

Part 3 : Electrokinetics

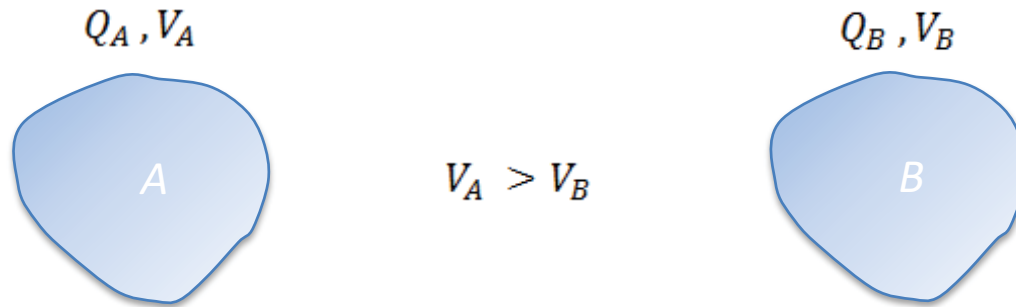
Electrokinetic

This topic is fundamental in medicine because it underlies essential physiological phenomena such as:

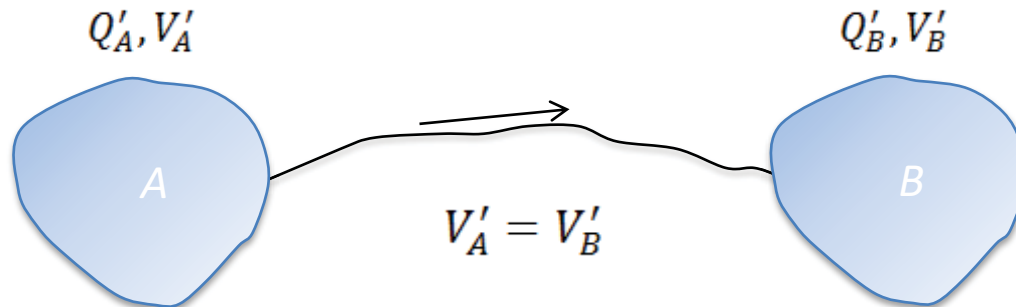
- The propagation of nerve impulses.
- The electrical activity of the heart.
- Membrane ion transport, and the functioning of medical devices such as ECGs and bioelectric sensors.

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1. Rupture of an electrostatic equilibrium → Electric current



When connecting A and B with a conductive wire,



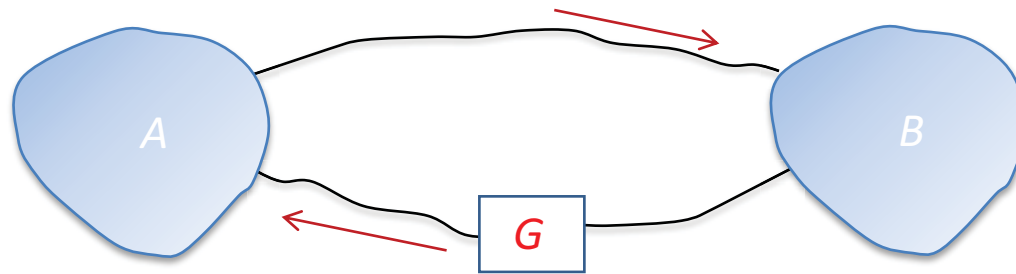
During this movement, the total charge of conductors A and B is conserved:

$$\begin{aligned}Q_A + Q_B &= Q'_A + Q'_B \\ \Delta Q_A &= Q'_A - Q_A, \quad \Delta Q_B = Q'_B - Q_B \\ -\Delta Q_A &= \Delta Q_B\end{aligned}$$

$V_A > V_B \rightarrow$ Electric current can be thought of as the transport of positive charges from the higher potential to the lower potential.

