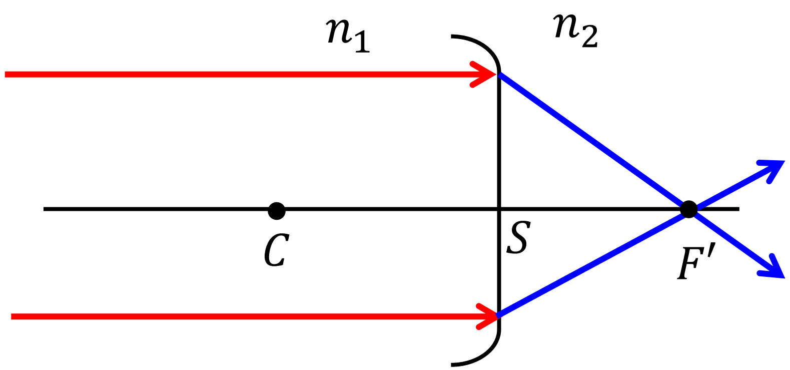
A dioptre will diverge if the image focal is virtual; the center of curvature C is located in the medium of the lowest refractive index.

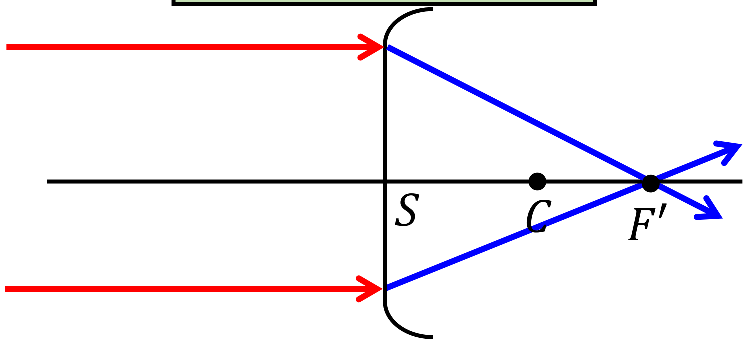
$\Longrightarrow$ ***The vergence of the diopter*** $V = \frac{n_2 - n_1}{\overline{CS}} < 0$

Diopter Convergent

$\overline{SC} < 0$ et $n_1 > n_2$

$\overline{SC} > 0$ et $n_1 < n_2$





A dioptre will be convergent if the image focal is real. the center of curvature C is located in the medium of the highest refractive index.

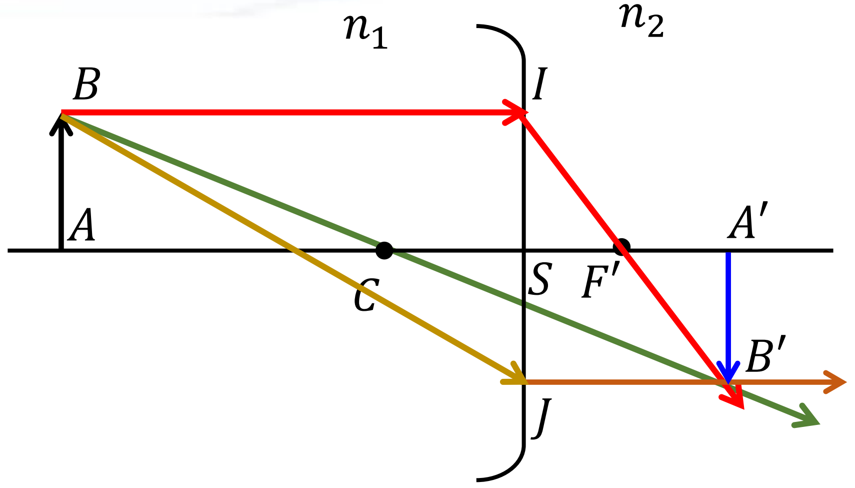
- **Magnification in the origin of C:**

From the triangles ***ABC*** and ***A'B'C***, we have:

$$\frac{A'B'}{A'C} = \frac{AB}{AC} \quad \Longrightarrow \frac{A'B'}{AB} = \frac{A'C}{AC}$$

$$\boldsymbol{\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{A'C}{AC}}$$

$\boldsymbol{n_1 > n_2}$



- **Magnification in the origin of S:**

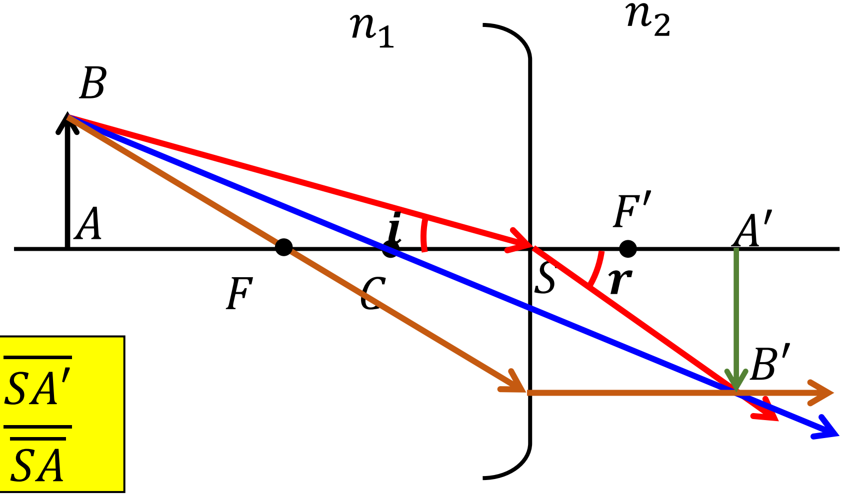
$$\operatorname{tg} i = \frac{\overline{AB}}{\overline{SA}} \quad \text{and} \quad \operatorname{tg} r = \frac{\overline{A'B'}}{\overline{SA'}}; \qquad n_1 \sin i = n_2 \sin r$$

$$\sin i \cong i\,; \sin r \cong r \implies n_1 i = n_2 r$$

$$\operatorname{tg} i \cong i\,; \operatorname{tg} r \cong r \quad \implies n_1 \frac{\overline{AB}}{\overline{SA}} = n_2 \frac{\overline{A'B'}}{\overline{SA'}}$$

