جامعة الجيلالي بوتعامة خميس مليانة

Djilali Bounaama University of Khemis Miliana

كلية الطب

Faculty of Medicine

BIOPHYSICS

Pr. A. DIAF

https://diafahmed.weebly.com/cours-et-td.html

Classroom : rvz62m2y

What is “Biophysics”

Biophysics is a specialized sub area of biology

It is the science of physical principles of life itself and of biological systems. Biophysics is an interdisciplinary science that explains the laws and principles of physics which govern various biological processes. Biophysics spans all levels of biological organization from molecular scale to whole organism

General Introduction.

What Is Biophysics?

Biophysics is the field that applies the theories and methods of physics to understand how biological systems work.

- **Study of the action of physical agents: vibrations, radiation, electricity, sounds...**
- **Study of physical methods and techniques for diagnosis and therapy.**
- **Study of physical phenomena in the organism: biophysics of fluid exchange and flow, biophysics of blood circulation, respiration, acid-base balance.**

General Introduction.

- Bioelectricity refers to electricity produced by living beings. It is a specific field of electricity related to the effects of living organisms. The main elements are neurons and the brain.

- It is also known as developmental bioelectricity and studies the regulation of behaviors in cells, tissues, and organs resulting from endogenous electrical signals.

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Module de Physique-Biophysique

Chapitre 1 : Electricité et phénomènes Bioélectriques (12h)

Chapitre 2 : Biophysique des solutions et applications médicales (20 h)

Chapitre 3 : Optique géométrique (6h)

Chapitre 4 : Rayonnements et bases biophysiques de l’imagerie moderne (22h

Physics-Biophysics

Chapter 1: Electricity and Bioelectrical Phenomena

Chapter 2: Biophysics of Solutions and Medical Applications

Chapter 3: Geometric Optics

Chapter 4: Radiations and Biophysical Foundations of Modern Imaging

CHAPITER 1

Electricity and Bioelectrical Phenomena

Fondamental Physical constant

c	velocity of light in vacuum	$2.997\,924\,58 \cdot 10^{8}$ m/s
h	Planck's constant	$6.626\,069 \cdot 10^{-34}$ J/s
$\hbar$	($= h/2\pi$)	$1.054\,571 \cdot 10^{-34}$ J/s
e	electronic charge	$1.602\,176 \cdot 10^{-19}$ C
μ_e	electron magnetic moment	$-928.476\,362 \cdot 10^{-26}$ J/T
μ_B	Bohr magneton	$927.400\,899 \cdot 10^{-26}$ J/T
μ_N	nuclear magneton	$5.050\,783\,17 \cdot 10^{-27}$ J/T
m_e	electron mass	$9.109\,381\,88 \cdot 10^{-31}$ kg
m_p	proton mass	$1.672\,621\,58 \cdot 10^{-27}$ kg
m_N	neutron mass	$1.674\,927\,16 \cdot 10^{-27}$ kg
k_B	Boltzmann's constant	$1.380\,650 \cdot 10^{-23}$ J/K
N_A	Avogadro's constant	$6.022\,142 \cdot 10^{23}$
R	molar gas constant	$N_A \cdot k_B = 8.314\,472$ J/mol·K
F	Faraday constant	96 485.3415 C/mol
g_e	g electron factor	$-2.002\,319$
α	fine structure constant ($e^2/4\pi\varepsilon_0\hbar c$)	$7.297\,352\,533 \cdot 10^{-3}$

Fondamental Physical constant

TABLE D.1 SI Units

Base Quantity	SI Base Unit Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

TABLE D.2 Some Derived SI Units

Quantity	Name	Symbol	Expression in Terms of Base Units	Expression in Terms of Other SI Units
Plane angle	radian	rad	m/m	
Frequency	hertz	Hz	s^{-1}	
Force	newton	N	$kg\cdot m/s^2$	J/m
Pressure	pascal	Pa	$kg/m\cdot s^2$	N/m^2
Energy; work	joule	J	$kg\cdot m^2/s^2$	$N\cdot m$
Power	watt	W	$kg\cdot m^2/s^3$	J/s
Electric charge	coulomb	C	$A\cdot s$	
Electric potential	volt	V	$kg\cdot m^2/A\cdot s^3$	W/A
Capacitance	farad	F	$A^2\cdot s^4/kg\cdot m^2$	C/V
Electric resistance	ohm	Ω	$kg\cdot m^2/A^2\cdot s^3$	V/A
Magnetic flux	weber	Wb	$kg\cdot m^2/A\cdot s^2$	$V\cdot s$
Magnetic field intensity	tesla	T	$kg/A\cdot s^2$	
Inductance	henry	H	$kg\cdot m^2/A^2\cdot s^2$	$T\cdot m^2/A$

TABLE A.2 Symbols, Dimensions, and Units of Physical Quantities

Quantity	Common Symbol	Unit[a]	Dimensions[b]	Unit in Terms of Base SI Units
Acceleration	**a**	m/s^2	L/T^2	m/s^2
Amount of substance	n	mole		mol
Angle	θ, ϕ	radian (rad)	1	
Angular acceleration	$\boldsymbol{\alpha}$	rad/s^2	T^{-2}	s^{-2}
Angular frequency	ω	rad/s	T^{-1}	s^{-1}
Angular momentum	**L**	$kg \cdot m^2/s$	ML^2/T	$kg \cdot m^2/s$
Angular velocity	$\boldsymbol{\omega}$	rad/s	T^{-1}	s^{-1}
Area	A	m^2	L^2	m^2
Atomic number	Z			
Capacitance	C	farad (F)	Q^2T^2/ML^2	$A^2 \cdot s^4/kg \cdot m^2$
Charge	q, Q, e	coulomb (C)	Q	$A \cdot s$
Charge density				
Line	λ	C/m	Q/L	$A \cdot s/m$
Surface	σ	C/m^2	Q/L^2	$A \cdot s/m^2$
Volume	ρ	C/m^3	Q/L^3	$A \cdot s/m^3$
Conductivity	σ	$1/\Omega \cdot m$	Q^2T/ML^3	$A^2 \cdot s^3/kg \cdot m^3$
Current	I	AMPERE	Q/T	A
Current density	**J**	A/m^2	Q/T^2	A/m^2
Density	ρ	kg/m^3	M/L^3	kg/m^3
Dielectric constant	κ			
Displacement	**r**, **s**	METER	L	m

Electric dipole moment	**p**	C · m	QL	A · s · m
Electric field	**E**	V/m	ML/QT^2	$kg \cdot m/A \cdot s^3$
Electric flux	Φ_E	V · m	ML^3/QT^2	$kg \cdot m^3/A \cdot s^3$
Electromotive force	$\mathcal{E}$	volt (V)	ML^2/QT^2	$kg \cdot m^2/A \cdot s^3$
Energy	*E, U, K*	joule (J)	ML^2/T^2	$kg \cdot m^2/s^2$
Entropy	*S*	J/K	$ML^2/T^2 \cdot K$	$kg \cdot m^2/s^2 \cdot K$
Force	**F**	newton (N)	ML/T^2	$kg \cdot m/s^2$
Frequency	*f*	hertz (Hz)	T^{-1}	s^{-1}
Heat	*Q*	joule (J)	ML^2/T^2	$kg \cdot m^2/s^2$
Inductance	*L*	henry (H)	ML^2/Q^2	$kg \cdot m^2/A^2 \cdot s^2$
Magnetic dipole moment	$\boldsymbol{\mu}$	N · m/T	QL^2/T	$A \cdot m^2$
Magnetic field	**B**	tesla (T) (= Wb/m^2)	M/QT	$kg/A \cdot s^2$
Magnetic flux	Φ_B	weber (Wb)	ML^2/QT	$kg \cdot m^2/A \cdot s^2$
Mass	*m, M*	KILOGRAM	M	kg
Molar specific heat	*C*	J/mol · K		$kg \cdot m^2/s^2 \cdot mol \cdot K$
Moment of inertia	*I*	$kg \cdot m^2$	ML^2	$kg \cdot m^2$
Momentum	**p**	kg · m/s	ML/T	kg · m/s
Period	*T*	s	T	s
Permeability of space	μ_0	N/A^2 (= H/m)	ML/Q^2T	$kg \cdot m/A^2 \cdot s^2$
Permittivity of space	ϵ_0	$C^2/N \cdot m^2$ (= F/m)	Q^2T^2/ML^3	$A^2 \cdot s^4/kg \cdot m^3$
Potential	*V*	volt (V) (= J/C)	ML^2/QT^2	$kg \cdot m^2/A \cdot s^3$
Power	$\mathcal{P}$	watt (W) (= J/s)	ML^2/T^3	$kg \cdot m^2/s^3$

