Chapter II: Dynamics of a Material Point

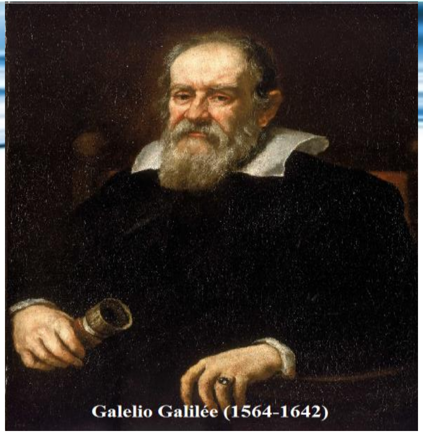
II.1. Objective :

The purpose of kinematics is to study the movements of particles as a function of time, without taking into account the causes that cause them.

Dynamics is the science that studies (or determines) the causes of the motions of these particles.

- Why do bodies near the surface of the earth fall with constant acceleration?
- Why does the earth move around the sun in an elliptical orbit ?
- Why do atoms bind together to form molecules?
  (Pourquoi les atomes se lient-ils entre eux pour former des molécules ?)
- Why does a spring oscillate when it is stretched?
  (Pourquoi un ressort oscille-t-il lorsqu'il est tendu ?)

II.2. The Law of Inertia (Galileo's law of Inertia):



Called Newton's first law, which reads as follows:

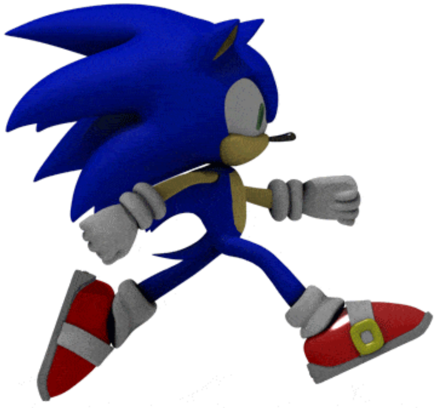
"Every body preservs in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by impressed forces".

Or

a free particle always moves with constant velocity, or without acceleration.

- ***In other words:*** *If no force acts on an object or if the resultant force is zero:*

- An object at rest remains at rest.



- A moving object contained to move at a constant velocity.

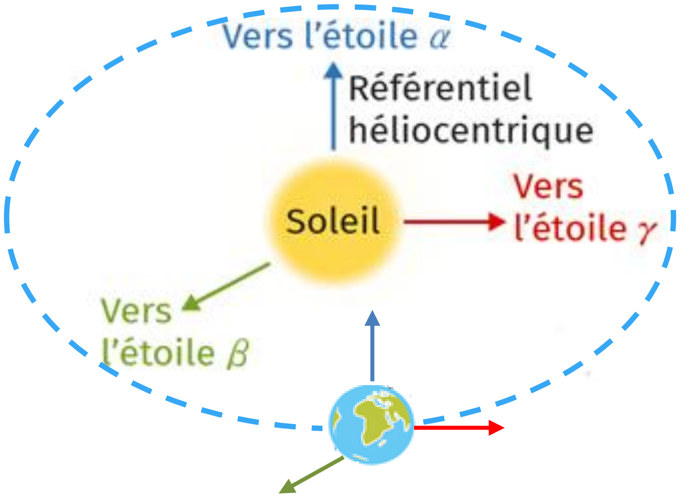
II.3. Inertial frame of reference (Galilean frame of reference):

Is defined as a frame of reference in which Newton's first law holds.

According to this definition, there is no such thing as an inertial frame of reference; Only approximate frames of reference are available.

Examples:

- For most experiments on Earth, the ground-bound frame of reference is a good inertial frame.
- whereas for the motion of the planets, this ground-bound frame of reference is not an inertial frame.
- **Copernican Frame of Reference (Heliocentric):** is the frame of reference centered on the center of mass of the solar system and whose three axes point to three distant stars.
- **Geocentric frame of reference:** is the frame of reference centered on the center of mass of the earth and whose axes are parallel to those of the Copernican frame of reference.



Remarks:

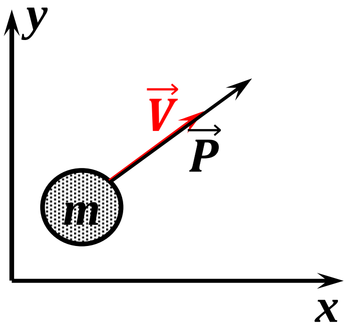
- Any coordinate system that moves at a constant velocity relative to an inertial frame of reference, can it self be considered as an inertial frame of reference.
- The velocities and accelerations of bodies, measured in Galilean reference frames, are said to be absolute, and those measured in non-Galilean reference frames are said to be relative.

II.4.Momentum (Quantity of motion:

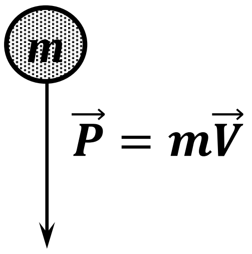
II.4.1. Definition: The momentum of a particle of mas of "m“ and moving at velocity $\vec{V}$ is defined by :

$$\vec{P} = m\vec{V} \qquad [\vec{P}] = Kg.m/s$$

- **2D Motion:**



falling mass



- The principle of inertia can then be stated as follows:

"A free particle moves with a constant momentum in a Galilean frame of reference"

Remark:

$$\frac{d\vec{P}}{dt} = \frac{d(m\vec{V})}{dt} = m\frac{d\vec{V}}{dt} = m\vec{a} = \vec{F}$$

$\Longrightarrow$ The derivative of the momentum vector of a body is equal to the sum of the external forces applied to that body:

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

II.4.2. Conservation of momentum:

A system is said to be isolated if it is not subject to any external (interaction) forces.

$$\vec{F} = \vec{0} \Longrightarrow m\frac{d\vec{V}}{dt} = \vec{0} \Rightarrow \frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \vec{P} = Cte$$

- **For a system of two particles with m_1 and m_2 isolated masses:**

The total momentum of the system at time t is:

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = m_1\vec{V}_1 + m_2\vec{V}_2$$

