General Introduction

The course **"Ordinary Differential Equations"** constitutes a fundamental module of the Master's program in *Mathematical Analysis and Applications*, Semester 1. It builds on the knowledge acquired at the undergraduate level, deepening the theoretical and qualitative study of differential equations, both in classical contexts and in abstract functional spaces. This discipline occupies a central place in the analysis of dynamic phenomena in applied mathematics, physics, engineering, and numerous scientific fields, as it allows modeling and understanding the evolving behavior of various systems.

The main objective of this course is the study of **Cauchy problems in infinite-dimensional Banach spaces**. This general framework requires the use of sophisticated analytical methods, among which *fixed-point theory* plays a crucial role. The latter serves as a fundamental tool for proving the existence and uniqueness of solutions, and it allows extending classical results obtained in finite-dimensional spaces to more abstract contexts. In parallel, the course addresses issues related to **the stability of solutions**, providing students with a deep understanding of the behavior of differential systems under various initial conditions and constraints.

The program begins with the **fundamentals of differential equations**, introducing the notion of a solution and the different types of solutions, including local, maximal, global, and saturated solutions. This classification allows for understanding the possible behaviors of solutions and precisely defining the contexts in which they are studied. Special attention is given to the **qualitative analysis of ordinary differential equations in finite dimension**, relying on classical theorems such as *Peano* and *Cauchy–Lipschitz*, which respectively guarantee the existence and uniqueness of solutions under appropriate conditions.

A natural extension of this study concerns **differential equations in infinite-dimensional spaces**, where the notions of convergence, continuity, and completeness play a critical role. Qualitative analysis in this framework allows for understanding complex phenomena and prepares students for advanced applications in functional analysis and dynamical systems. The theoretical tools developed here are essential for addressing scientific and technical problems involving continuous models and infinite-dimensional spaces.

A significant part of the course is devoted to **differential equations under constraints**, where the solution must remain within a given set. This study involves geometric concepts such as the *Bouligand–Severi tangent cone* and other notions of tangent cones, which formalize the conditions that an admissible trajectory must satisfy. The *Nagumo theorem* constitutes a key result in this context, providing a necessary and sufficient criterion for the viability of solutions within the constraint set.

Finally, the course addresses the **notion of stability**, an essential element for the study of dynamical systems. Students will analyze general properties of stability, the stability of linear differential systems, as well as stability in the sense of *Lyapunov*, which provides a robust theoretical framework to evaluate the persistence of solutions under small initial perturbations. This part enables understanding the long-term behavior of systems and anticipating the sensitivity of solutions, which is crucial in scientific modeling and technical applications.

In summary, this module offers a rigorous and comprehensive approach to ordinary differential equations, combining **theoretical foundations**, **qualitative analysis**, and **advanced concepts such as constraints and stability**. It provides a solid base for students, allowing them to pursue advanced studies in *functional analysis*, *dynamical systems*, and other applied areas of mathematics. Upon completing this course, students will be able to formulate, analyze, and solve complex differential problems, understand stability conditions, and integrate abstract tools into the study of dynamical systems.