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Obtaining a steady current :

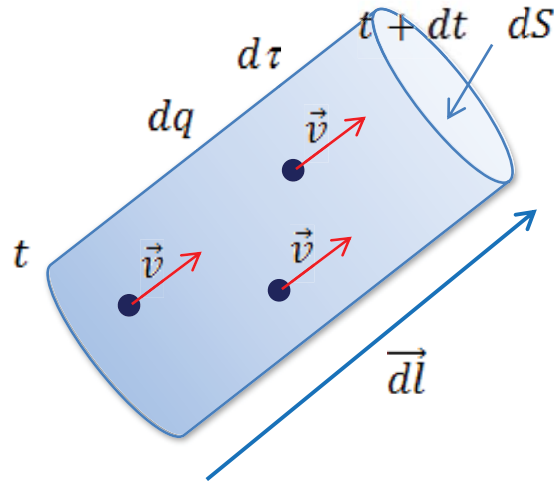


The generator does not create charges; it makes them circulate while maintaining a constant voltage (potential difference) between its terminals.

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2. Electric Current

The electrical charge Q flows through section S of a conductor during time t (supplied in steady state). The current intensity is:

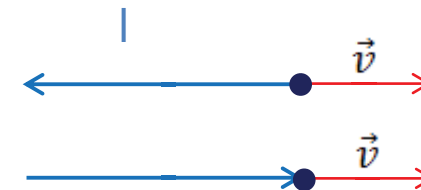


$$I = \frac{dq}{dt}$$

$$[I] = \frac{[q]}{[t]} = \frac{C}{s} = \text{Ampere (A)}$$

By convention, the direction of current is that of the vector $q\vec{v}$

$q < 0$, electrons
 $q > 0$, cations



Stationary state

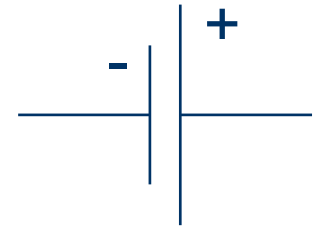
For a given circuit, if the potentials V_i at different points in the circuit are constant over time, then the electric current is the same throughout the circuit the system is said to be in a stationary state.

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➤ Voltage generator

In simple terms, a voltage generator can be defined as a device that maintains a constant potential difference (ddp) between its terminals.

The schematic representation of a voltage generator is :



➤ Electrical receiver

An electrical receiver is a general term for a device capable of converting electrical energy into chemical, mechanical, thermal energy, etc ...

- Resistance and resistivity

➤ Ohm's law

Consider a given conductor (at constant temperature). If the ratio between the potential difference (ddp) $[V_A - V_B]$ between two points A and B at electric current I is constant, the conductor is said to be ohmic, and it is possible to write:

$$R = \frac{(V_A - V_B)}{I}$$

R is the electrical resistance

➤ Concept of Resistivity

Application to a homogeneous cylindrical conductor

$$\rho = \frac{R S}{L}$$

ρ : resistivity

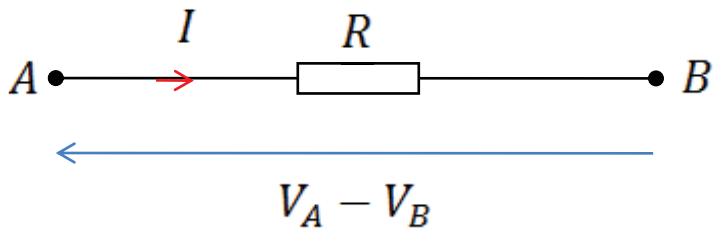
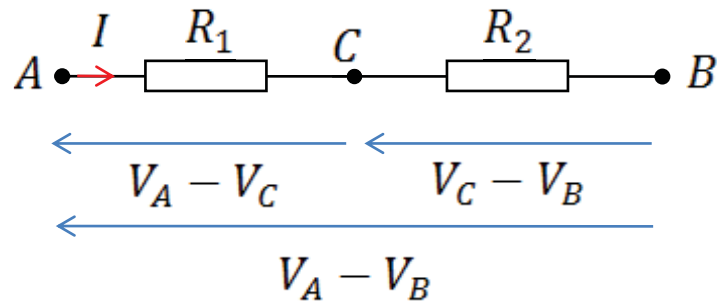
S : Conductor cross-section