At the moment t' we have: $\overrightarrow{P'} = \overrightarrow{P'}_1 + \overrightarrow{P'}_2 = m_1\overrightarrow{V'}_1 + m_2\overrightarrow{V'}_2$

Isolated System $\Longrightarrow$ Total momentum is retained:

$$\vec{P} = \overrightarrow{P'} \Longrightarrow \vec{P}_1 + \vec{P}_2 = \overrightarrow{P'}_1 + \overrightarrow{P'}_2 \Longrightarrow \overrightarrow{P'}_1 - \vec{P}_1 = \vec{P}_2 - \overrightarrow{P'}_2$$

$$\Longrightarrow \overrightarrow{\Delta P_1} = -\overrightarrow{\Delta P_2}$$

- **For an isolated system of interacting "n" particles:** $\vec{P}_T = \sum_{i=1}^{n} \vec{P}_i = Cte$

Example:

A rifle of mass m of 0.8 **kg** fires a bullet of mass of 0.016 kg with a velocity of 700 m/s.

Calculate the recoil velocity of the rifle.

Solution:

The system consists of two bodies: Rifle + Bullet

Principle of conservation of momentum: $\vec{\boldsymbol{P}}_{\boldsymbol{Before}} = \vec{\boldsymbol{P}}_{\boldsymbol{After}}$

Before Shooting: Total momentum is zero

After Shooting: Total momentum: $\vec{\boldsymbol{P}}_{\boldsymbol{After}}$=$\vec{P}_R + \vec{P}_B$

$$\vec{P}_R + \vec{P}_B = \vec{0} \Longrightarrow m_f\vec{V}_F + m_B\vec{V}_B = \vec{0}$$

By projection:

$$m_R(-V_R)0 + m_B V_B = 0 \Longrightarrow V_R = \frac{m_B}{m_R}V_B$$

N.A:

$$V_R = \frac{0{,}016}{0{,}8}700 = 14\text{m/s}$$

II.5. Newtonian Definition of Force:

- Any cause capable of modifying the momentum vector of a material point, in a Galilean frame of reference, is called “ ***FORCE*** ".

- So, force is a mathematical notion that, by definition, is equal to the derivative of momentum with respect to time.

- We defined the average force, during a time interval Δt, as:

$$\vec{F}_{ave} = \frac{\overrightarrow{\Delta P}}{\Delta t}$$

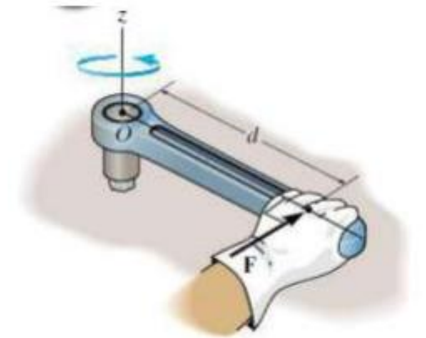
- The instantaneous force is therefore given by:

$$\vec{F}_{inst} = \vec{F} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta P}}{\Delta t} = \frac{d\vec{P}}{dt} = m\frac{d\vec{V}}{dt}$$

$$[\vec{F}] = Kg.ms^{-2} = Newton\ (N)$$

II.5.1. Moment of a Force about a Point (Torque):

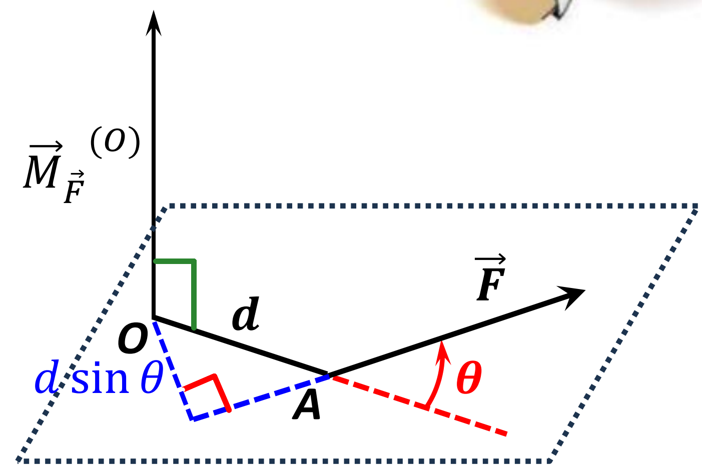
A moment of a force is the tendency of that force to cause a rotation of a body about an axis,



- ***Vector Expression***

The moment of the force $\vec{F}$ about the point ***O***, denoted $\vec{M}_{\vec{F}}^{(O)}$, is:

$$\boldsymbol{\vec{M}_{\vec{F}}^{(O)} = \overrightarrow{OA} \wedge \vec{F}}$$



$$\left\| \vec{M}_{\vec{F}}^{(O)} \right\| = \|\overrightarrow{OA}\| \|\vec{F}\| \sin\theta \; = F.d\sin\theta$$

$$\left[\vec{M}_{\vec{F}}^{(O)}\right] = N.m$$

- ***In other words:***

The magnitude of the moment of a force about a point is (the magnitude of the force) × (the perpendicular distance of the line of action of the force from the point).

Example:

Find the moment of $\vec{F}$ about P when $\theta = 35\circ, F = 8N$ and $d = 14m$.

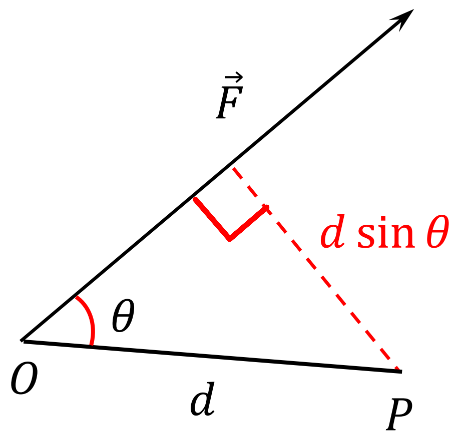
Solution:

$$\overrightarrow{M}_{\vec{F}}^{(P)} = \overrightarrow{PO} \wedge \vec{F}$$

$$\Longrightarrow \left\| \overrightarrow{M}_{\vec{F}}^{(P)} \right\| = \|\overrightarrow{PO}\| \|\vec{F}\| \sin\theta \quad ; \quad \|\overrightarrow{PO}\| = d$$

