General Introduction

The course **Differential Equations**, offered in **Semester 05** within the **Fundamental Teaching Unit**, constitutes a core component of the undergraduate mathematics curriculum. Differential equations represent one of the most powerful tools in mathematics, providing a natural framework for modeling change and describing the evolution of systems over time. They arise in a wide variety of scientific fields, including physics, engineering, biology, economics, and social sciences, which makes their study essential for both theoretical development and practical applications.

The primary objective of this course is to introduce students to the **fundamental concepts, methods, and theorems** governing the theory of **ordinary differential equations (ODEs)**. Emphasis is placed on the *qualitative study* of solutions rather than on computational techniques alone. The course aims to develop a rigorous understanding of the behavior of solutions, the conditions ensuring their existence and uniqueness, and their dependence on initial conditions. These notions form the theoretical foundation of modern analysis and are crucial for advanced studies in mathematics and applied sciences.

The course begins with an in-depth study of **first-order differential equations**, which serve as the cornerstone of the theory. Fundamental results are established in a rigorous analytical setting, including local and global existence theorems and uniqueness results. Particular attention is devoted to the continuous dependence of solutions on initial data, a key concept for understanding the stability and predictability of mathematical models. Through this study, students gain insight into how small variations in initial conditions can influence the behavior of solutions.

The course then extends to **higher-order differential equations**, demonstrating how they can be systematically reduced to systems of first-order equations. This approach provides a unified framework that allows the effective use of tools from linear algebra. By studying differential equations through the lens of systems, students develop a deeper understanding of their structural and geometric properties.

A major part of the course is dedicated to **linear systems of differential equations**. The theory of the *matrix exponential* is introduced as a fundamental method for solving linear systems, along with the study of second-order systems and the notion of the resolvent. These concepts are essential for analyzing the dynamics of linear systems and constitute a foundation for further studies in areas such as control theory, numerical analysis, and mathematical modeling. Throughout this chapter, the strong interaction between linear algebra and differential equations is emphasized.

In the final chapter, the course introduces the basic **notions of stability**, which play a central role in the qualitative theory of differential equations. Stability analysis allows students to understand the long-term behavior of solutions and to determine whether a system approaches equilibrium, exhibits oscillatory behavior, or becomes unstable. These ideas are fundamental in the study of dynamical systems and are of great importance in real-world applications where robustness and reliability are critical.

This course relies on prior knowledge of **Real Analysis, Linear Algebra, and Topology**, which provide the analytical and conceptual tools necessary for a rigorous treatment of differential equations. By integrating these prerequisite subjects, the course strengthens students' mathematical maturity and coherence in analysis.

With a workload corresponding to **6 credits** and a **coefficient of 4**, this course is designed to offer a solid theoretical foundation while preparing students for advanced studies in differential equations, dynamical systems, and applied mathematics. Upon successful completion, students will have acquired the essential analytical skills and conceptual understanding required to model, analyze, and interpret complex phenomena using ordinary differential equations.