L : length considered

6. Resistor Combinations

There are two types of resistor configurations:

a) Series connection



$$V_A - V_B = (V_A - V_C) + (V_C - V_B)$$

$$RI$$

$$R_1I + R_2I$$

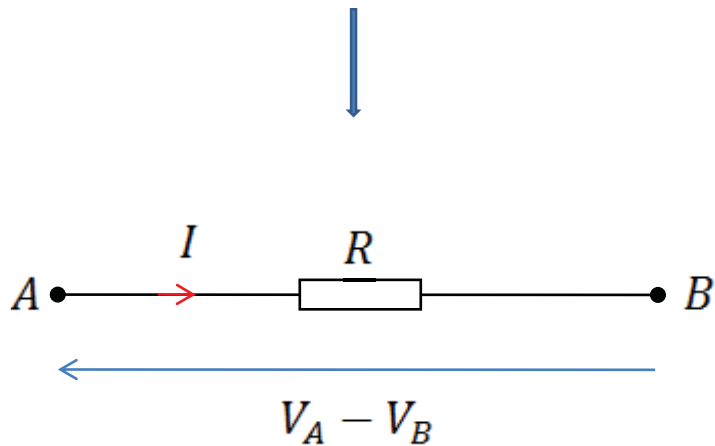
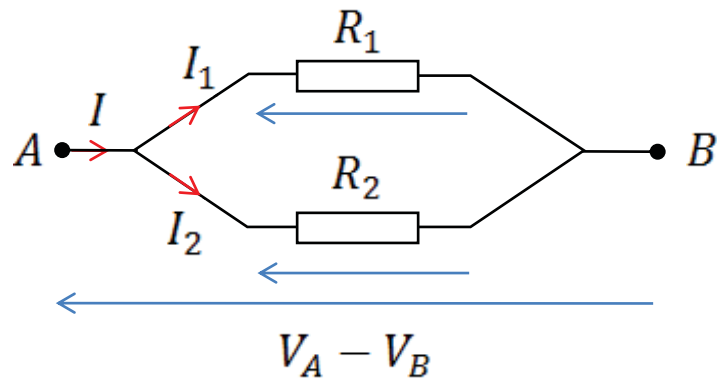
$$I \neq 0$$

$$R = R_1 + R_2$$

For n resistors :

$$R = \sum_{i=1}^n R_i$$

b) Parallel connection



$$I = I_1 + I_2$$
$$\frac{V_A - V_B}{R} = \frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2}$$

$$V_A \neq V_B$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

For n resistors :

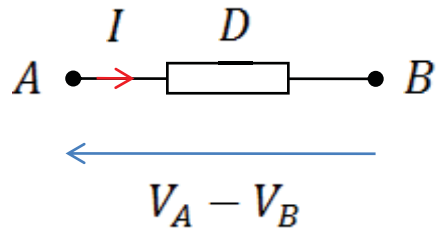
$$\frac{1}{R} = \sum_{i=1}^n \frac{1}{R_i}$$

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7. Electrical power and energy (Joule's law)

a) Electrical power

Let:



$$U_{AB} = V_A - V_B$$

The power received by D is:

$$P = U_{AB} \cdot I, \quad [P] = \text{Watt}(W)$$

If $P > 0$ then D receives this power,
If $P < 0$ then D supplies this power.

The Case : $D = R \rightarrow U_{AB} = R \cdot I$

$$P = R \cdot I^2 = \frac{U_{AB}^2}{R}$$

Power dissipated by the Joule effect

b) Electrical energy

If during time t , D consumes P , it receives:

$$W = P \cdot t, \quad [W] = [P][t] = W \cdot s = \text{Joule}(J)$$

$$1kWh = 10^3 W \times 3600 s = 3.6 \times 10^6 J$$

$$W = U_{AB} \cdot I \cdot t = U_{AB} \cdot Q$$

The Case : $D = R \rightarrow U_{AB} = R \cdot I$

$$W = R \cdot I^2 \cdot t \quad \text{Joule Law}$$

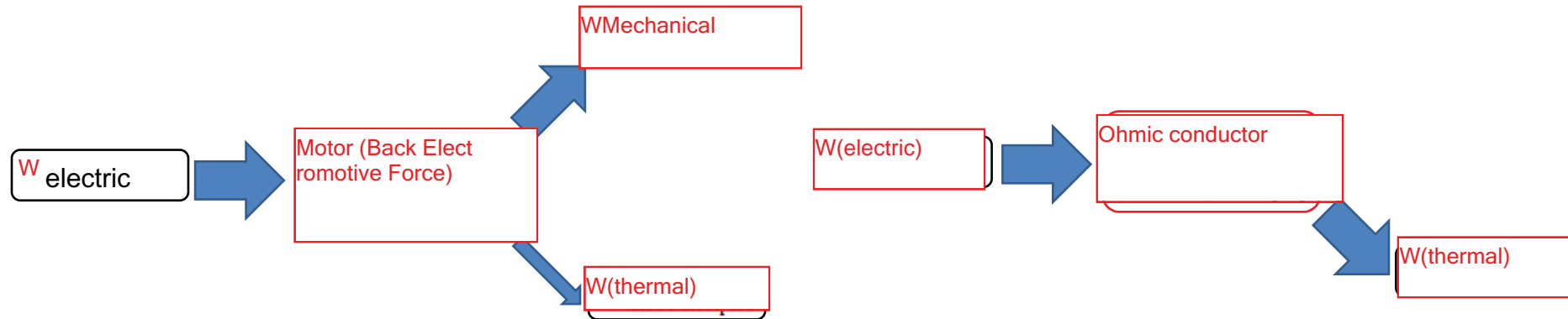
Principle of conservation of energy:

received Energy = Lost Energy + Useful energy

$$W_r = W_p + W_u$$

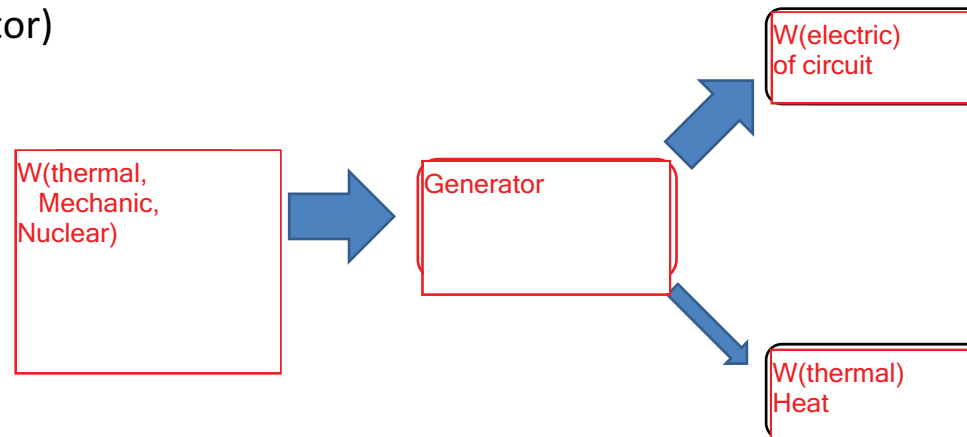
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c) Joule effect in receptors (motor, ohmic conductor)



d) Joule effect in a generator

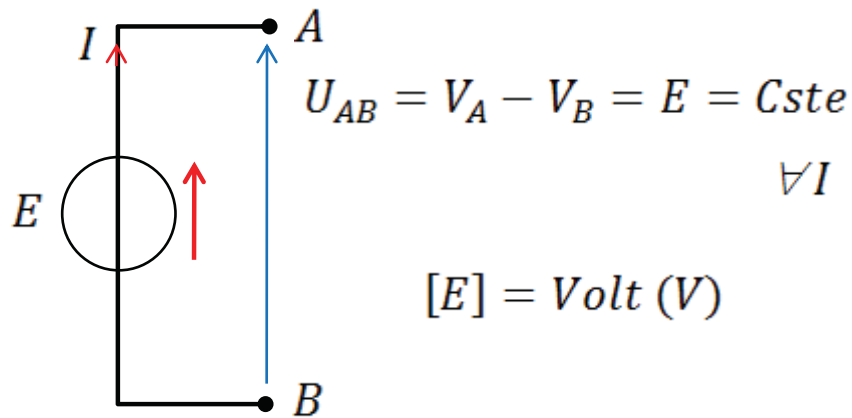
(Voltage generator)



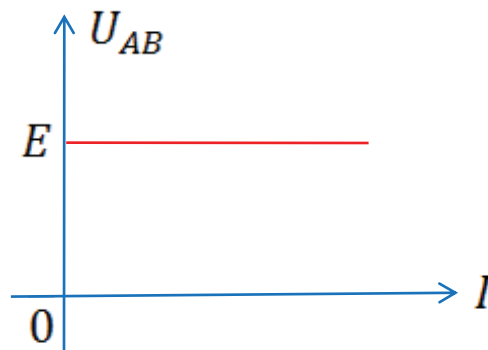
8. Electrical Generators and Receptors

The flow of a direct current in a circuit requires a source of energy capable of maintaining a constant potential difference: the electrical generator.