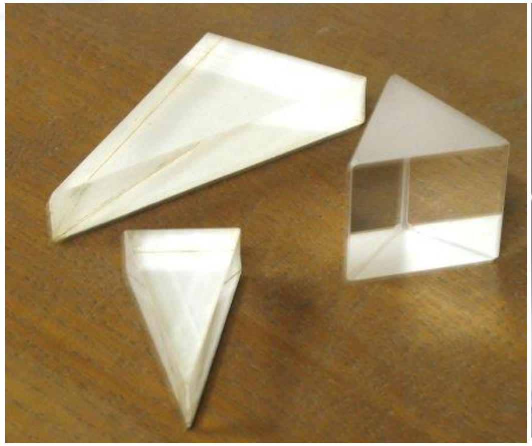
$$\implies \frac{n_1}{n_2}\frac{\overline{SA'}}{\overline{SA}} = \frac{\overline{A'B'}}{\overline{AB}}$$

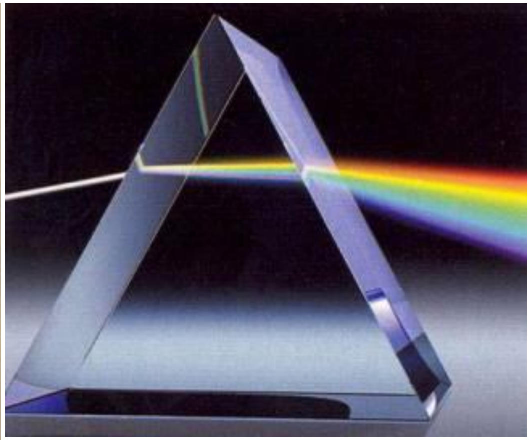
$$\implies \gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}\frac{\overline{SA'}}{\overline{SA}}$$



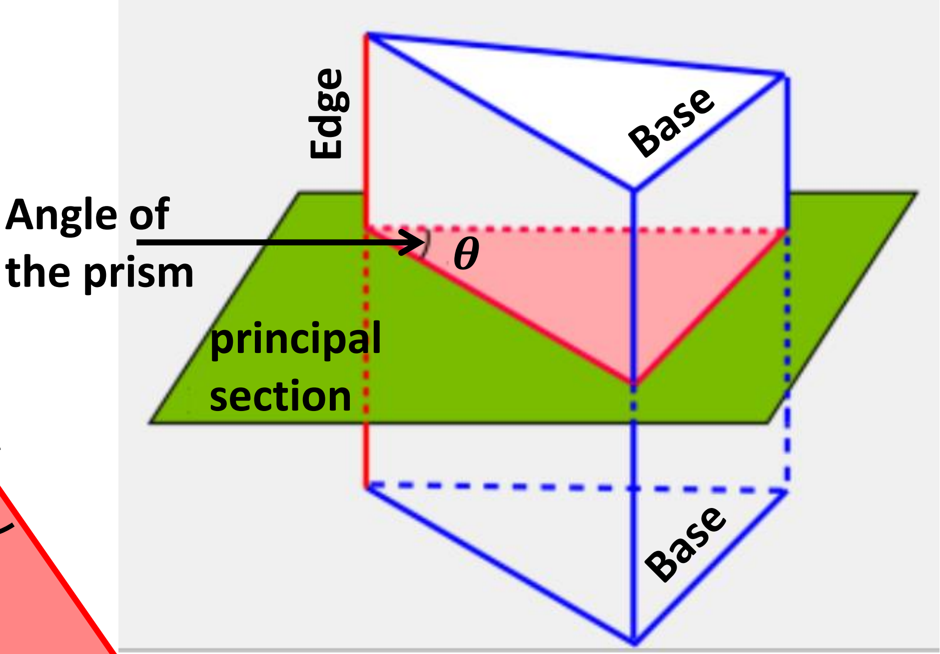
1.4. Refraction in a prism:

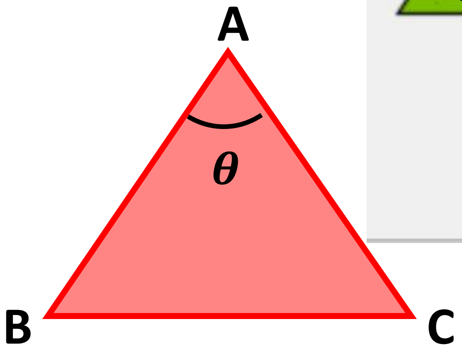
Definitions and propositions:





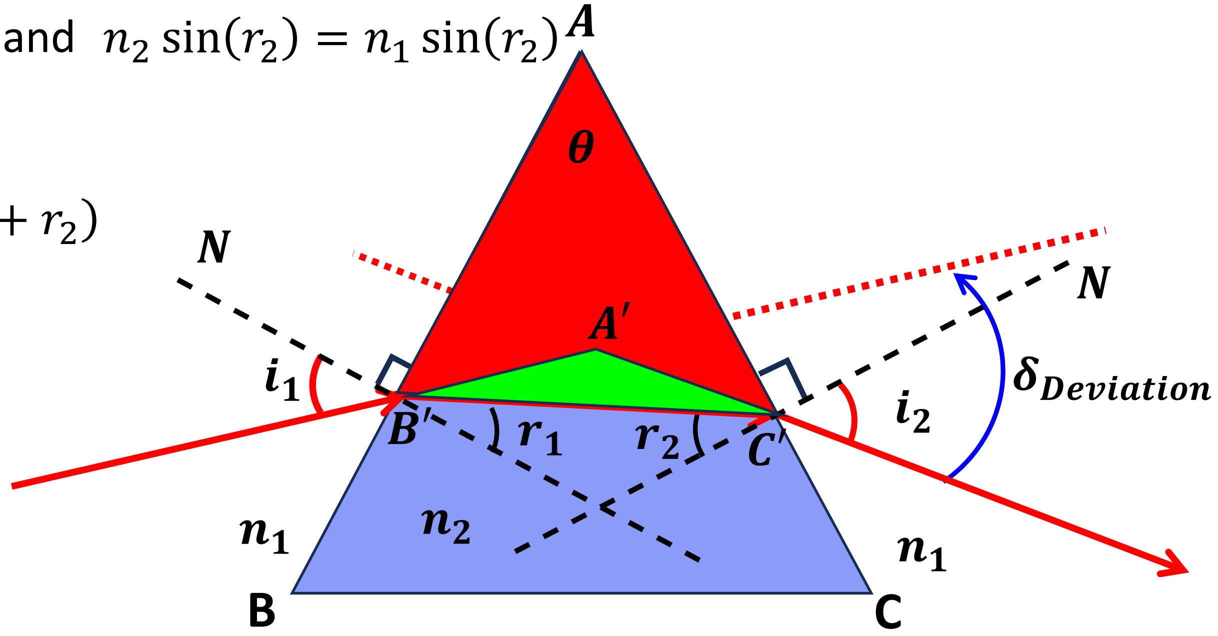
- A prism is a transparent and homogeneous medium bounded by two equal and parallel bases and three non-parallel plane diopters that constitute the faces of the prism;
- Each pair of diopters intersects with each other along a straight line which is the edge of the prism.
- Any plane perpendicular to the edge is called a plane of **principal section;**
- The intersection of the principal section with the two diopters defines an angle called **the angle of the prism θ**
- In practice, we will only take into consideration the representation of the prism with that limited to the intersection of its principal section with the edge of the prism, i.e. to a triangle ABC .
- To simplify the study, we will agree that the prism is immersed in air, and therefore that its refractive index n_P is greater than 1.