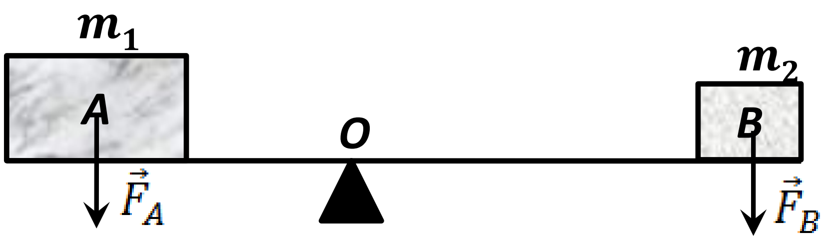
$$\Longrightarrow \left\| \overrightarrow{M}_{\vec{F}}^{(P)} \right\| = F.d \sin\theta$$

$$= 8.14. \sin 35° = 64{,}24\ Nm$$



II.5.2. Center of Inertia or Barycenter: (Center of Gravity)

In equilibrium, the sum of the moments of the forces about "O" equal zero:



(Clockwise moments will equal anticlockwise moments),

$$\sum \vec{M}_{\vec{F}_i}^{(O)} = \vec{0} \Rightarrow \vec{M}_{\vec{F}_A}^{(O)} + \vec{M}_{\vec{F}_B}^{(O)} = \vec{0} \qquad \Rightarrow \overrightarrow{OA} \wedge \vec{F}_A + \overrightarrow{OB} \wedge \vec{F}_B = \vec{0}$$

$$\Rightarrow \overrightarrow{OA} \wedge m_1\vec{g} + \overrightarrow{OB} \wedge m_2\vec{g} = \vec{0} \qquad \Rightarrow \left(m_1\overrightarrow{OA} + m_2\overrightarrow{OB}\right) \wedge \vec{g} = \vec{0}$$

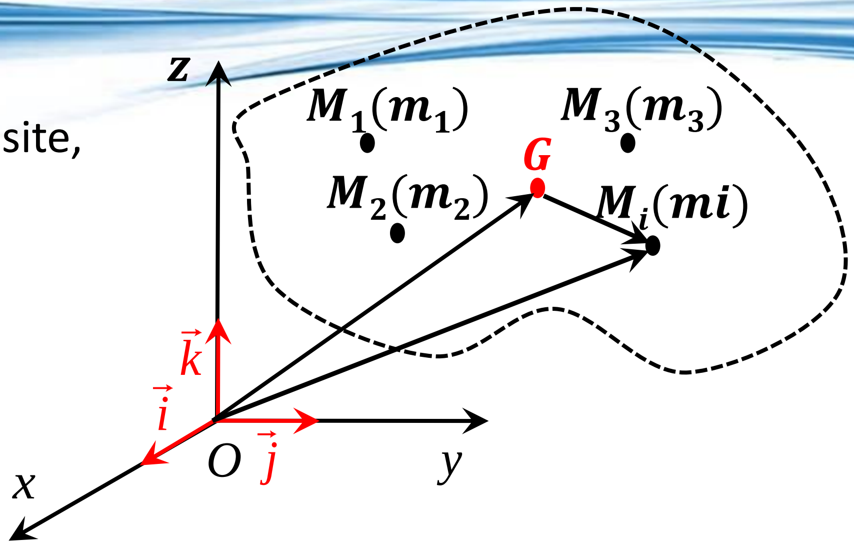
$$\boxed{\Rightarrow \boldsymbol{m_1\overrightarrow{OA} + m_2\overrightarrow{OB} = \vec{0}}}$$

For a system of m masses (G is a center of gravity):

$$m_1\overrightarrow{GM_1} + m_2\overrightarrow{GM_2} + \cdots m_n\overrightarrow{GM_n} = \vec{0} \qquad \Longrightarrow \sum_i m_i\overrightarrow{GM_i} = \vec{0}$$

On the other hand, according to the diagram opposite,

with ***G*** *is a center of gravity,* we have:



$$\overrightarrow{OG} + \overrightarrow{GM_i} = \overrightarrow{OM_i} \quad \Longrightarrow \overrightarrow{GM_i} = \overrightarrow{OM_i} - \overrightarrow{OG}$$

$$\sum_i m_i \overrightarrow{GM_i} = \vec{0} \quad \Longrightarrow \sum_i m_i \left(\overrightarrow{OM_i} - \overrightarrow{OG}\right) = \vec{0}$$

$$\Longrightarrow \sum_i m_i \overrightarrow{OM_i} = \sum_i m_i \overrightarrow{OG} \quad \Longrightarrow \overrightarrow{OG} = \frac{\sum_i m_i \overrightarrow{OM_i}}{\sum_i m_i}$$

$\sum_i m_i = M$, With M is the total mass of the system.

$$\Longrightarrow \overrightarrow{OG} = \frac{1}{M} \sum_i m_i \overrightarrow{OM_i}$$

This last relation gives the center of inertia of a system consisting of masses m_i located at the points M_i

- For a continuous environment, the sum becomes integral: $\overrightarrow{OG} = \frac{1}{M} \iiint \overrightarrow{OM} dM$

II.5.3.Newton’s Laws of Motion

- ***Newton's First Law:***

Newton’s first law states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

$$\vec{F} = \vec{0}, \qquad \vec{V} = Cst$$

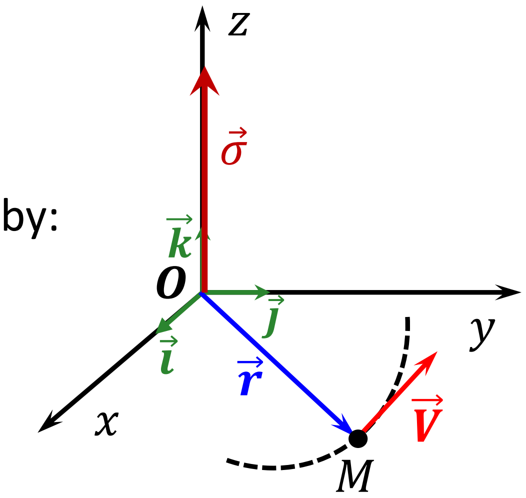
- ***Newton’s Second Law (Fundamental Principle of Dynamics):***

In a Galilean frame of reference, the sum of the external forces applied to a system is equal to the derivative of the momentum vector of the center of inertia of that system.