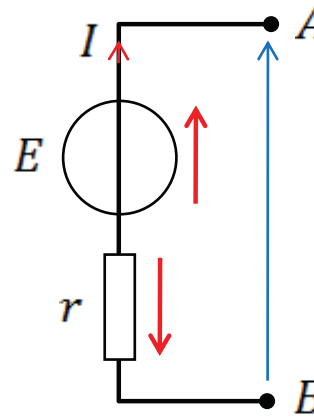
Ideal generator : $r \approx 0$



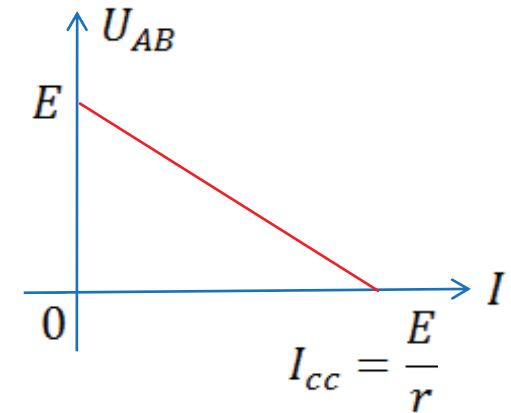
$[E] = \text{Volt (V)}$



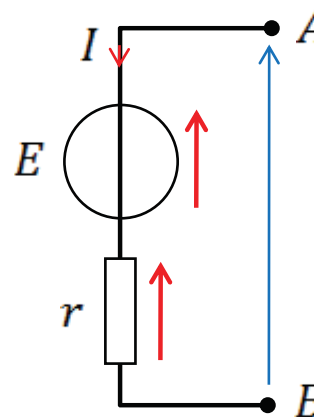
Reel Generator $r \neq 0$



$U_{AB} = V_A - V_B = E - r \cdot I$



Reel receptor: $r \neq 0$



$U_{AB} = V_A - V_B = E + r \cdot I$

Power balance :

$$U_{AB} = E - r.I$$

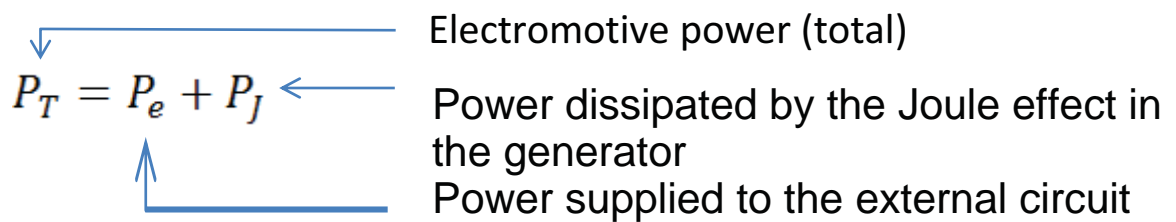
$$U_{AB}I = E.I - r.I^2 \rightarrow E.I = U_{AB}I + r.I^2$$

$P_T = P_e + P_J$

Electromotive power (total)