TABLE A.2 Continued

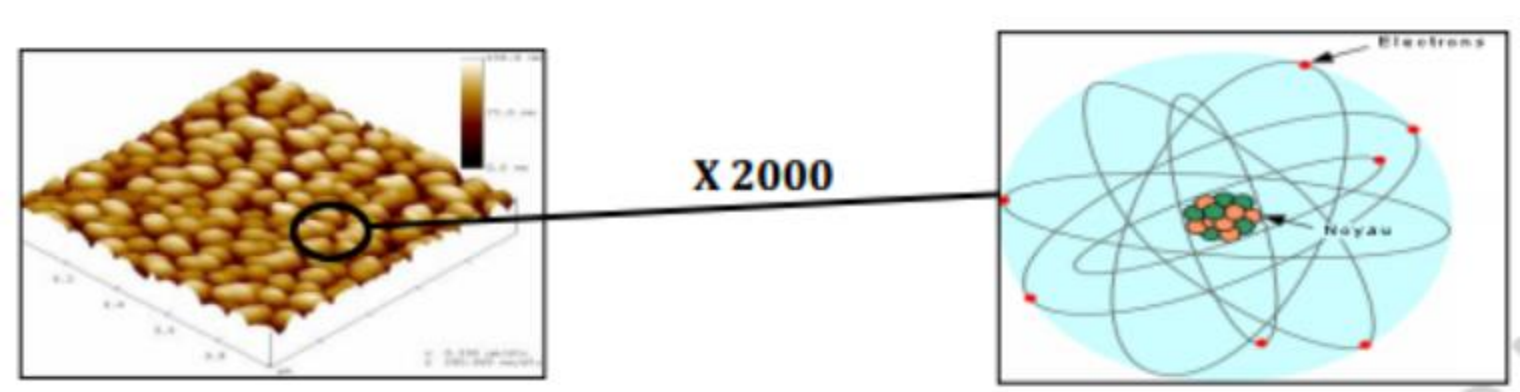
Quantity	Common Symbol	Unit[a]	Dimensions[b]	Unit in Terms of Base SI Units
Pressure	P	pascal (Pa) = (N/m^2)	M/LT^2	$kg/m \cdot s^2$
Resistance	R	ohm (Ω)(= V/A)	ML^2/Q^2T	$kg \cdot m^2/A^2 \cdot s^3$
Specific heat	c	$J/kg \cdot K$	$L^2/T^2 \cdot K$	$m^2/s^2 \cdot K$
Speed	v	m/s	L/T	m/s
Temperature	T	KELVIN	K	K
Time	t	SECOND	T	s
Torque	τ	$N \cdot m$	ML^2/T^2	$kg \cdot m^2/s^2$
Volume	V	m^3	L^3	m^3
Wavelength	λ	m	L	m
Work	W	joule (J)(= $N \cdot m$)	ML^2/T^2	$kg \cdot m^2/s^2$

List of Prefixes in the Metric System

Prefix	Symbol	Numerical Value	Exponential Value
quetta	Q	1,000,000,000,000,000,000,000,000,000,000	10^{30}
ronna	R	1,000,000,000,000,000,000,000,000,000	10^{27}
yotta	Y	1,000,000,000,000,000,000,000,000	10^{24}
zetta	Z	1,000,000,000,000,000,000,000	10^{21}
exa	E	1,000,000,000,000,000,000	10^{18}
peta	P	1,000,000,000,000,000	10^{15}
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^{9}
mega	M	1,000,000	10^{6}
kilo	k	1,000	10^{3}
hecto	h	100	10^{2}
deka	da	10	10^{1}
no prefix		1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}
zepto	z	0.000000000000000000001	10^{-21}

1- Generalities and reminder

On a microscopic scale, matter is composed of a collection of atoms or molecules linked together by Van der Waals forces



Atom: The atom consists of a very dense central part (the nucleus), around which electrons orbit.

The nucleus of the atom is composed of protons and neutrons, and the peripheral part consists of an electron cloud

The following table gives the physical characteristics of the particles found in the atom.

	Charge in (C)	Mass in (Kg)
Electron	$(-e = -1,6\ 10^{-19})$	$9,11\ 10^{-31}$
Proton	$(+e = +1,6\ 10^{-19})$	$1,67.\ 10^{-27}$
Neutron	0	$1,67.\ 10^{-27}$

Atomic charge:

The electrical charge of an atom can be deduced from the algebraic sum of all its elementary charges.

Chemical element symbol

Total number of nucleons

$$^{A}_{Z}X_{N}$$

Number of protons

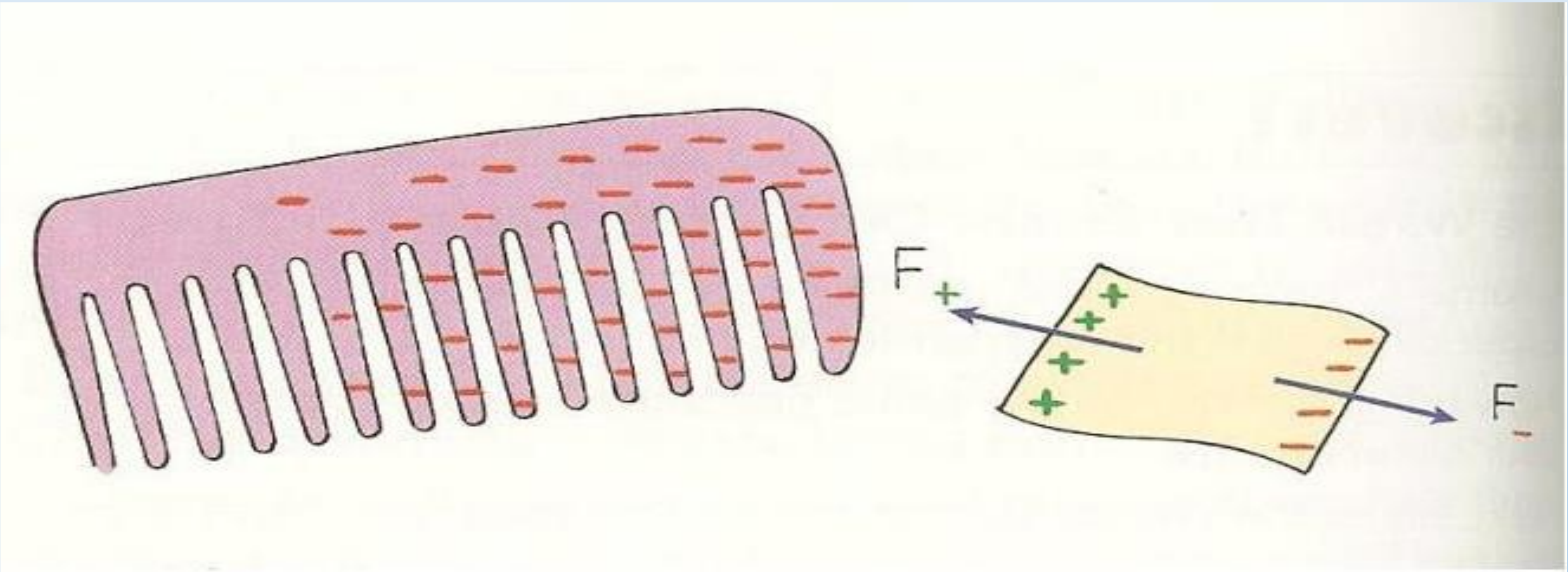
Number of neutrons

A=Z+N

2- Electrostatics

A charged comb attracts bits of paper because charges are displaced in the paper.