A prism is therefore defined by its angle θ and its refractive index n_P.

The aim of this study is to calculate the angle of deviation of light ray and the refractive index in the prism



- *By appling the Snell Descartes laws:* $n_1 \sin(i_1) = n_2 \sin(r_1)$ and $n_2 \sin(r_2) = n_1 \sin(r_2)$
- **Calcul of the angle of deviation δ:** *In triagle A'B'C', we have:*

$$\pi - \delta + (i_1 - r_1) + (i_2 - r_2) = \pi \quad \Longrightarrow \delta = (i_1 + i_2) - (r_1 + r_2)$$

$$\Longrightarrow \boldsymbol{\delta = (i_1 + i_2) - \theta}$$

- ***In triagle AB'C', we have:***

$$\frac{\pi}{2} - r_1 + \frac{\pi}{2} - r_2 + \theta = \pi \qquad \Longrightarrow \boldsymbol{r_1 + r_2 = \theta}$$

Cases where the angles are small: $i_1 \cong i_2 = i \;;\; r_1 \cong r_2 = r \;;$

$$\Longrightarrow \begin{cases} \theta = 2r \\ \delta = 2i - \theta \end{cases} \Longrightarrow \begin{cases} r = \dfrac{\theta}{2} \\ i = \dfrac{\delta + \theta}{2} \end{cases}$$

Appling the Snell-Descartes law: $n_1 \sin(i) = n_2 \sin(r)$

$$\Longrightarrow \boldsymbol{n_2 = n_1 \frac{\sin\left(\frac{\delta + \theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}}$$

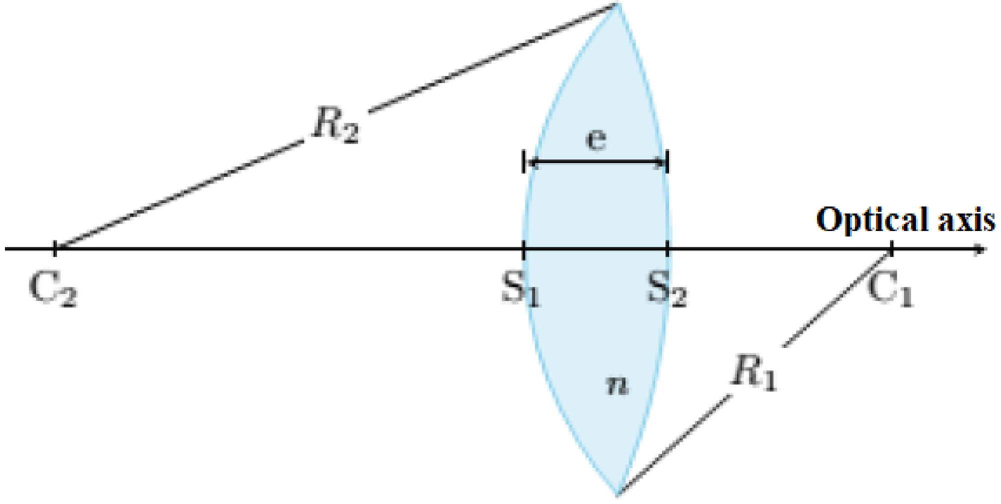
Thin lenses:

Description:

- A thin lens is a transparent medium, usually glass, formed by the combination of two spherical diopters, or a spherical diopter and a planar diopter, with large radii of curvature (R1, R2) compared to the thickness of the lens (e).

- If we note $R_1 = \overline{S_1C_1}$ and $R_2 = \overline{S_2C_2}$ the radii of curvature of the two diopters, S_1 and S_2 their respective centers and "e" the thickness of the lens, then:

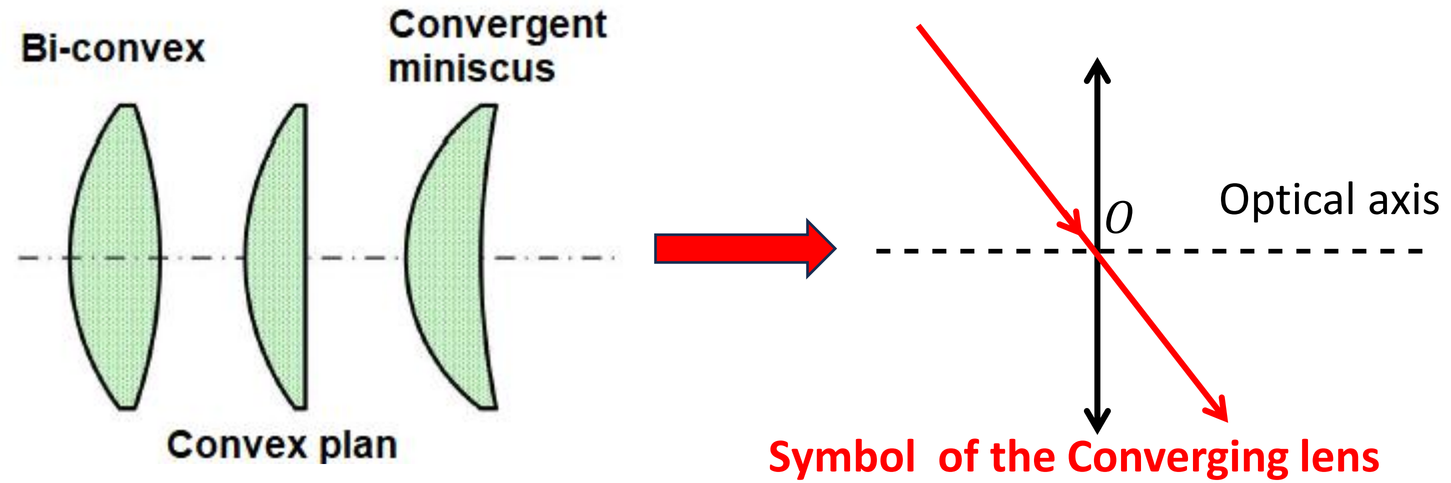
$$\boldsymbol{e \ll R_1, e \ll R_2}$$



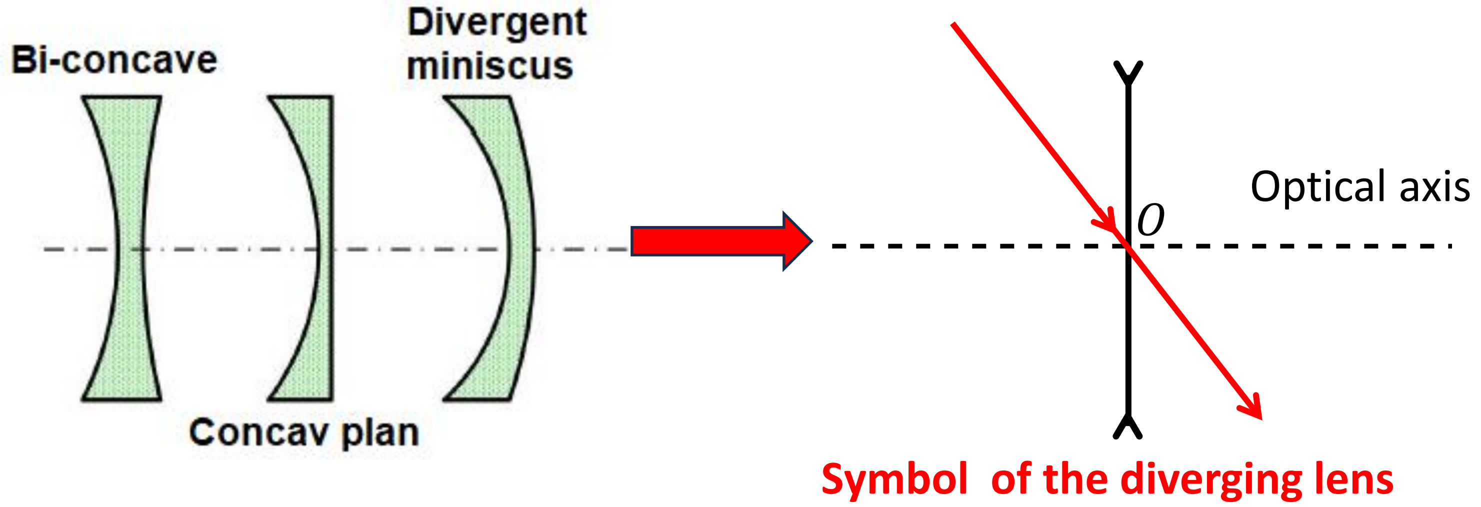
Optical lens

Types of thin lenses: Two types of thin lenses are distinguished:

1. Converging lenses with thin edges:



2. Divergent lenses with thick edges:



Characteristics of thin lenses:

- ***Optical Center:***

➢ The point O is the center of the lens, it is called the **optical center**.

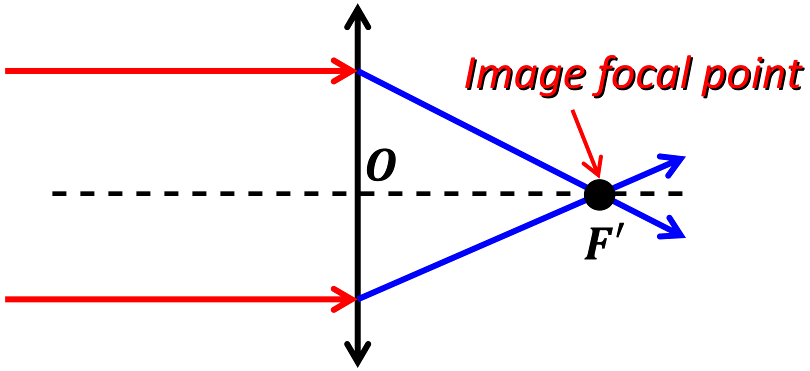
➢ The ray passing through the optical center is not deflected.

- ***Focal points of a thin lens***

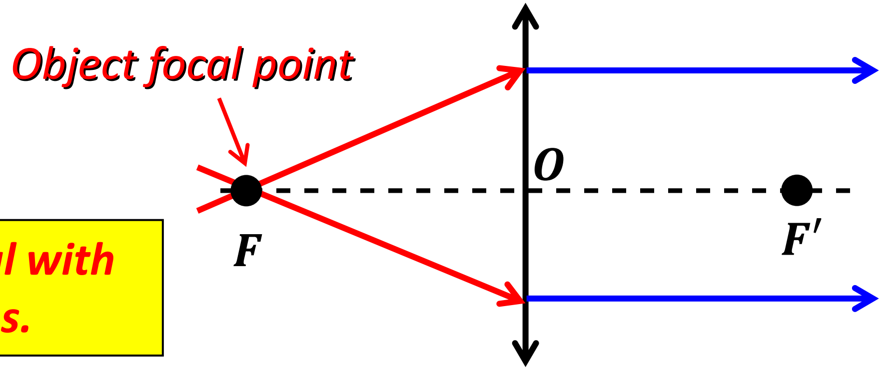
a) Converging lens:

A converging lens has two focal points, called the focal point and the image focal point,

- ***Any incident ray parallel to the optical axis emerges through F', (Image focal point).***



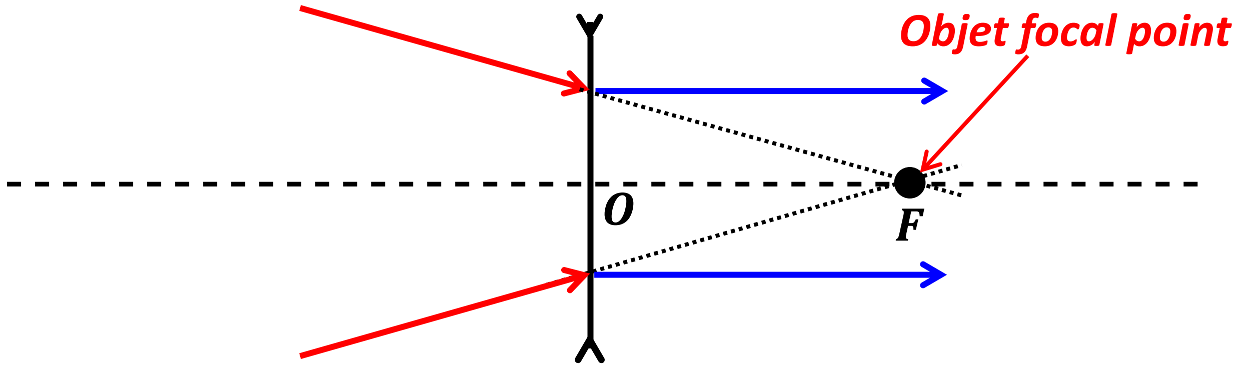
- ***Any incident ray passing through F, (object focal point), emerges parallel to the optical axis.***