Power dissipated by the Joule effect in the generator

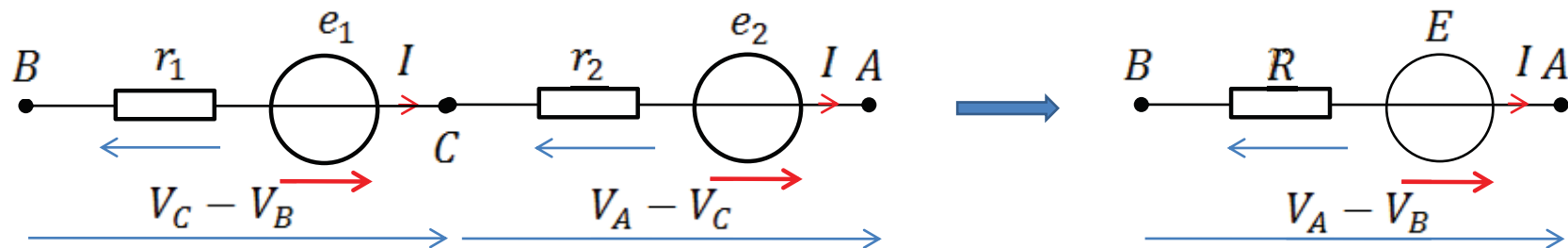
Power supplied to the external circuit

The diagram shows the equation $P_T = P_e + P_J$ with three arrows pointing to the terms. A blue arrow points from the text 'Electromotive power (total)' to the term $E.I$ in the equation above. A blue arrow points from the text 'Power dissipated by the Joule effect in the generator' to the term $r.I^2$. A blue arrow points from the text 'Power supplied to the external circuit' to the term $U_{AB}I$.

9. Connecting generators.

a) Series connection

Let there be N generators connected in series:



$$V_A - V_B = (V_A - V_C) + (V_C - V_B)$$

$$V_A - V_B = e_1 - r_1 \cdot I + e_2 - r_2 \cdot I$$

$$V_A - V_B = (e_1 + e_2) - (r_1 + r_2) \cdot I$$

$$V_A - V_B = E - R \cdot I$$

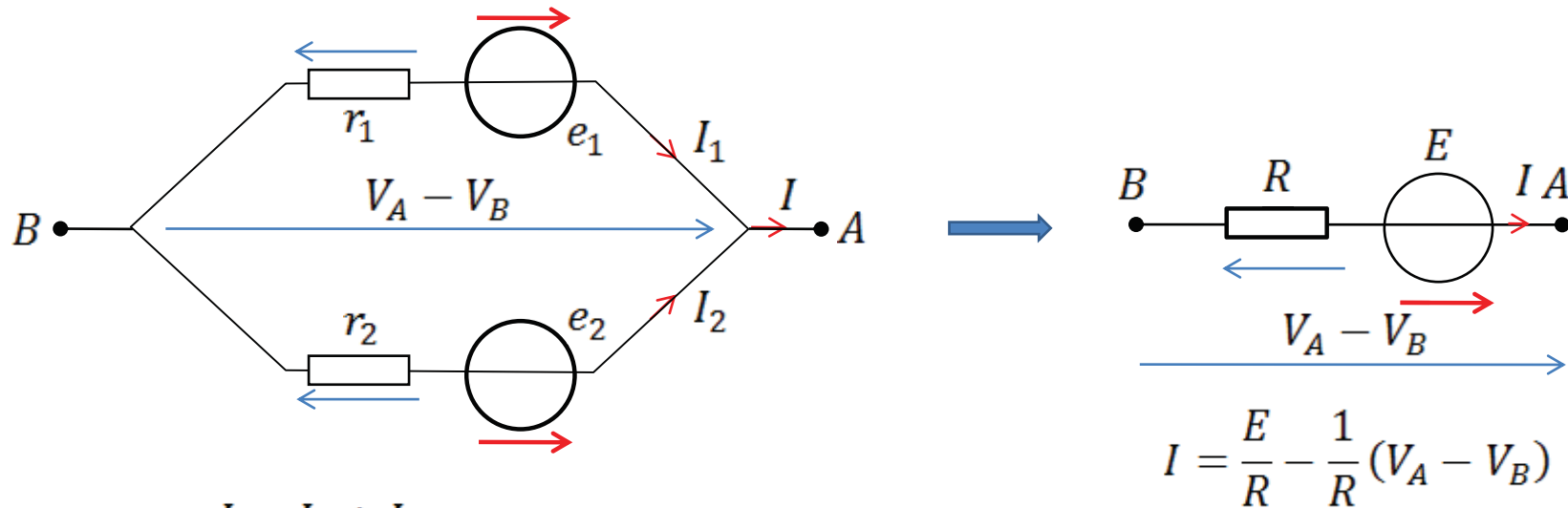
$$E = e_1 + e_2 \quad , \quad R = r_1 + r_2$$