Electrically polarized

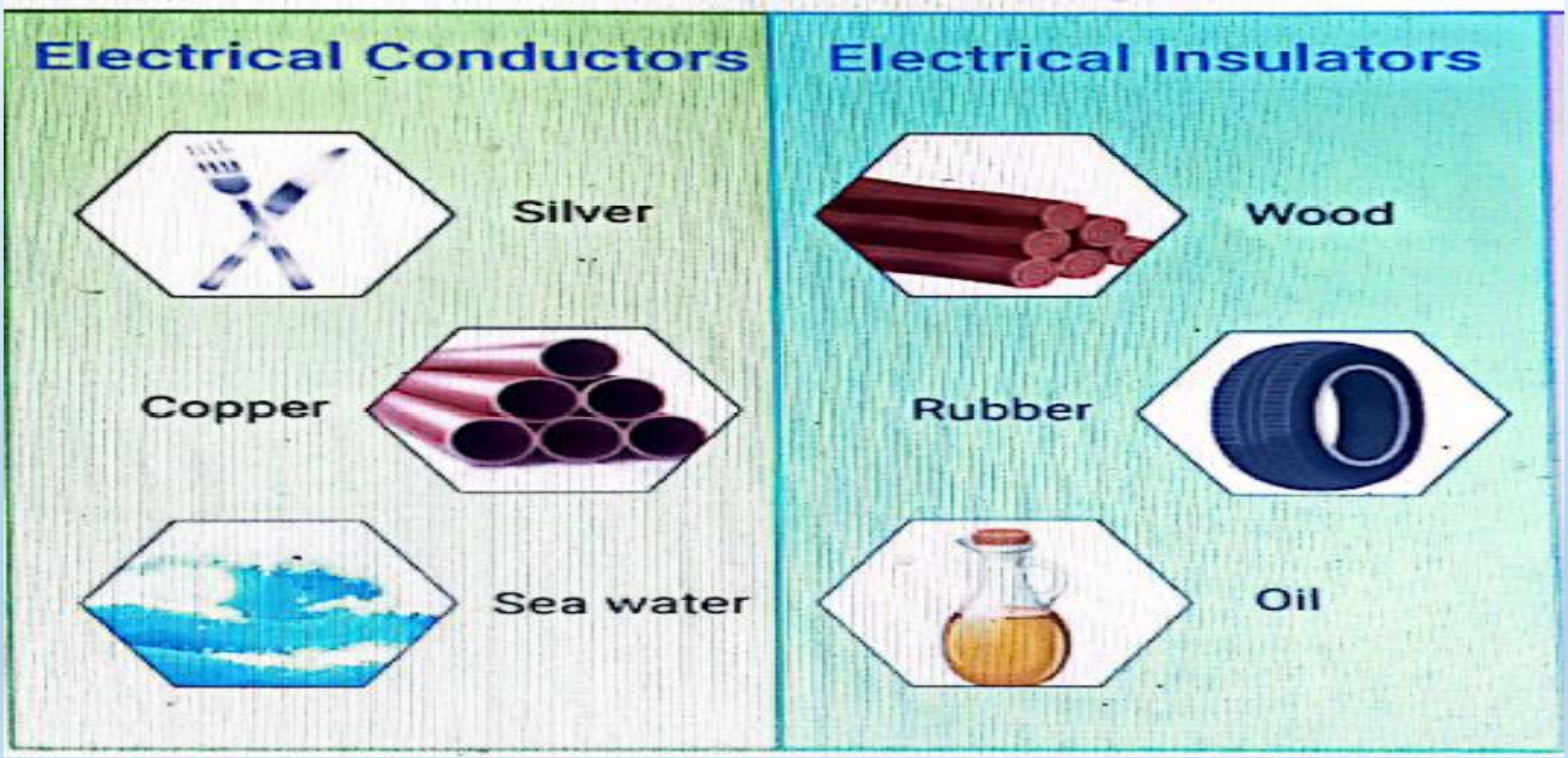


Conductor and insulator

A material is said to be a perfect conductor if, when it becomes electrified, the charge carriers can move freely throughout the entire volume occupied by the material.

It will be a perfect insulator if the charge carriers cannot move freely and remain localized at the place where they were deposited.

Conductor and insulator



Electrostatics: Applications in Biophysics

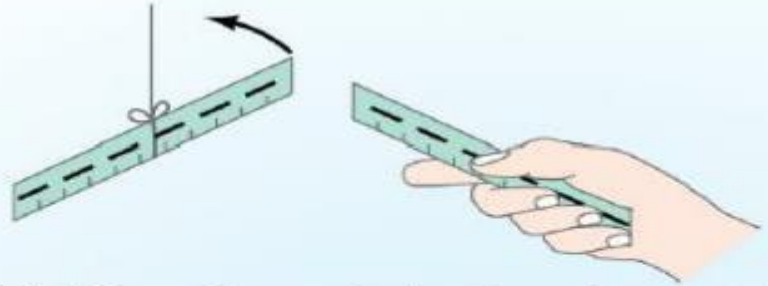
- **Biological membranes**
- Biological membranes are insulating materials. placed between two conducting materials., thus forming a capacitor.

They allow for charge separation.
They contain ionic channels that help maintain a difference in charges between the inside and the outside (thanks to chemical energy), resulting in an electrostatic field at the membrane.
This field influences ionic exchanges at the membrane level.

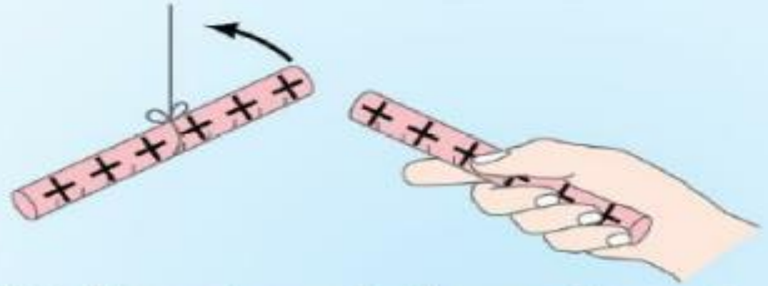
- **Proteins**
- Proteins are made up of amino acids, some of which are charged. The surface of a protein shows a charge distribution that is not uniform (even if the overall charge may be zero).
This distribution guides the intermolecular interactions that are essential for signaling pathways.
For example, phosphorylation is a modification of the surface of a protein after activation, which leads to a negative charge that modifies interactions with other partners of this protein.

Positive and negative charge

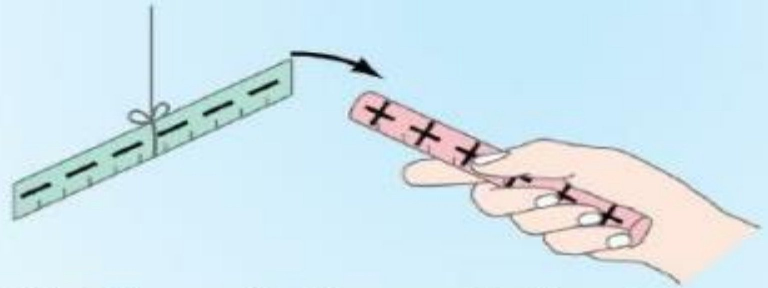
- Like charge repel one another and unlike charges attract one another where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.



(a) Two charged plastic rulers repel



(b) Two charged glass rods repel



(c) Charged glass rod attracts charged plastic ruler

Charging by Friction or Contact

- Two ways electrical charge can be transferred
  - Friction – electrons are being transferred as one material rubs against another
  - Contact – electrons will transfer simply by two materials touching

1-2 Electric force

Coulomb's Law

In 1785, Coulomb established the fundamental law of *electric force* between two stationary, charged particles.

1-2Electric force

Coulomb's Law

The electrostatic force of a charged particle exerts on another is proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$\therefore F = K \frac{q_1 q_2}{r^2}$$

1-2Electric force

$$\therefore F = K\frac{q_1 q_2}{r^2}$$

- where K is the coulomb constant = 9×10^9 N.m^2/C^2.

- The above equation is called ***Coulomb's law***, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

Electric force

Electric force

Permittivity constant of free space

- The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

- where ε_0 is known as the *Permittivity constant of free*

ε_0 *space.* (generally, we have $\varepsilon = \varepsilon_0 \times \varepsilon_r$)

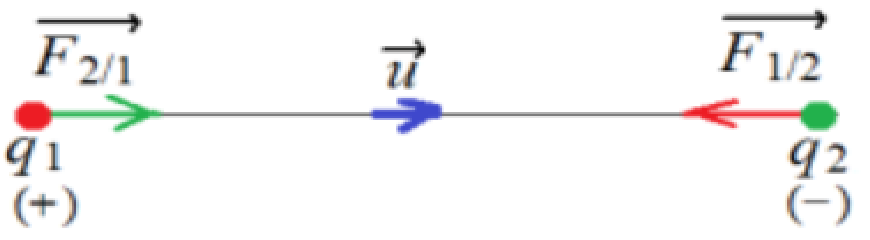
- $= 8.85 \times 10^{-12}\ C^2/N.m^2$

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2 / C^2$$

1-2 Electric force

Vectorial form of the electric force

$$\overrightarrow{F_{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{u}$$



Here $\vec{u}$ is a unit vector

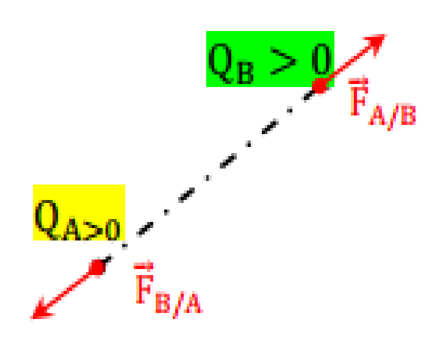
Characteristics of Coulomb force: Coulomb force is a vector quantity, also characterised by

a)The point of application: The charge.