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{d\left(m\vec{V}\right)}{dt} = m\frac{d\vec{V}}{dt} = m\vec{a} \qquad (\boldsymbol{m = cts})$$

➢ ***Angular Momentum Theorem for a particle:***

Consider a particle M of mass m, moving in plan (O, x, y) with velocity vector $\vec{V}$ relative to inertial frame R.



The particle M has the momentum $\vec{P} = m\vec{V}$ relative to R.

The angular momentum $\vec{\sigma}$ (or $\vec{L}$) of M with respect to O is given by:

$$\vec{\sigma} = \overrightarrow{OM} \wedge \vec{P}$$

$$\Longrightarrow \vec{\sigma} = \vec{r} \wedge m\vec{V} = m\vec{r} \wedge \vec{V} \qquad \left(\vec{\sigma} \perp (\vec{r}, \vec{V})\right)$$

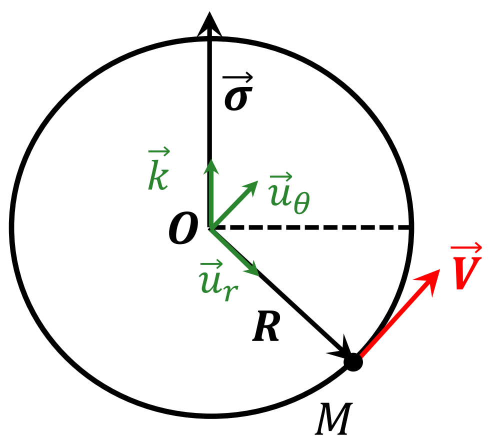
❖ ***In the case of a circular motion with constant velocity angular ω , we have:***

$$\vec{r} = R\vec{u}_r$$

$$\vec{V} = R\omega\vec{u}_\theta$$

$$\Longrightarrow \vec{\sigma} = mR^2\omega(\vec{u}_r \wedge \vec{u}_\theta)$$

$$\Longrightarrow \vec{\sigma} = mR^2\omega\vec{k}$$



❖ ***In the case of a planar curvilinear motion (Polar coordinates):***

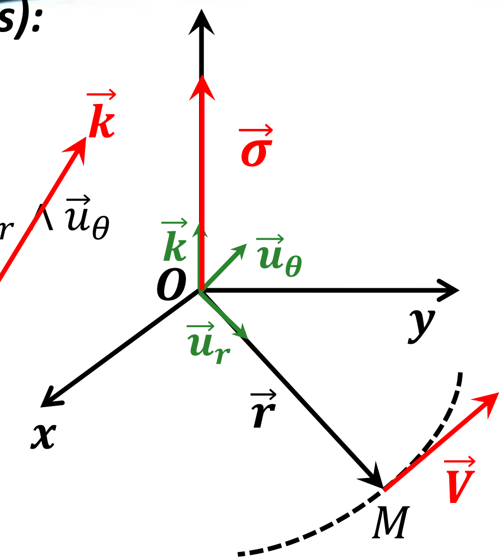
$$\overrightarrow{OM} = \vec{r} = r\vec{u}_r \qquad \vec{V} = V_r\vec{u}_r + V_\theta\vec{u}_\theta$$

$$\vec{\sigma} = m.\vec{r} \wedge \vec{V} = m.r\vec{u}_r \wedge (V_r\vec{u}_r + V_\theta\vec{u}_\theta) = m.rV_r\vec{u}_r \wedge \vec{u}_r + m.rV_\theta\vec{u}_r \wedge \vec{u}_\theta$$

0 0

$$\Rightarrow \vec{\sigma} = m.rV_\theta\vec{k}$$

$$V_\theta = r\frac{d\theta}{dt} \quad \Rightarrow \vec{\sigma} = mr^2\frac{d\theta}{dt}\vec{k}$$



❑ ***The derivative of $\vec{\sigma}$ with respect to time is given by:***

$$\frac{\overrightarrow{d\sigma}}{dt} = \frac{d\left(\vec{r} \wedge m\vec{V}\right)}{dt} = \frac{\overrightarrow{dr}}{dt} \wedge m\vec{V} + \vec{r} \wedge m\frac{\overrightarrow{dV}}{dt} = \vec{V} \wedge m\vec{V} + \vec{r} \wedge \frac{\overrightarrow{dP}}{dt} = \vec{r} \wedge \vec{F}$$

0

$\vec{F}$: is the resultant force

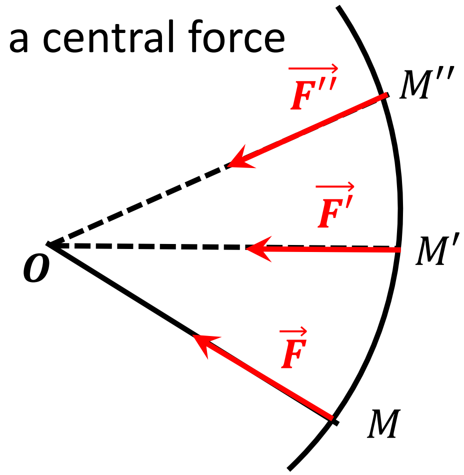
$$\Rightarrow \frac{\overrightarrow{d\sigma}}{dt} = \overrightarrow{M}_{\vec{F}}^{(O)}$$

→ ***(Moment of Force $\vec{F}$)***

***Theorem**: the derivative, with respect to time, of the angular momentum of a particle is equal to the moment of the force applied to it when both are measured with respect to the same point.*

❖ *In case of central Force:*

A force whose direction always passes through a fixed point is called a central force



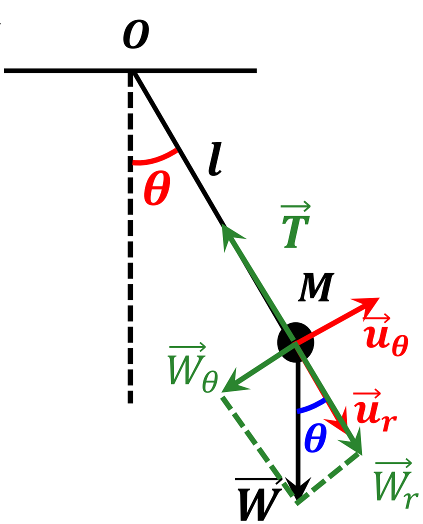
$$\vec{F} \parallel \overrightarrow{OM} \quad \Longrightarrow \frac{d\vec{\sigma}}{dt} = \overrightarrow{OM} \wedge \vec{F} = 0 \quad \Longrightarrow \vec{\sigma} = Cte$$

Exercise: *(Simple Pendulum)*

Find the differential equation to write the equation of motion

of a simple pendulum θ(t).