For N generatord :

$$E = \sum_{i=1}^N e_i \quad , \quad R = \sum_{i=1}^N r_i$$

b) Parallel connection

Let there be N generators connected in series:



$$I = I_1 + I_2$$

$$V_A - V_B = e_1 - r_1 \cdot I_1 = e_2 - r_2 \cdot I_2$$

$$I_1 = \frac{e_1}{r_1} - \frac{1}{r_1} (V_A - V_B) \quad , \quad I_2 = \frac{e_2}{r_2} - \frac{1}{r_2} (V_A - V_B)$$

$$E = \left(\frac{1}{r_1} + \frac{1}{r_2} \right)^{-1} \left(\frac{e_1}{r_1} + \frac{e_2}{r_2} \right)$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

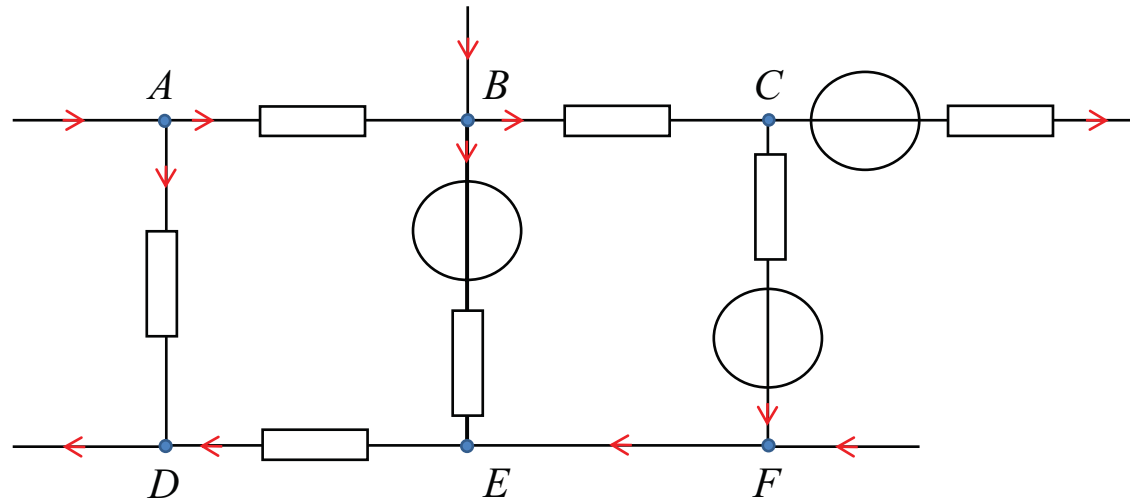
For N identical generators

$$E = e \quad , \quad R = \frac{r}{N}$$

10. Analysis of electrical networks

a) Definitions

Network : is a complex circuit consisting of a set of dipoles (resistors, generators, receivers...)
connected together.



Note: a node is defined as a point where at least three dipoles of the network meet

Branch: a branch is a section of the network located between two nodes.

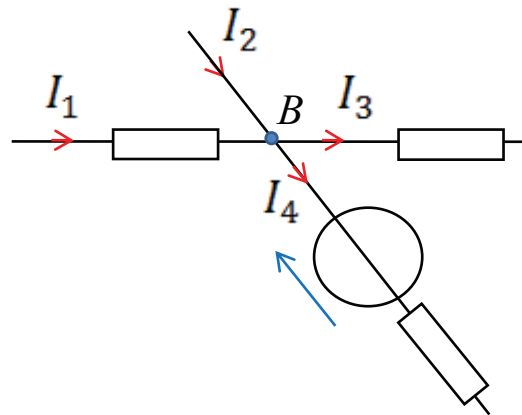
Mesh: a mesh of the network consists of a set of branches forming a closed circuit such as ABEDA

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b) Kirchhoff law

Kirchhoff's laws help us determine the current intensities flowing in the various branches of the network. Indeed, they lead to linear equations with respect to the currents.