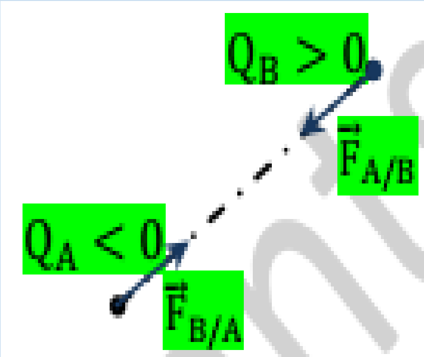
b)The direction of Coulomb forces: the direction depends on the nature of the two charges. Two situations may arise:

1-2 Electric force

- If the two charges (Q_A) and (Q_B) are of the same nature (positive-positive) or (negative-negative), the two charges will repel each other; these are called repulsive forces, see the diagram opposite.



- If, on the other hand, the two charges (Q_A) and (Q_B) are of different types (positive-negative) or (negative-positive), the two charges will attract each other; these are called attractive forces.



1-2 Electric force

Example 01 : The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= 8.2 \times 10^{-8}\ \text{N}$$

Using Newton's law of gravitation and Table 23.1 for the particle masses, we find that the gravitational force has the

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)$$

$$= \times\,\frac{(9.11 \times 10^{-31}\,\mathrm{kg})(1.67 \times 10^{-27}\,\mathrm{kg})}{(5.3 \times 10^{-11}\,\mathrm{m})^2}$$

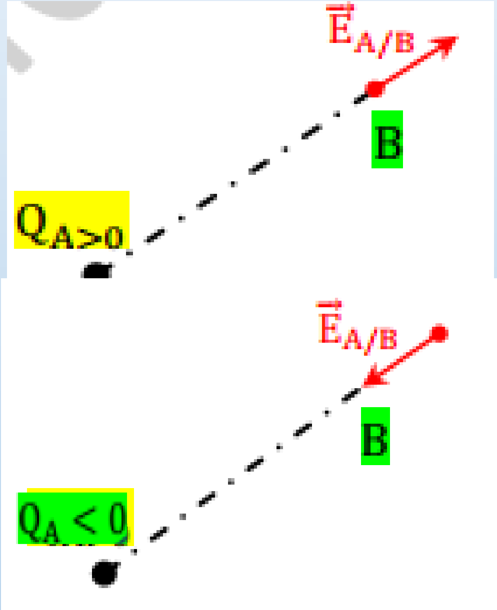
$$= \boxed{3.6 \times 10^{-47}\,\mathrm{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces.

1-3 Electric field

An electric field is any region in space where an electric charge is subject to the action of an electric force.

When a point charge q_A, is placed at the origin, the Coulomb force on another point charge, q_B, at the position r=AB is given by



$$\vec{F}_C = k \times \frac{Q_A \times Q_B}{(\overline{AB})^2} \vec{u}$$

With

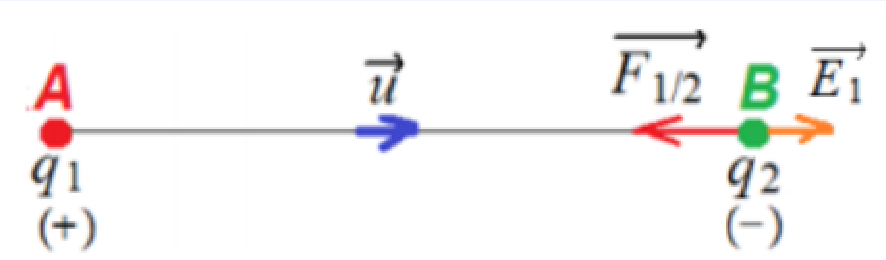
$$\vec{E}_{A/B} = k \times \frac{Q_A}{(\overline{AB})^2} \vec{u}$$

-The SI Unit is the Newton/Coulomb=N/C, Alternative Unit : Volt/Meter

-The direction of the field is the same as that of the force if the influenced charge is positive.

-Otherwise, the direction of the field is opposite to that of the electric force.

1-3 Electric field



$$\vec{F}_{1/2} = -F_{1/2}\vec{u} = -K\frac{|q_1 \times q_2|}{r^2}\vec{u}$$

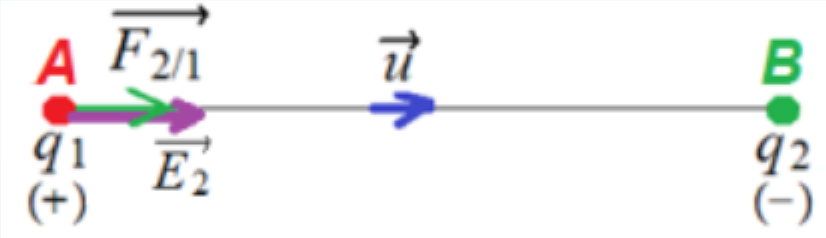
$$= -K\frac{q_1 \times q_2}{r^2}\vec{u}$$

$$\Rightarrow \vec{F}_{1/2} = -q_2\left(K\frac{q_1}{r^2}\vec{u}\right) = -q_2\vec{E}_1$$

$$\vec{E}_1 = K\frac{q_1}{r^2}\vec{u}$$

$$[E_1] = N/C$$

Output field



$$\vec{F}_{2/1} = F_{2/1}\vec{u} = K\frac{|q_1 \times q_2|}{r^2}\vec{u}$$

$$= K\frac{q_1 \times q_2}{r^2}\vec{u}$$

$$\Rightarrow \vec{F}_{2/1} = q_1\left(K\frac{q_2}{r^2}\vec{u}\right) = q_1\vec{E}_2$$

$$\vec{E}_2 = K\frac{q_2}{r^2}\vec{u}$$

Input field

1-3 Electric field

Analogy with mechanics

In analogy with mechanics, if in the vicinity of the earth, where the gravity field reigns, we place a mass m, it will be subject to its weight. We have:

$$\vec{W} = m\,\vec{g}$$

Forces	$\vec{F}_g = G \times \dfrac{m_A \times m_B}{(\overline{AB})^2}\vec{u}$	$\vec{F}_{élec} = K \times \dfrac{Q_A \times Q_B}{(\overline{AB})^2}\vec{u}$

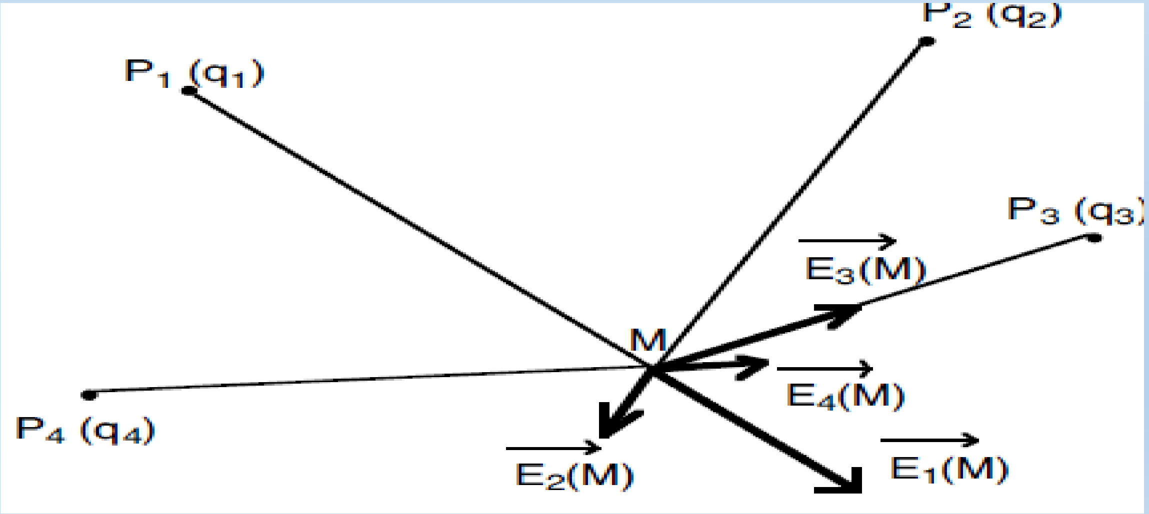
- $G = 6{,}67\ 10^{11} (S.I)$
- $K = 9.0\ 10^{9} (S.I)$

1-3 Electric field

Case of n charges :