I- We apply the Newton's second law : $\sum \vec{F}_{ext} = m\vec{a} \quad \Rightarrow \vec{W} + \vec{T} = m\vec{a}$



By projection:

$\vec{u}_r$: $\quad W_r - T = ma_r$

$\vec{u}_\theta$: $\quad -W_\theta = ma_\theta$

$$\Rightarrow \begin{cases} mgcos\theta - T = -ml\left(\frac{d\theta}{dt}\right)^2 \dots\dots\dots..(1) \\ -mg\,sin\theta = ml\frac{d^2\theta}{dt^2} \dots\dots\dots\dots\dots(2) \end{cases}$$

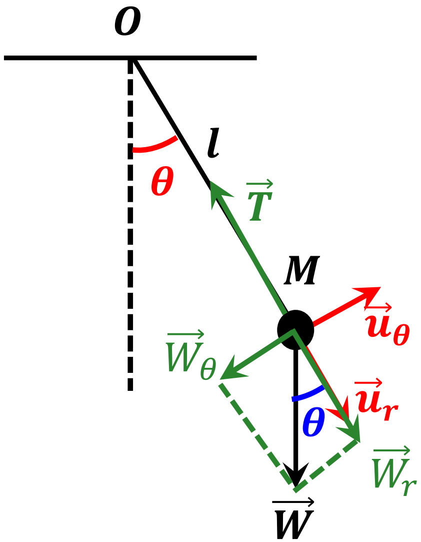
$$(2) \Leftrightarrow ml\frac{d^2\theta}{dt^2} + mgsin\theta = 0 \quad \Rightarrow l\frac{d^2\theta}{dt^2} + \frac{g}{l}sin\theta = 0$$

$$\vec{a} = \left(\frac{d^2r(t)}{dt^2} - r(t)\left(\frac{d\theta(t)}{dt}\right)^2\right)\vec{u}_r + \left(2\frac{dr(t)}{dt}\frac{d\theta(t)}{dt} + r(t)\frac{d^2\theta(t)}{dt^2}\right)\vec{u}_\theta$$

$$(sin\theta \approx \theta) \Longrightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

II- Let's apply the angular momentum theorem with respect to O :

$$\frac{\overrightarrow{d\sigma}}{dt} = \vec{M}_{\vec{F}}^{(O)} = \vec{M}_{\vec{W}}^{(O)} + \vec{M}_{\vec{T}}^{(O)}$$



We have: $\vec{\sigma} = \overrightarrow{OM} \wedge m\vec{v} = l\,\vec{u}_r \wedge ml\frac{d\theta}{dt}\vec{u}_\theta = ml^2\frac{d\theta}{dt}\vec{k}$ ***(circular motion)***

$$\Rightarrow \frac{\overrightarrow{d\sigma}}{dt} = ml^2\frac{d^2\theta}{dt^2}\vec{k} \;\ldots\ldots\ldots\ldots\ldots..(1)$$

On the other hand, we have:

- $\vec{M}_{\vec{T}}^{(O)} = \overrightarrow{OM} \wedge \vec{T} = l\,\vec{u}_r \wedge (-T\vec{u}_r) = \vec{0}$
- $\vec{M}_{\vec{W}}^{(O)} = \overrightarrow{OM} \wedge \vec{W} = l\,\vec{u}_r \wedge (mg\cos\theta\,\vec{u}_r - mg\sin\theta\,\vec{u}_\theta) = -lmg\sin\theta\vec{k}$

$$\Rightarrow \vec{M}_{\vec{W}}^{(O)} + \vec{M}_{\vec{T}}^{(O)} = -lmg\sin\theta\vec{k} \ldots\ldots\ldots(2)$$

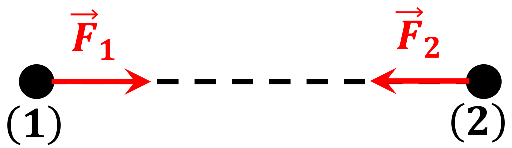
$$(1) = (2) \Leftrightarrow ml^2\frac{d^2\theta}{dt^2}\vec{k} = -lmg\sin\theta\vec{k} \quad \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

For small oscillations, we have: $\sin\theta \approx \theta$

$$\boxed{\Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0}$$

❑ ***Newton's Third Law (3rd law of dynamics: Principle of action and reaction):***

Let two particles (1) and (2) interacting with each other, the action of (1) on (2) $(\vec{F}_1)$ is equal and opposite to that exerted by (2) on (1) $(\vec{F}_2)$.



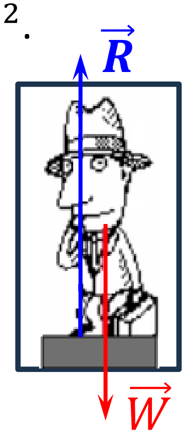
In the other word:

If a particle (1) exerts a force $(\vec{F}_1)$ on a particle (2), then (2) exerts a force $(\vec{F}_2)$ on (1) in the opposite direction with the same magnitude.

$$\vec{F}_1 = -\vec{F}_2 \quad (\|\vec{F}_1\| = \|\vec{F}_2\|)$$

Example:

A person of mass 85 kg is standing in a lift which is accelerating downwards at $0.45\ ms^{-2}$. Draw a diagram to show the forces acting on the person and calculate the force the person exerts on the floor of the lift.