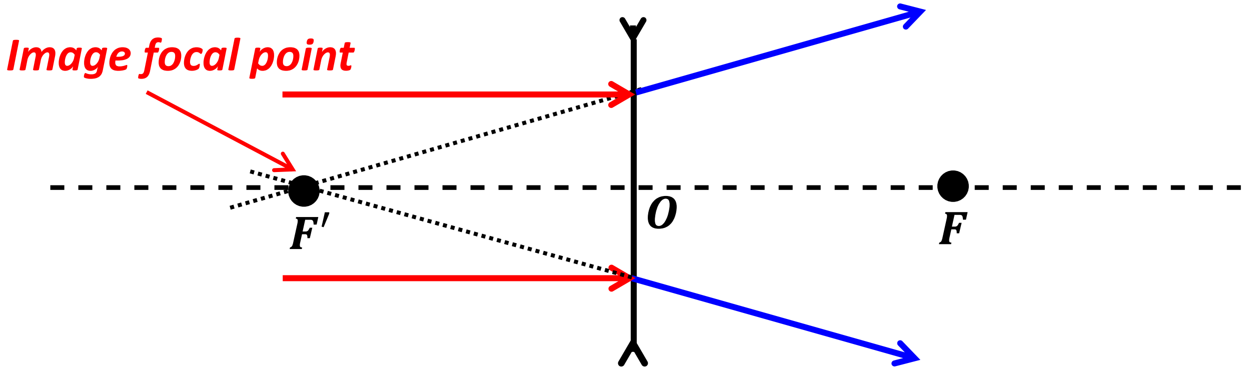
These focal points F and F' are symmetrical with respect to the optical center of the lens.

a) Diverging lens : A diverging lens also has two focal points, whose positions are reversed compared to those of a converging lens:"

- ***Any incident ray whose extension passes through F, Principal object focal point, emerges parallel to the optical axis.***



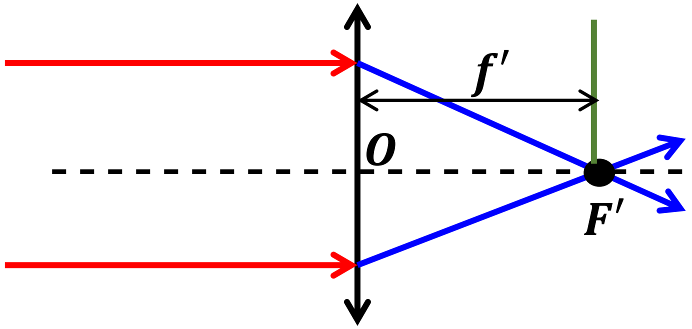
- ***Any incident ray parallel to the optical axis emerges in such a way that their extension passes throughF', Image focal point.***

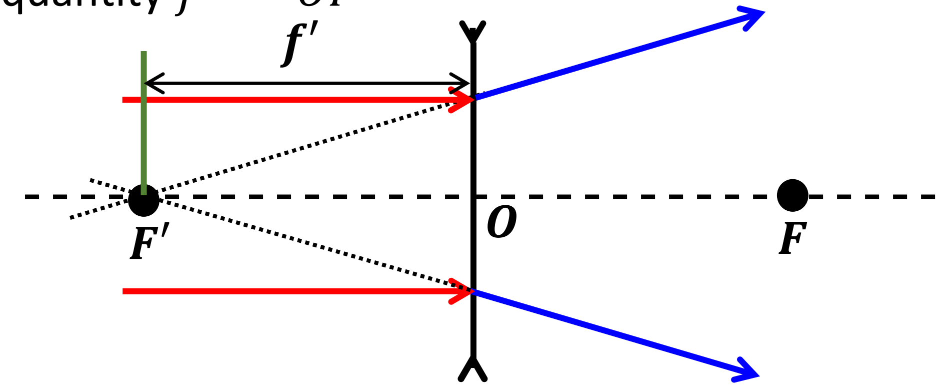


Characteristic quantities of a thin lens:

- ***Focal length:*** The optical axis is oriented in the direction of the incident light. We define algebraic quantities:

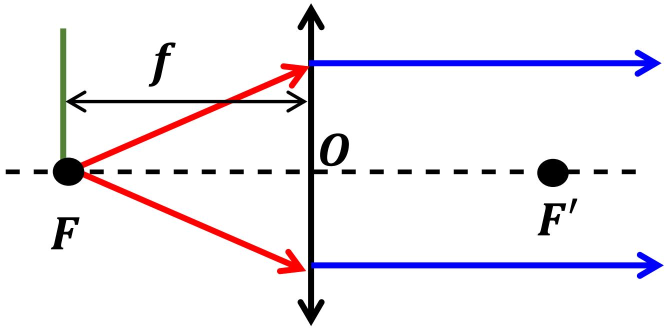
a) Image focal distance : We called image focal distance the quantity $f' = \overline{OF'}$

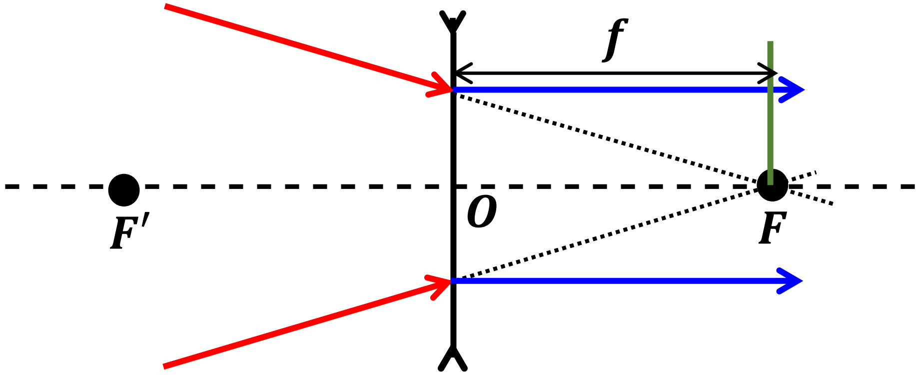




❑ if $f' > 0$, The lens is convergent. Si $f' < 0$, the lens is divergent.

b) Object focale distance: We called object focal distance the quantity $f = \overline{OF}$





Si $f > 0$, the lens is divergent. Si $f < 0$, la Lens is convergent.