- Here, we calculate the electric field strength for electric charges distributed in space. When an electric charge, q_i , is placed at position r_i (i = 1,2,……., n), with the superposition principle, the electric field strength at r is given by

$$\vec{E}(M) = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \vec{u}_i$$

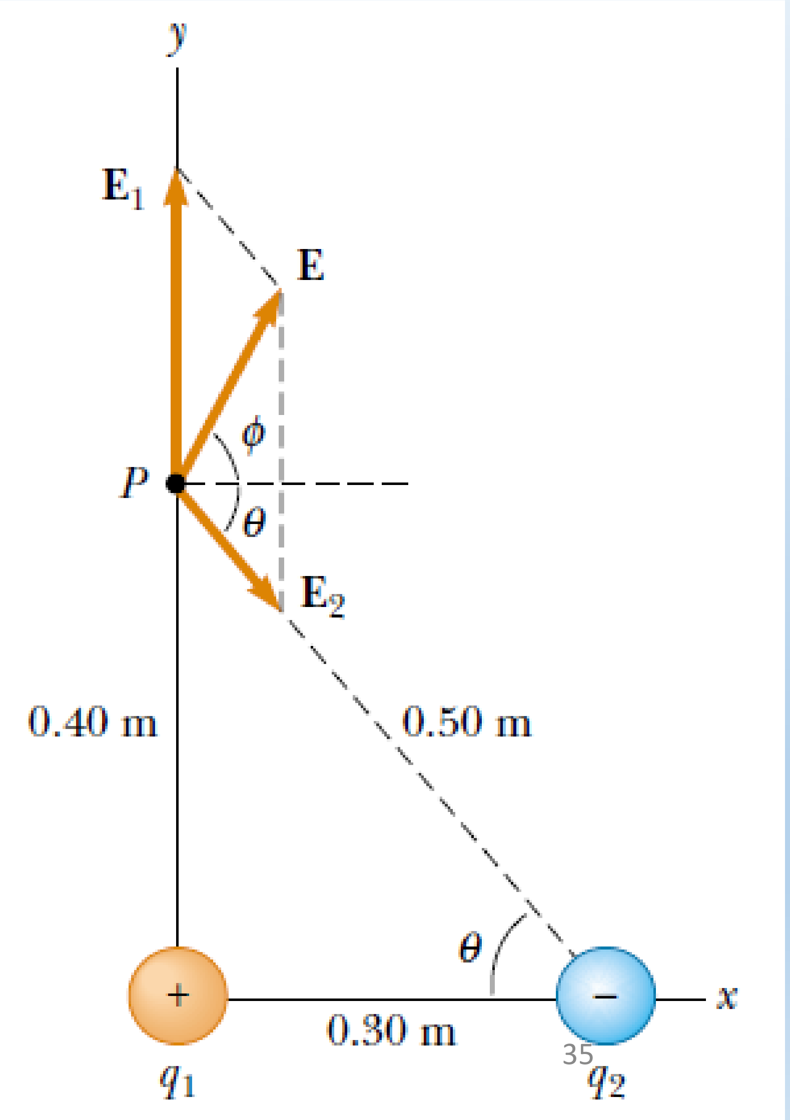


1-3 Electric field

<u>Example Electric Field Due to Two Charges</u>

A charge q_1= 7.0 μC is located at the origin, and a second charge q_2= -5.0 μC is located on the *x* axis, 0.30 m from the origin.
Find the electric field at the point *P*, which has coordinates (0, 0.40) m.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0μC charge and $\mathbf{E}_2$ due to the -5.0μC charge are shown in the Figure. Their magnitudes are



$$E_1 = k_e \frac{|q_1|}{r_1^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$

$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$

$$= 1.8 \times 10^5\ \text{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j}\ \text{N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j})\ \text{N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

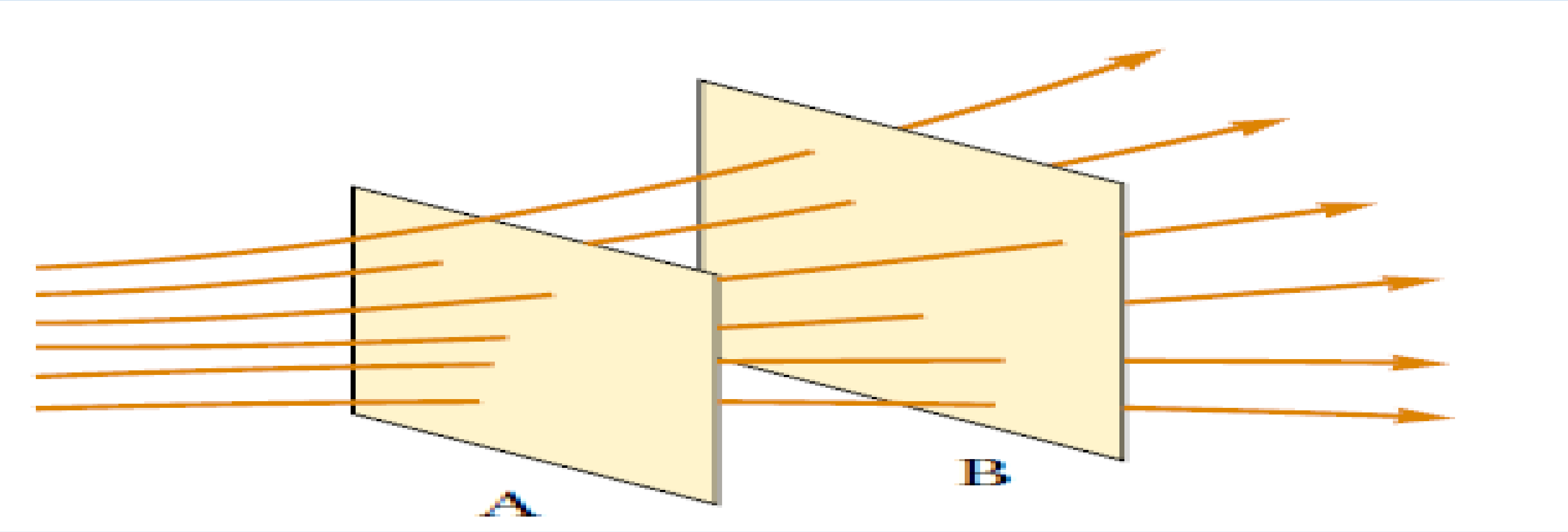
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j})\ \text{N/C}$$

From this result, we find that $\mathbf{E}$ has a magnitude of 2.7×10^5 N/C and makes an angle ϕ of 66° with the positive x axis.

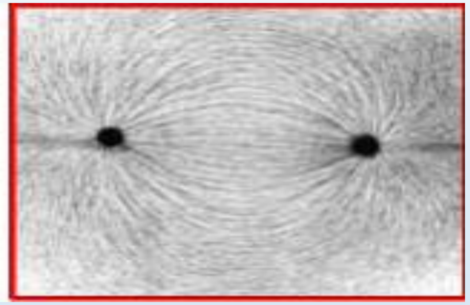
Electric field lines

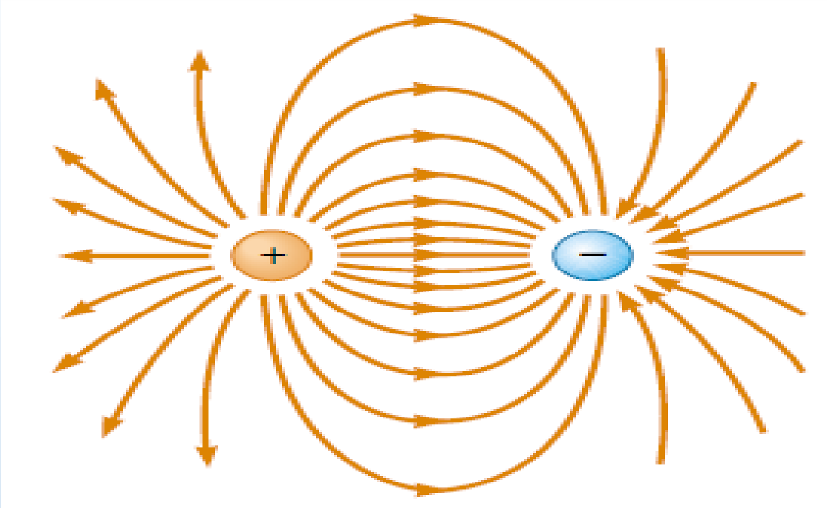
We can visualize the field using electric field lines (lines of electric force), which help us to understand the field easily. We can refer to a line of electric force as an electric field line, and take the tangent to an electric field line at an arbitrary point as being parallel to the direction of the electric field at this point.

It can be shown that electric field lines never cross each other.