Solution: using Newton's second law gives: $\sum \vec{F}_{ext} = m\vec{a}$

$$\Longrightarrow \vec{R} + \vec{W} = m\vec{a}$$

By projection: $W - R = ma \quad \Longrightarrow R = W - Ra \quad = mg - ma$

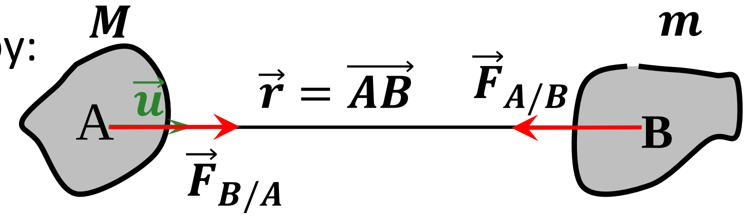
$$R = 795{,}6\ N$$

II.6. Some laws of forces:

II.6.1. Newton's Law of Universal Gravitation (1666):

This law explains the motions of the planets around the sun.

The force of attraction between $\boldsymbol{M}$ and $\boldsymbol{m}$ is given by:



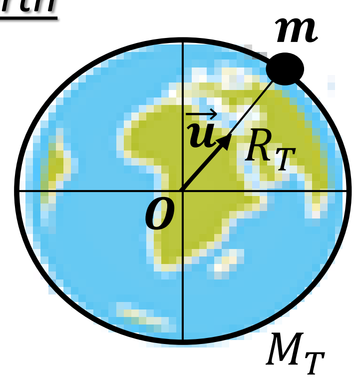
$$\vec{F}_{A/B} = -\frac{GMm}{r^2}\vec{u} \qquad \left(\vec{F}_{A/B} = -\vec{F}_{B/A}\right)$$

With:

$G = 6{,}67259.10^{-11}\, m^3 Kg^{-1} s^{-2}$: Universal gravitational constant

$$r = \left\|\overrightarrow{AB}\right\| \qquad \Longrightarrow \vec{F}_{A/B} = -\frac{GMm}{r^2}\frac{\overrightarrow{AB}}{\left\|\overrightarrow{AB}\right\|} = -\frac{GMm}{r^3}\vec{r}$$

Special case: *The weight of an object placed on the surface of the earth*



$$\vec{F} = -\frac{GM_T m}{{R_T}^2}\vec{u}$$

We posit : $\vec{g} = -\frac{GM_T}{{R_T}^2}\vec{u} \quad \Longrightarrow \vec{F} = m\vec{g}$

$\vec{g}$: Gravitational Field of Earth,

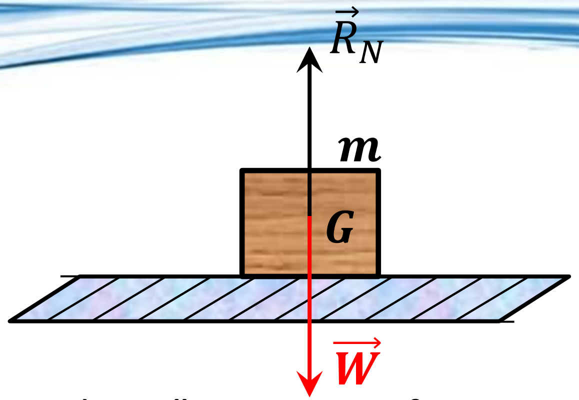
$(M_T = 5{,}9737 \times 10^{24}\ Kg\ ;R_T = 6371\ km\ \ ;G = 6{,}67259.10^{11}\ m^3Kg^{-1}s^{-2})$

- At the surface level of the earth: $g = g_0 = \frac{GM_T}{{R_T}^2} = 9{,}820251\ m.s^{-2}$
- At an altitude $\boldsymbol{h}$ of the earth's surface: $g = \frac{GM_T}{(R_T+h)^2} = \frac{GM_T}{(R_T+h)^2}\frac{{R_T}^2}{{R_T}^2}$

$$\Longrightarrow g = \frac{GM_T}{{R_T}^2}\left(\frac{R_T}{R_T + h}\right)^2 = g_0\left(\frac{R_T}{R_T + h}\right)^2$$

(Neglecting the rotational speed of the earth upon itself).

II.6.2. Contact forces:



❑ Support Reaction:

- The force that a mass m, placed on a horizontal support, undergoes from the support is called the ***"support force"***
- The support reaction on $\boldsymbol{m}$ is distributed over the entire "support-object" contact surface

  $\vec{R}_N$: Represents the resultant of all actions exerted on the contact surface.

- In equilibrium : $\vec{R}_N + \vec{W} = 0 \Longrightarrow \vec{R}_N = -\vec{W}$

❑ Frictional forces:

- Frictional forces are forces that appear:

  - Either when an object is moving (Soit lors de mouvement d'un objet),

  - Or that object is subjected to a force that tends to want to move it

    (Cet objet est soumis à une force qui tend à vouloir de le déplacé).

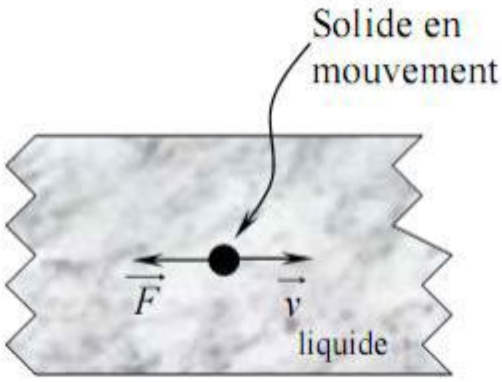
- We distinguish two types of friction forces:

- Viscous friction (contact: solid – fluid).
- Solid friction (contact: solid-solid).

❑ Viscous friction:

Viscous friction is related to the movement of an object $\boldsymbol{M}$ in a fluid medium (air, liquid or other)

At low velocities, the friction (in magnitude) is proportional to the velocity at which the object is moving.