Kirchhoff's first law (node rule)



The sum of the currents entering this node is equal to the sum of the currents leaving it:

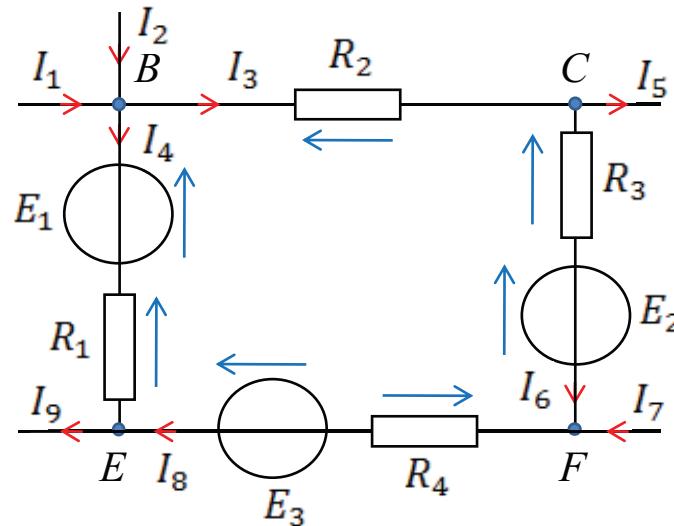
$$\sum_i I_i \text{ Incoming} = \sum_j I_j \text{ Outgoing}$$

In this case :

$$I_1 + I_2 = I_3 + I_4$$

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Kirchhoff's second law :loop rule (mesh)



For a circuit loop, the algebraic sum of the emfs is equal to the algebraic sum of the potential differences in the loop :

$$\sum_i E_i = \sum_j R_j \cdot I_j$$

In this case :

$$E_1 - E_2 + E_3 = R_2 I_3 + R_3 I_6 + R_4 I_8 - R_1 I_4$$

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Application Example:

Calculate the current intensity flowing in the various branches of the network.

1°/node law :

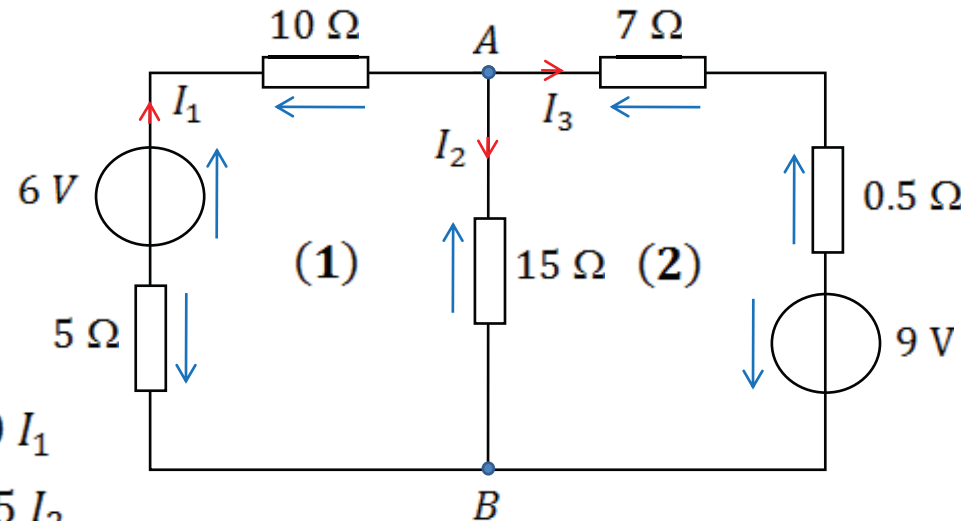
$$\text{Node A} : I_1 = I_2 + I_3$$

$$\text{Node B} : \textit{idem}$$

2°/ Loop rule :

$$\text{Mesh 1 : } 6 = 5 I_1 + 15 I_2 + 10 I_1$$

$$\text{Mesh 2 : } 9 = 7 I_3 + 0.5 I_3 - 15 I_2$$



$$\begin{cases} 5 I_1 + 5 I_2 = 2 \\ -15 I_2 + 7.5 (I_1 - I_2) = 9 \end{cases} \longrightarrow \begin{cases} I_1 + I_2 = 0.4 \\ 7.5 I_1 - 22.5 I_2 = 9 \end{cases}$$

$$I_1 = 0.6 A \quad , \quad I_2 = -0.2 A \quad , \quad I_3 = 0.8 A$$

The actual current of intensity I_2 flows in the direction opposite to the arrow.