Electric field lines

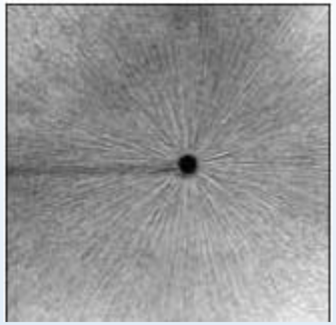




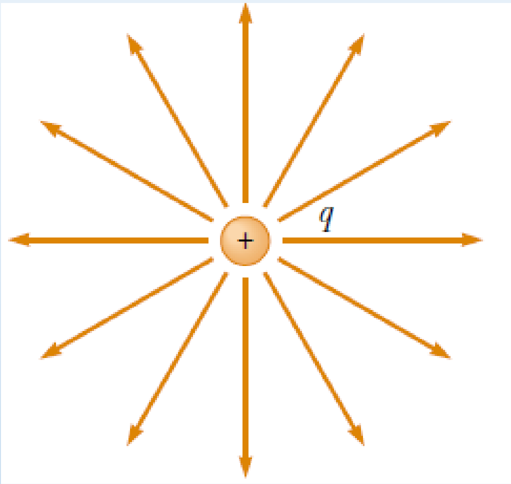
Rules for drawing electric field lines

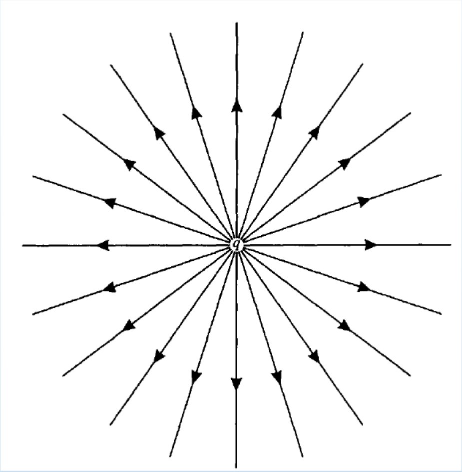
- The lines must begin on a positive charge and terminate on a negative charge.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

Electric field lines

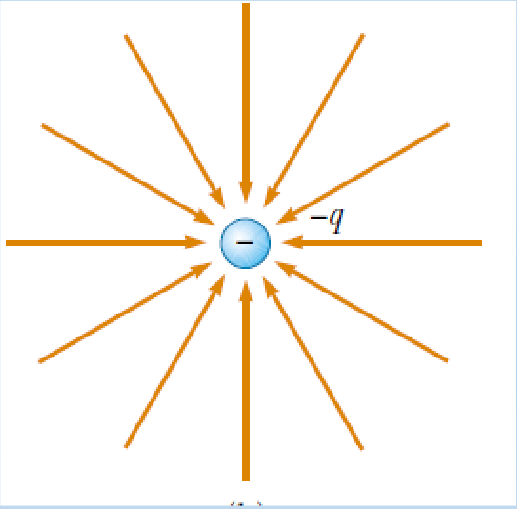


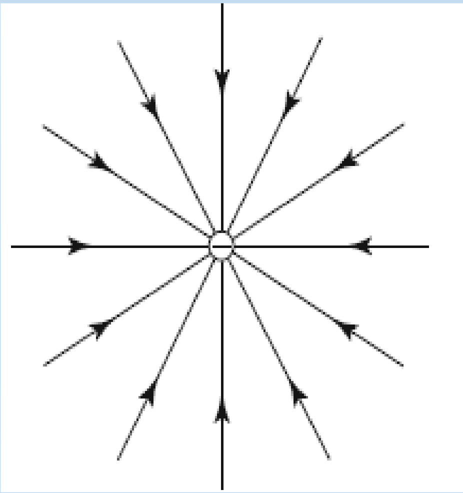
Positive point charge





Negative point charge



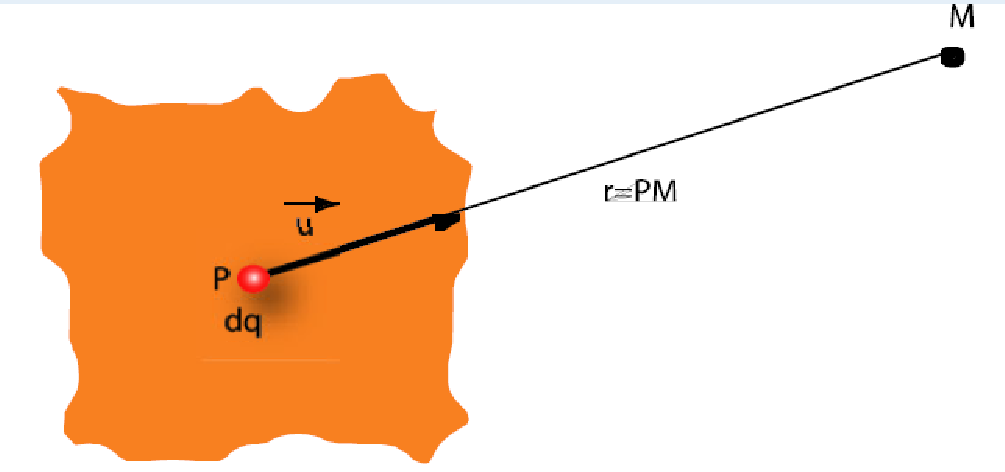


Electric field with surface charge distribution

In this case, we have:

$$\vec{E}(M) = \int \overrightarrow{dE}(M)$$

with $$\overrightarrow{dE}(M) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u}$$



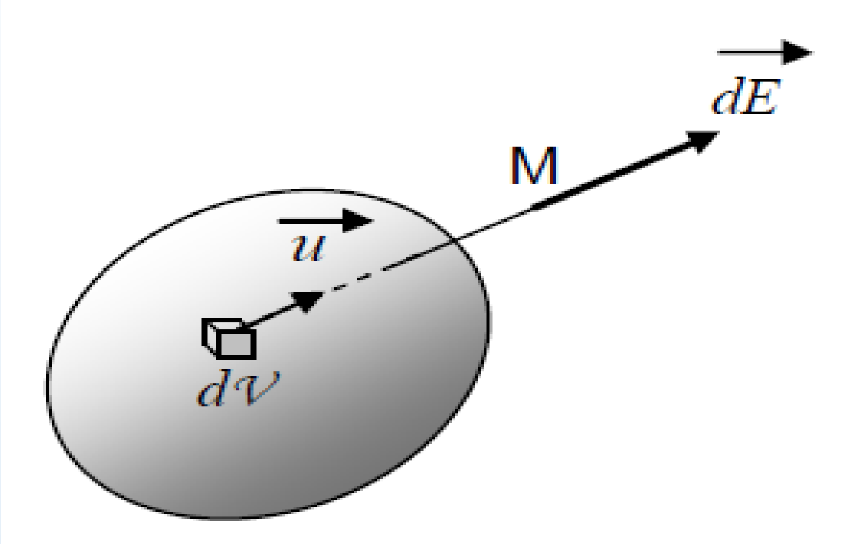
If σ is the surface charge density (units: $C.m^{-2}$) with $\sigma = \frac{dq}{dS}$

Then the electric field is given by: $$\vec{E}(M) = \iint_{Surface} \frac{1}{4\pi\varepsilon_0}\frac{\sigma}{r^2}\vec{u}\,dS$$

Electric field with volume charge distribution

We define ρ as the volume charge density (units: C.m^{-3}). The electrostatic field created by such a distribution is

$$\vec{E}(M) = \int \overrightarrow{dE}(M)$$



with

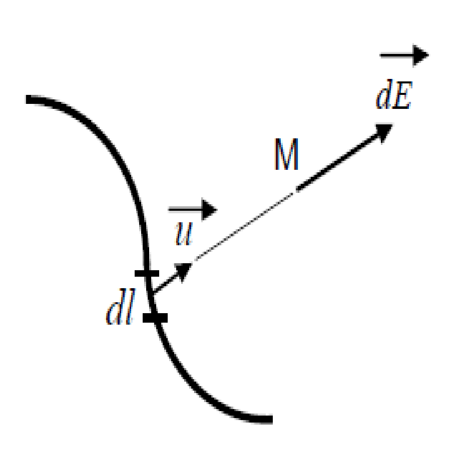
$$\overrightarrow{dE}(M) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u}$$

with $\rho = \frac{dq}{dv}$

Then the electric field is given by:

$$\vec{E}(M) = \iiint_{\text{Volume}} \frac{1}{4\pi\varepsilon_0}\frac{\rho}{r^2}\vec{u}\, dv$$

Electric field with linear charge distribution



We define $\lambda = \frac{dq}{dl}$ as the linear charge density

(units: C. m^{-1}). The

Electrostaticfield created by such a distribution is

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \int_l \frac{\lambda\, dl}{r^2}\, \vec{u}$$

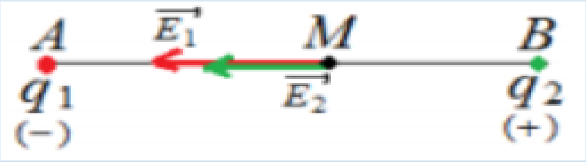
Example 2

Two charges q_1 and q_2, placed respectively at points A and B, are separated by a distance of 5 cm. On the line (AB), where can a third charge be placed so that it experiences no force? Can this charge be either positive or negative? Given: q_1 = -8 µC; q_2 = 3 µC.

Solution: . Principle: The net electric force on a test charge is zero when the electric fields produced by q_1 and q_2 at that point cancel each other $\vec{E_M} = \vec{E_1} + \vec{E_2} = \vec{0}$ 0). This condition is independent of the sign or magnitude of the test charge). So any nonzero test charge (positive or negative) will experience zero force if the electric field is zero at that point.