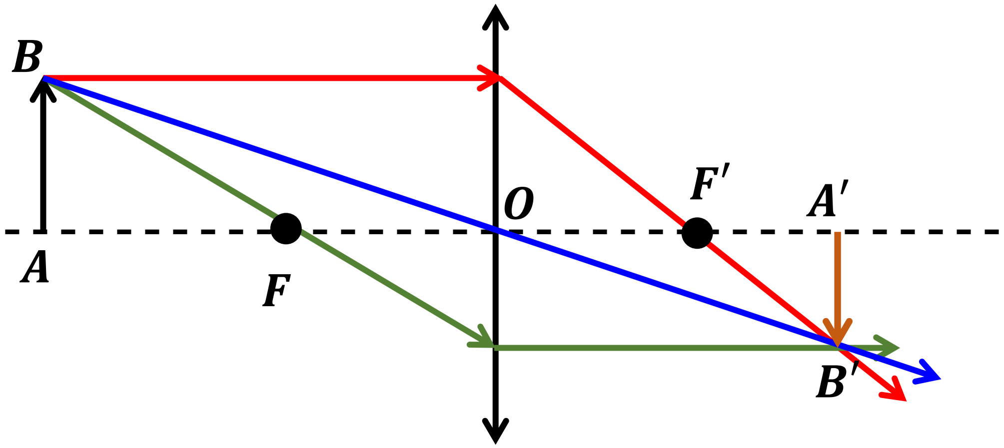
The vergence: The vergence C of a lens is the inverse of its image focal distance: $C = \frac{1}{f'}$

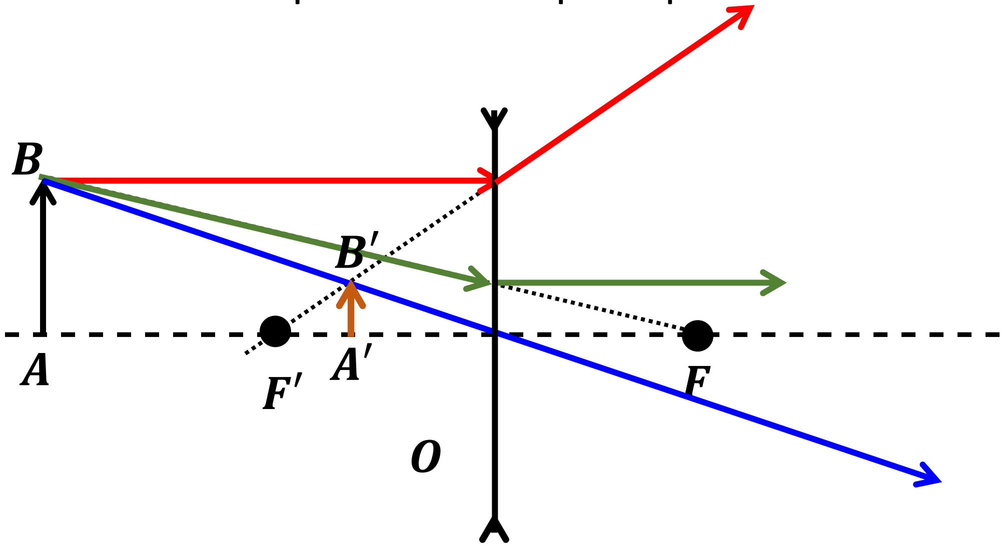
Vergence is expressed in diopters (δ). The image focal distance f' is expressed in meters (m)

Image construction :

To graphically determine the position of the image of an object by a lens, it is sufficient to apply the following rules:

- A ray passing through the optical center of a lens is not deflected.
- A ray parallel to the principal axis of a lens emerges through the principal focus image F'
- A ray passing through the principal object focus emerges from the lens parallel to its principal axis.





Characteristics of the image formed by a thin converging lens:

An object (AB) of height of 2cm is placed perpendicular to the optical axis of a converging lens of focal length $f = 2cm$

We take the object AB at different distances OA from the converging lens.

1st case: $\boldsymbol{OA > OF}$**:** We take $OA = 3cm$

- **The length of the image A'B' in relation to that of the object AB:**

The image A'B' is larger than the object AB (A’B’=4cm, AB=2cm)

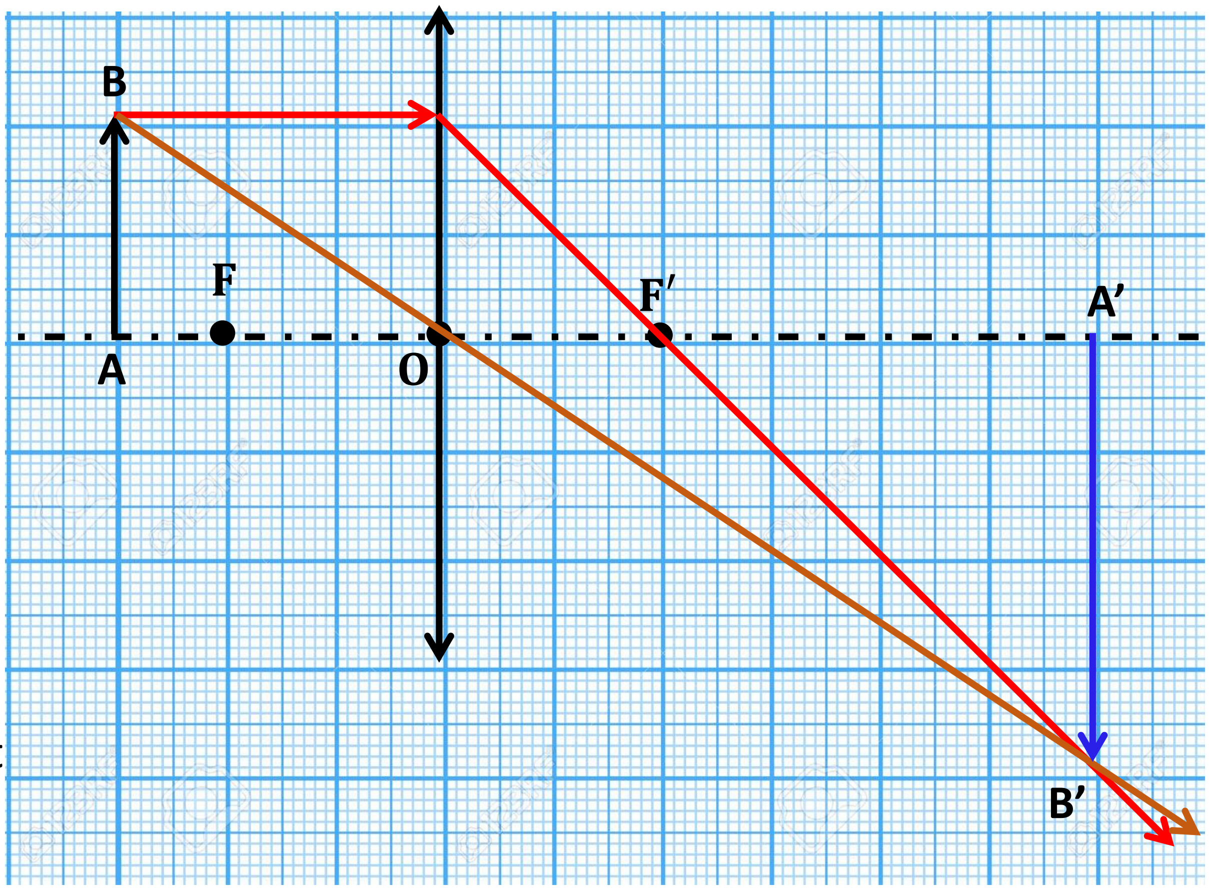
- **The position of** $A'B'$ **in relation to that of** AB**:**