Friction Force ← $\vec{F} = -k\vec{V}$ → **Object velocity**

↓

Positive constant

We give: $k = -K\eta$

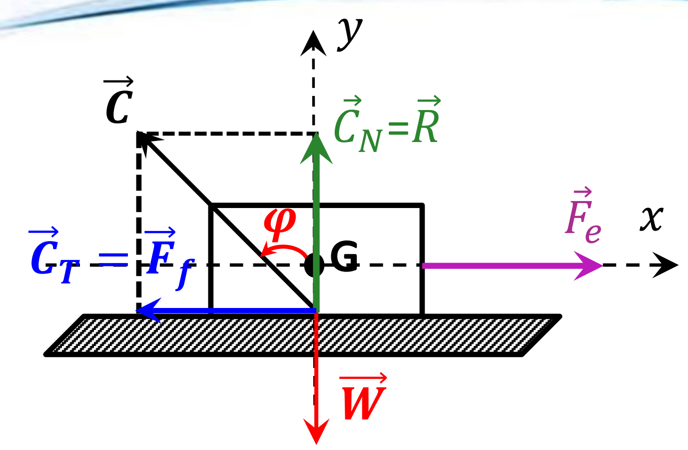
K: Depends on the geometric shape of the body

η: Fluid viscosity coefficient, depends on internal fluid friction,

Remark: For higher speeds, experiments have shown that the frictional forces in this case are given by:

$$\vec{F} = -kV^{n}\vec{u} \qquad \text{with } n \geq 2$$

❑ *Solid friction:*



$\vec{F}_e$: Force of entrainment $\vec{C}$: Contact force

$\vec{C}_N = \vec{R}$: Surface reaction force

$\vec{C}_T = \vec{F}_f$: Friction force (Sliding friction)

- The body is initially at rest;
- We increase gradually the value of $\vec{F}_e$
- Each time $\vec{F}_e$ e is increased, the value of the frictional force $\vec{F}_f$ increases until it reaches a maximum value $\vec{F}_{f0} = \vec{C}_{T0}$ which corresponds to the beginning of the object's slippage. $\Longrightarrow$ This position is called: Limit equilibrium state,

Applying the Newton's second law in this case: $\sum \vec{F}_{ext} = \vec{0} \quad \Longrightarrow \vec{W} + \vec{C} + \vec{F}_e = \vec{0}$

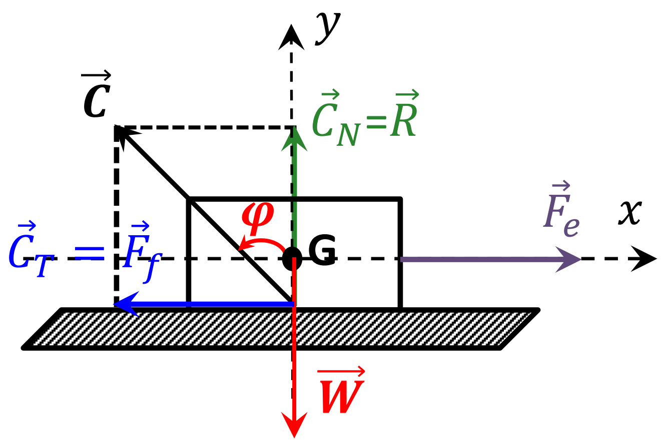
❖ ***By projection on the (Ox) and (Oy) axes:*** $\begin{cases} F_e - C_{T0} = 0 \\ C_{N0} - W = 0 \end{cases} \Longrightarrow \begin{cases} C_{T0} = F_e \\ C_{N0} = W \end{cases}$

- The static coefficient of friction is defined as:

$$\boldsymbol{\mu_s = tg\varphi = \frac{C_{T0}}{C_{N0}}}$$: characterizes the limit equilibrium state

- When $\vec{F}_e > \vec{F}_{f0}$, the object begins to move from its steady state with uniformly accelerated motion

  - Applying the Newton's second law in this case: $\sum \vec{F}_{ext} = m\vec{a} \Longrightarrow \vec{W} + \vec{C} + \vec{F}_e = m\vec{a}$



By projection on the (Ox) and (Oy) axes:

$$\begin{cases} F_e - C_T = ma \\ C_N - W = 0 \end{cases} \Longrightarrow \begin{cases} C_T = F_e - ma \\ C_N = W \end{cases}$$

- The dynamic coefficient of friction is then defined:

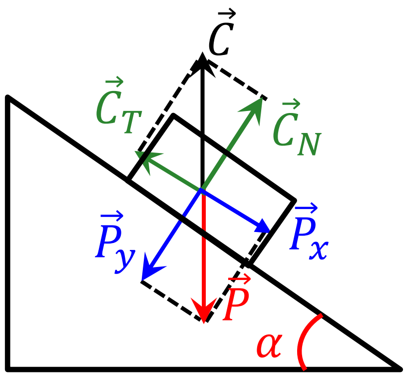
$$\mu_d = tg\varphi = \frac{C_T}{C_N} = \frac{F_e - ma}{mg}$$

<u>Remarks:</u>

- ❑ μ_s and μ_d depend on the nature of the surfaces in contact,
- ❑ μ_d is less than μ_S
- ❑ μ_d is substantially independent of speed
- ❑ μ_d is substantially independent of the surface area of the surfaces in contact and depends only on their nature

Application: Inclined Plane

- **At the limit equilibrium state:** $\sum \vec{F}_{ext} = \vec{0} \implies \vec{P} + \vec{C}_0 = \vec{0}$



By projection:

$$\begin{cases} P_x - C_{T0} = 0 \\ C_{N0} - P_y = 0 \end{cases} \implies \begin{cases} Psin\alpha_0 = C_T \dots\dots\dots\dots (1) \\ Pcos\alpha_0 = C_N \dots\dots\dots (2) \end{cases}$$

$$(1)/(2) \implies tg\alpha_0 = \frac{C_T}{C_N} = \mu_S$$

- **In the state of motion:** $\alpha_0 \to \alpha \quad (\alpha = \alpha_0 + d\alpha)$

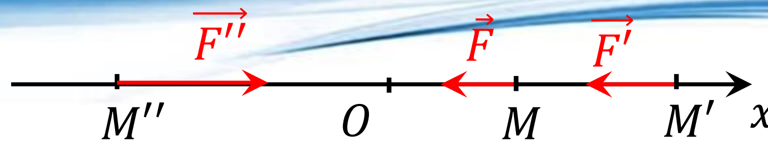
$$\sum \vec{F}_{ext} = m\vec{a} \implies \vec{P} + \vec{C} = m\vec{a}$$