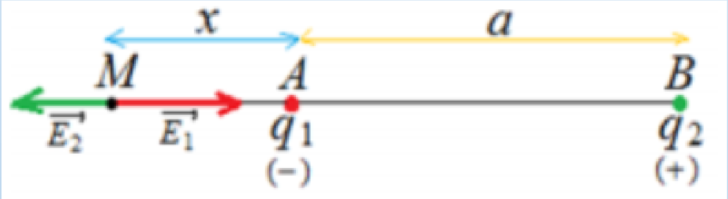
Solution:

- ⇒ q_3 will be subjected to: $\vec{F} = q_3 \vec{E}_M$
- For q_3 to be immobile ⇒ $\vec{F} = \vec{0} \Rightarrow \overrightarrow{E_M} = 0$
- **Case 1: M between A and B** $\vec{E}_1 + \vec{E}_2 \neq \vec{0}$ ⇒ q_3 can never be at rest.



- **Case 2: M at a distance x to the left of A:**

$$E_1 = K\frac{|q_1|}{x^2} \qquad E_2 = K\frac{|q_2|}{(x+a)^2}$$



- If $\vec{E}_1 + \vec{E}_2 = \vec{0}$ then $\Rightarrow E_1 = E_2$
- $\Rightarrow \frac{|q_1|}{x^2} = \frac{|q_2|}{(x+a)^2}$ $\Rightarrow 5x^2 + 80x + 200 = 0$ OR $\Rightarrow x = -12.89cm$ $x = -3.1cm$
- Rejected values because x must be positive (distance).

1.4 Electrical potential.

1-4-1Work of an Electric Force

The charge q_2 follows an arbitrary path between A and B.

$$W_{A\to B}(\vec{F}) = \int_A^B dW = \int_A^B \vec{F}.\vec{dl} = \int_A^B K\frac{|q_1 q_2|}{r^2}\ \vec{u}.\vec{dl}$$

dW: elementary work of $\vec{F}$ between points M and N.
The work of the force $\vec{F}$ is written

$$\vec{F} = F\vec{u} = K\frac{|q_1 q_2|}{r^2}\ \vec{u}$$

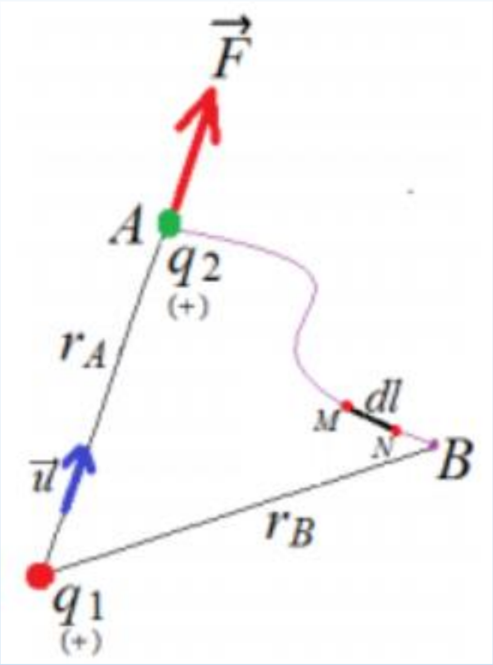
$$W_{A\to B}(\vec{F}) = K|q_1 q_2| \int_A^B \frac{1}{r^2}\vec{u}.\vec{dl}$$

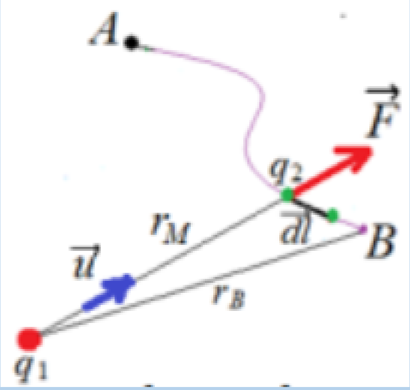
$$\vec{u}.\vec{dl} = \|\vec{u}\| \left\|\vec{dl}\right\| \cos\theta = dl\cos\theta \equiv dr$$

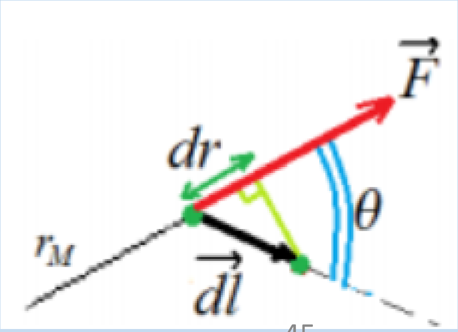
$$\Rightarrow W_{A\to B}(\vec{F}) = K|q_1 q_2| \int_A^B \frac{1}{r^2} dr$$

$$= K|q_1 q_2|[\frac{1}{r_A} - \frac{1}{r_B}] = K q_1 q_2[\frac{1}{r_A} - \frac{1}{r_B}]$$

W $_{A\to B}$ is independent of the chosen path







1-4Electrical Potential

- Electrical Energy and Potential

$$W_{A\to B}(\vec{F}) = \frac{Kq_1q_2}{r_A} - \frac{Kq_1q_2}{r_B}$$ this gives $$W_{A\to B}(\vec{F}) = E_p(A) - E_p(B)$$

Here $[E_p] = \text{Joules } (J)$

$$E_p = K\frac{q_1q_2}{r}$$

Here E_p is the electrostatic potential energy of the system composed of q_1 and q_2 separated by a distance r.

Relationship between potential energy and electric potential.

If q_1 is in the field E_2 produced by q_2 , we can side that q_1 has an electrostatic potential energy because of this field and we can write: $E_{p_1} = q_1 K\frac{q_2}{r} \equiv q_1 V_{2/1}$

$V_{2/1} = K\frac{q_2}{r}$ electric potential created by charge q_2 at the point where charge q_1 is located.

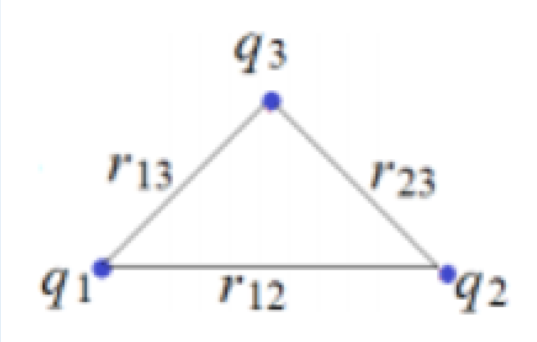
q_1 is the influenced charge.

$V_{2/1}$: is the external influencing electric potential

1-4Electrical Potential

Case of two charges : $E_p = E_{p_1} = E_{p_2} \Rightarrow E_p = \frac{1}{2}(E_{p_1} + E_{p_2})$

Case of three charges : $E_p = E_p(q_1,q_2) + E_p(q_1,q_3) + E_p(q_2,q_3)$

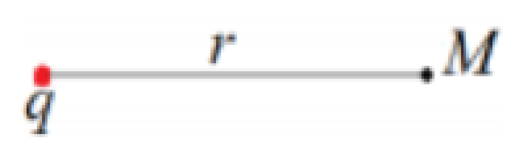


$$= K\frac{q_1 q_2}{r_{12}} + K\frac{q_1 q_3}{r_{13}} + K\frac{q_2 q_3}{r_{23}}$$

$$= \frac{1}{2}[q_1(K\frac{q_2}{r_{12}}) + q_2(K\frac{q_1}{r_{12}})] + \frac{1}{2}[q_1(K\frac{q_3}{r_{13}}) + q_3(K\frac{q_1}{r_{13}})] + \frac{1}{2}[q_2(K\frac{q_3}{r_{23}}) + q_3(K\frac{q_2}{r_{23}})]$$

1-4Electrical Potential

Generally: $V_M = K\frac{q}{r}$ $[V_M] = \text{Volts}\ (V)$



- Characteristics of electric potential:

- Electric potential is a scalar quantity, characterized solely by its value and sign.
- If the influencing charge is positive, the value of the electric potential will also be positive.
- If the influencing charge is negative, the value of the electric potential will be negative.
- In the International System, the unit of electric is the Volt.

1.4 Electrical potential.

- Relationship between electric potential and electric field. The electric field derives from the electric potential. The expression of the relationship between the electric field and the electric potential is given by:

$$\begin{aligned}\vec{E} &= E_x\vec{i} + E_y\vec{j} + E_z\vec{k} \\ &= -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k}\end{aligned}$$

$$\vec{E} = -\overrightarrow{grad}\, V$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

.