La distance OA’ est plus grande que la distance OA

$$(OA = 3cm\ , \qquad OA' = 6\ cm)$$

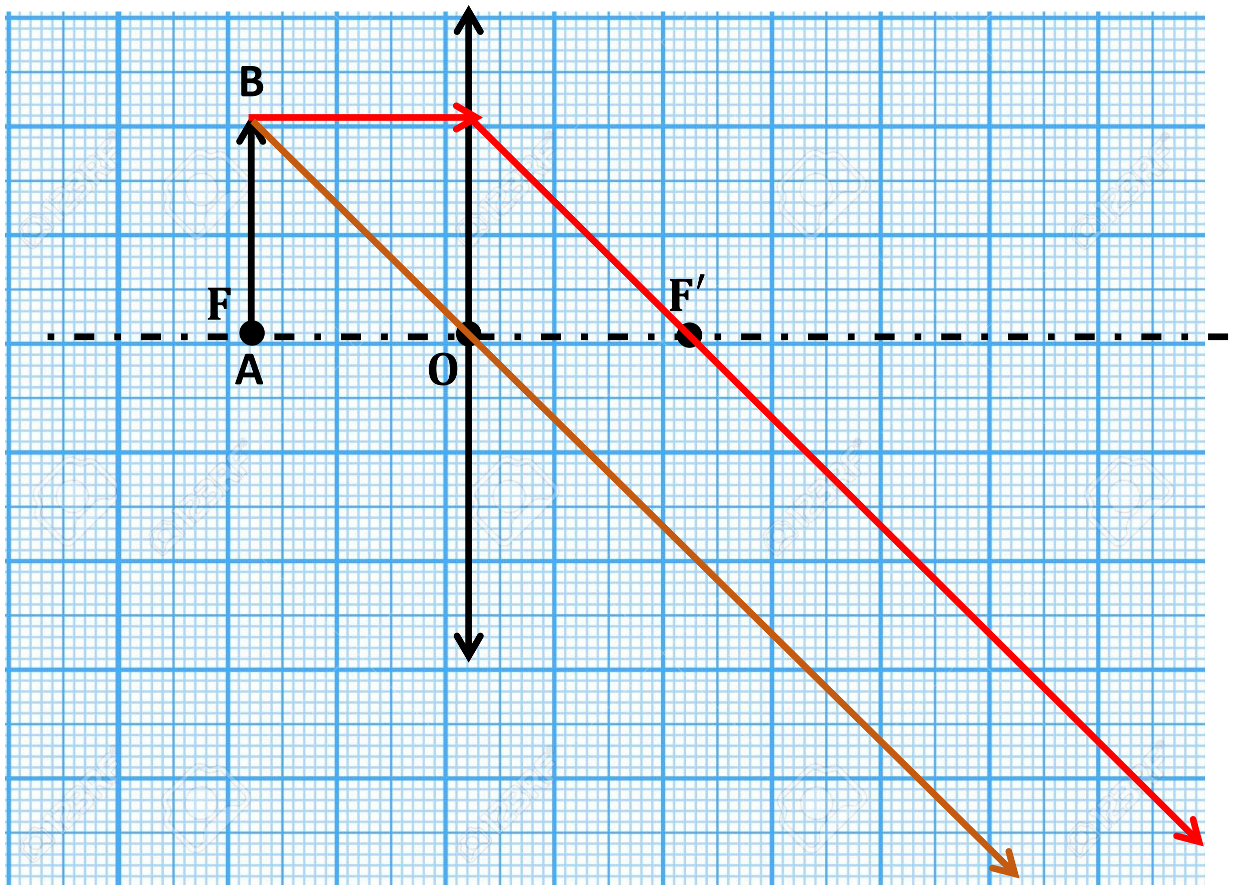
- **The image** $A'B'$ **is reversed.**
- **the A'B image is real because it is located after the lens (it is observable on the screen).**

We get a real image when the object is located before the object focal of the lens.



2nd case: $\boldsymbol{OA = OF}$**:** We take $OA = 2cm$

The light rays are parallel, so the image A'B' of the object AB is formed at infinity.



3 rd case: $OA < OF$: We take $OA = 1{,}5cm$

- **The length of the image A'B' in relation to that of the object AB:** The image A'B' is larger than the object AB