By projection:

$$\begin{cases} P_x - C_T = ma \\ C_N - P_y = 0 \end{cases} \implies \begin{cases} Psin\alpha - ma = C_T \dots (1) \\ Pcos\alpha = C_N \dots\dots\dots (2) \end{cases}$$

$$\mu_d = tg\alpha = \frac{C_T}{C_N} = \frac{Psin\alpha - ma}{Pcos\alpha} == \frac{gsin\alpha - a}{gcos\alpha}$$

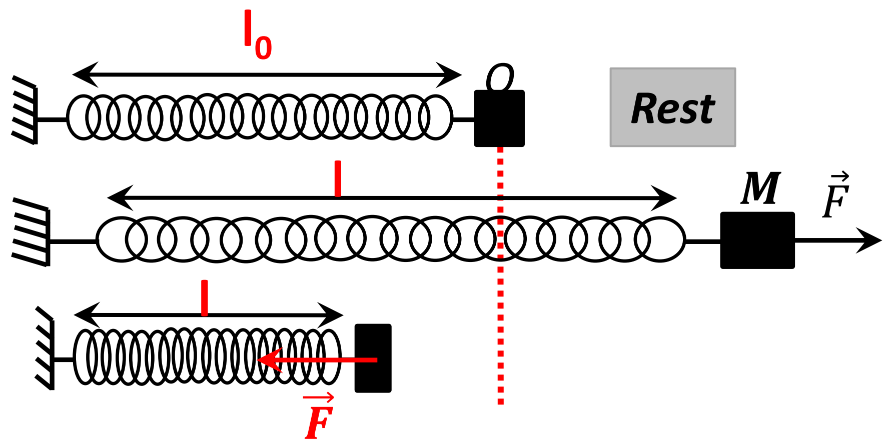
3. Elastic Strength:



$\vec{F} = -k\overrightarrow{OM} \implies$ proportional and opposite to the position vector $\overrightarrow{OM}$

k : Stiffness Constant

By projection on the axis (Ox): $\vec{F} = -kx\vec{\imath}$

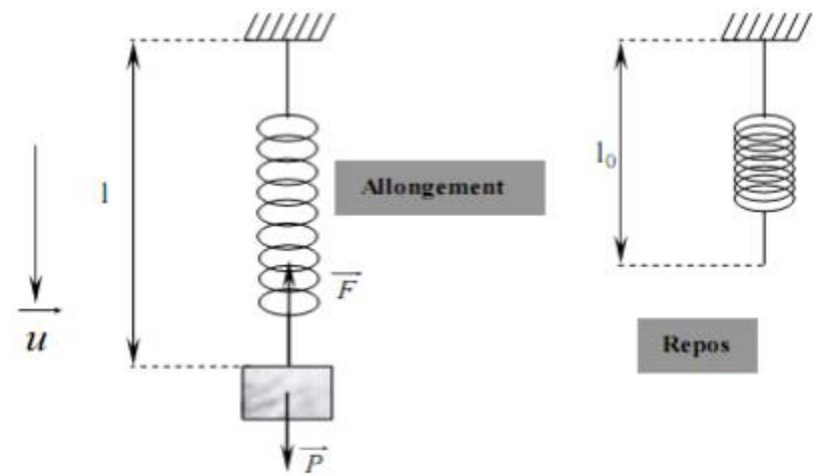


Example:

$$\vec{F} = -k\overrightarrow{OM} = -k(l - l_0)\vec{\imath}$$

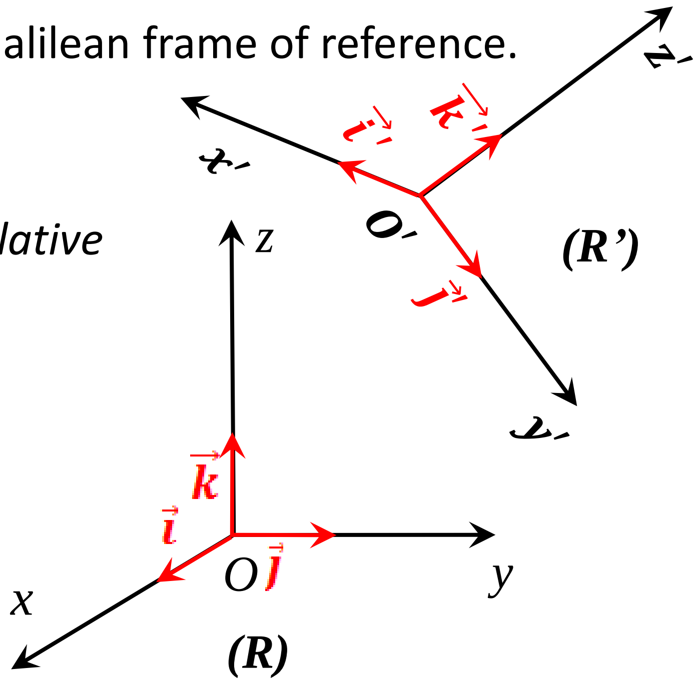
Or

$$\vec{F} = -k(l - l_0)\vec{u}$$



II.6. Fundamental Principle of Dynamics in a Non-Galilean Frame of Reference

- Let (R)a Galilean frame of reference and (R') a non-Galilean frame of reference.



- *R' is in moving relative to R.*

$\Longrightarrow$ *R is the absolute frame of reference and R' is the relative frame of reference*

$$\vec{a}_a = \vec{a}_r + \vec{a}_e + \vec{a}_C$$

$$\Longrightarrow \sum \vec{F}_{ext} = m\vec{a}_a \quad = m\vec{a}_r + m\vec{a}_e + \vec{a}_C$$

<u>In the R' coordinate system, the PFD is:</u>

$$m\vec{a}_r = m\vec{a}_a - m\vec{a}_e - \vec{a}_C \ = \sum \vec{F}_{ext} + \vec{F}_e + \vec{F}_C$$

$\vec{F}_e = -m\vec{a}_e$est la force d'inertie d'entraînement,

$\vec{F}_C = -m\vec{a}_C$ is the Coriolis force of inertia,

$\vec{F}_e$ et $\vec{F}_C$ are non-real forces, they depend on the motion of R'/R.