A'B'=4 cm , AB= 1cm

- **The position of A'B' in relation to that of AB:**

The distance OA' is greater than the distance OA

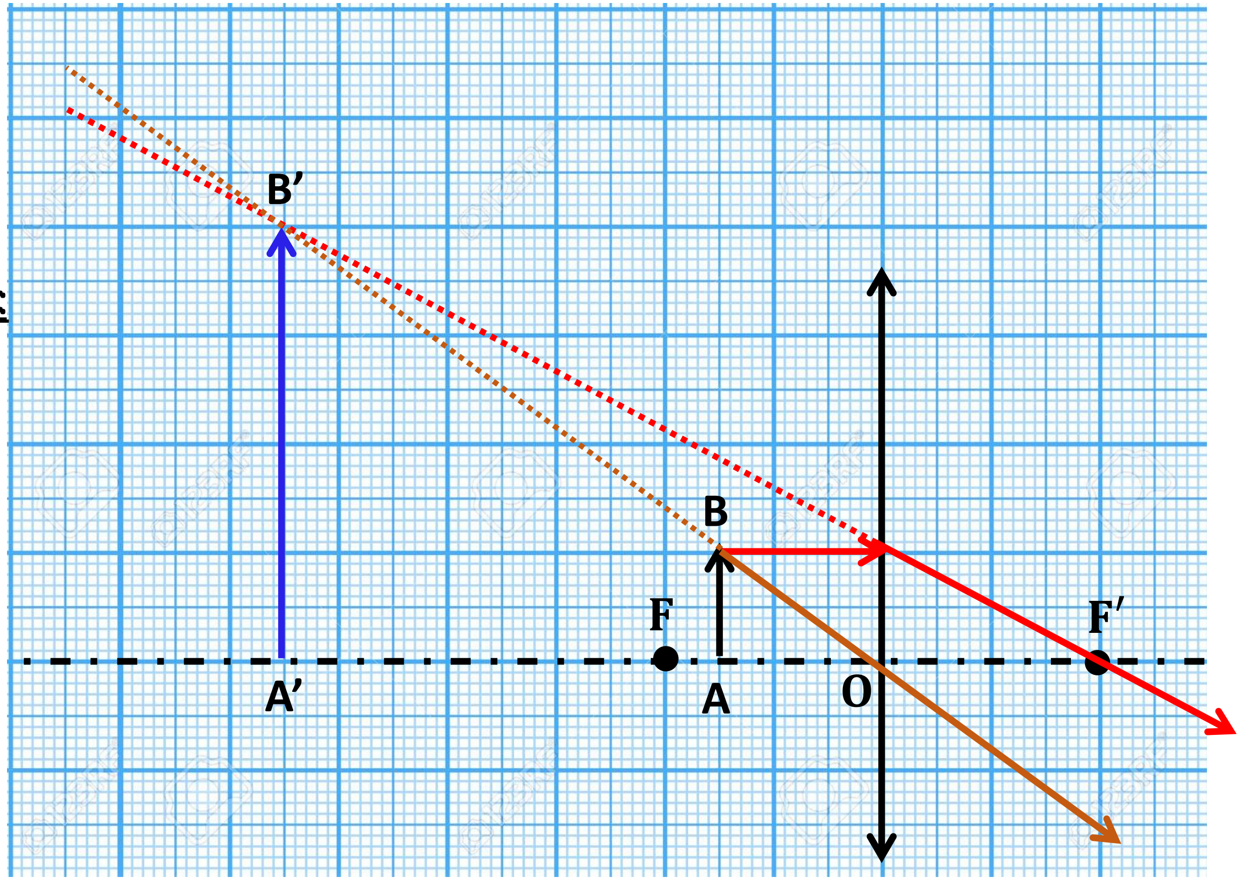
$(OA = 1{,}5\ cm\ , \quad OA' = 5{,}5\ cm)$

- **The length of the image A'B' in relation to that of the object AB:**

The image A'B' is larger than the object AB (A'B'=4cm, AB=1cm)

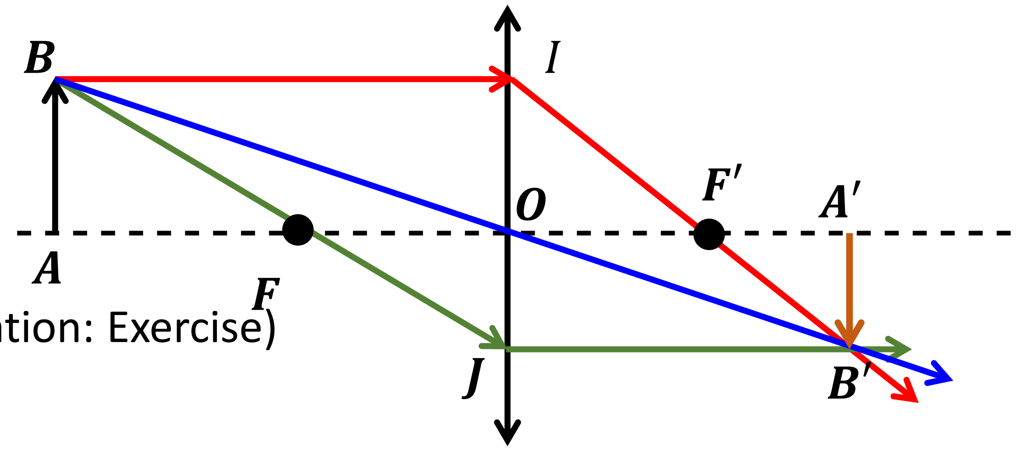
- **Image A'B' is straight (not reversed).**

- **the image A'B' is virtual because it is located before the lens (is not observable on the screen)**



Magnification: The ratio of object and image sizes defines the magnification γ.

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$



Conjugation relation:

The conjugation relation is given by: $\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}$ (Demonstration: Exercise)

Solution

$$\frac{\overline{A'B'}}{\overline{A'O}} = \frac{\overline{AB}}{\overline{AO}} \Rightarrow \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$ **(Relation of Thales)** **We can see from the diagram that:** $\overline{AB} = \overline{OI}$ et $\overline{A'B'} = \overline{OJ}$

$$\frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{A'B'}}{\overline{OI}} \text{ and } \frac{\overline{A'B'}}{\overline{A'F'}} = \frac{\overline{OI}}{\overline{OF'}} \Rightarrow \frac{\overline{A'B'}}{\overline{OI}} = \frac{\overline{F'A'}}{\overline{F'O}} \text{ And } \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OJ}}{\overline{AB}} = \frac{\overline{FO}}{\overline{FA}} \Rightarrow \frac{\overline{F'A'}}{\overline{F'O}} = \frac{\overline{FO}}{\overline{FA}} \Rightarrow \overline{FA}.\overline{F'A'} = \overline{FO}.\overline{F'O}$$

$$\Rightarrow \left(\overline{FO} + \overline{OA}\right).\left(\overline{F'O} + \overline{OA'}\right) = \overline{FO}.\overline{F'O} \Rightarrow \overline{FO}.\overline{F'O} + \overline{OA}.\overline{F'O} + \overline{FO}.\overline{OA'} + \overline{OA}.\overline{OA'} = \overline{FO}.\overline{F'O}$$

$$\Rightarrow \overline{OA}.\overline{F'O} + \overline{FO}.\overline{OA'} + \overline{OA}.\overline{OA'} = 0 \Rightarrow -\overline{OA}.\overline{OF'} + \overline{OF'}.\overline{OA'} + \overline{OA}.\overline{OA'} = 0 \Rightarrow -\overline{OA}.\overline{OF'} + \overline{OF'}.\overline{OA'} = -\overline{OA}.\overline{OA'} = 0$$

$$\Rightarrow -\frac{\overline{OA}.\overline{OF'}}{\overline{OA}.\overline{OA'}.\overline{OF'}} + \frac{\overline{OF'}.\overline{OA'}}{\overline{OA}.\overline{OA'}.\overline{OF'}} = -\frac{\overline{OA}.\overline{OA'}}{\overline{OA}.\overline{OA'}.\overline{OF'}} \Rightarrow -\frac{1}{\overline{OA'}.} + \frac{1}{\overline{OA}} = -\frac{1}{\overline{OF'}}$$

$$\boxed{\Rightarrow \frac{1}{\overline{OA'}.} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}}$$