1.4 Electrical potential.

.2 Case of n charges:

In the presence of several q_i charges, the electrostatic field is the sum of the electric potentials produced by each charge q_i:

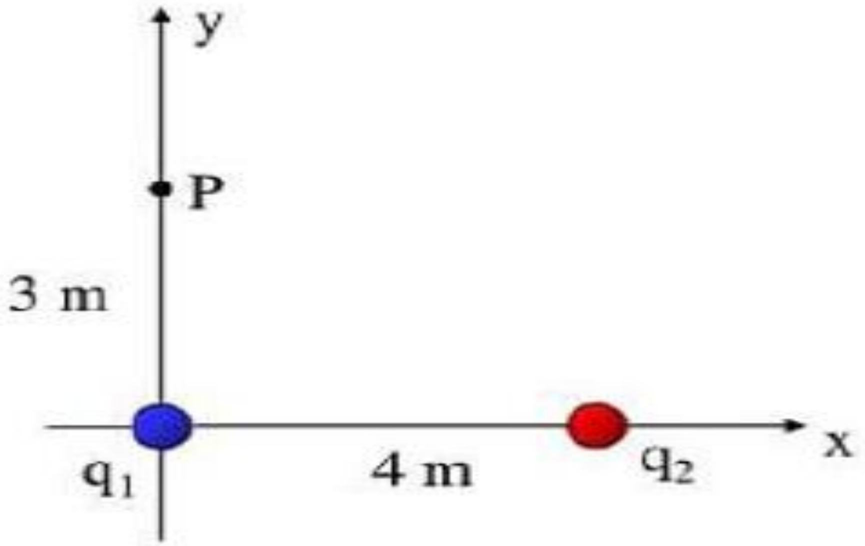
$$V(M) = \sum_{i=1}^{n} V_i(M)$$

$$V(M) = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i} \quad + \quad V_0$$

Where r_i is the distance between the charge q_i and the point M.

1.4 Electrical potential.

Example: a 1 μC point charge is located at the origin and a -4 μC point charge 4 meters along the +x axis. Calculate the electric potential at a point P, 3 meters along the +y axis.



$$V_P = k\sum_i \frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

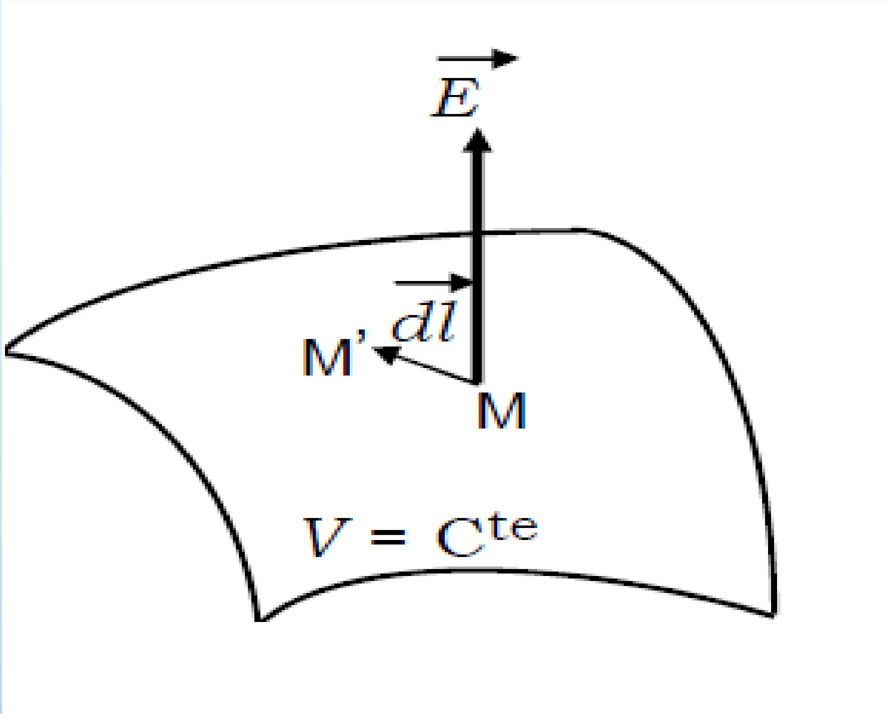
$$= 9\times10^9\left(\frac{1\times10^{-6}}{3} + \frac{-4\times10^{-6}}{5}\right)$$

$$= -4.2\times10^3\ V$$

1.4 Electrical potential.

- Equipotential surfaces:

A virtual surface composed of points with the same electric potential is called an equipotential surface.



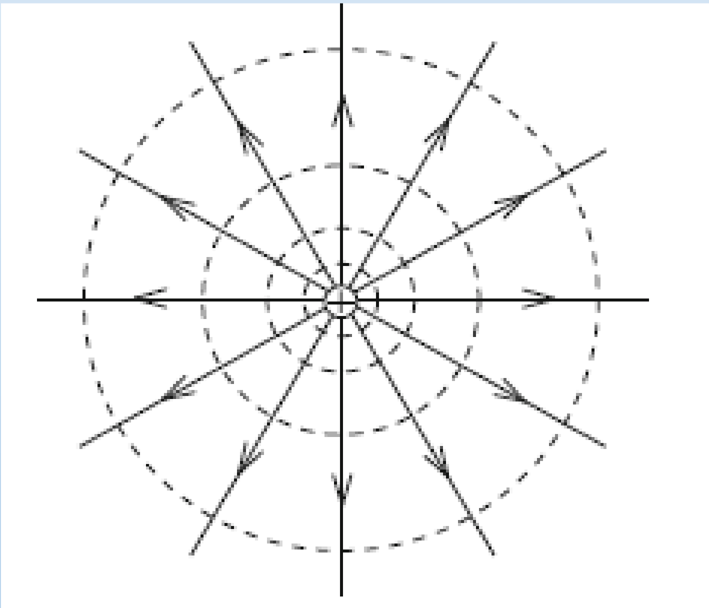
$$dV = -\vec{E}.\vec{dl} = -\vec{E}.\vec{MM'} = 0$$

That is, E vector is normal to the equipotential surface. This can also be expressed by saying that the electric field lines are normal to the equipotential surface.

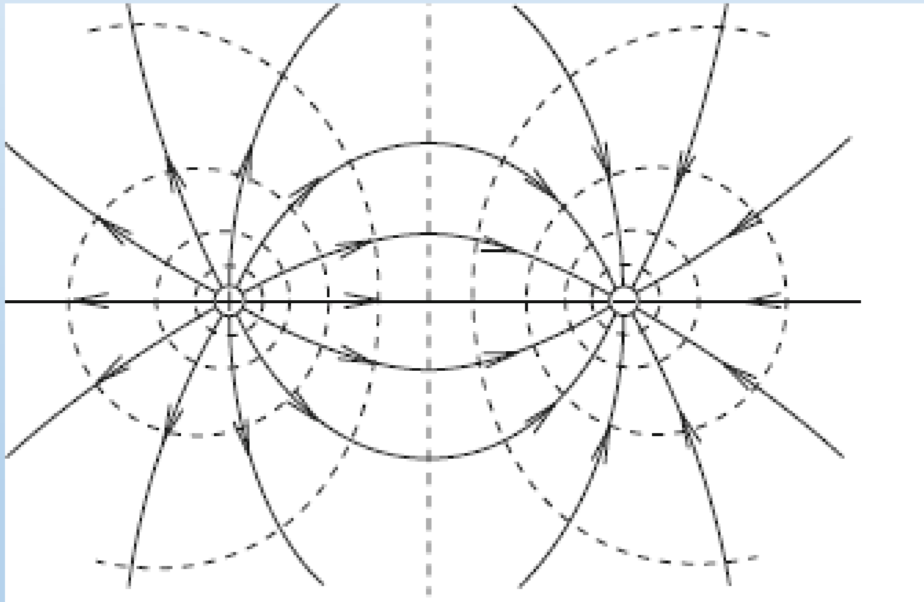
1.4 Electrical potential.

Equipotential surfaces:

Positive electric charge



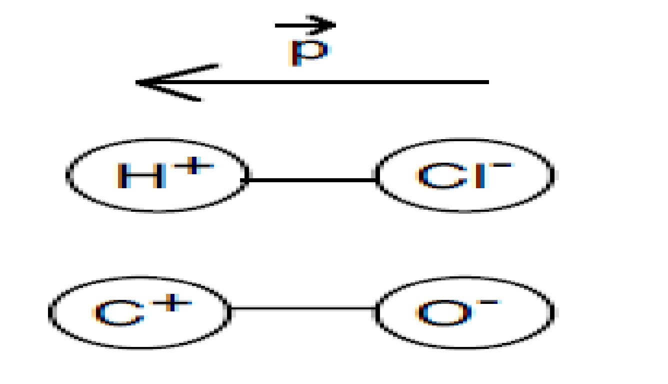
Pair of positive and negative charges

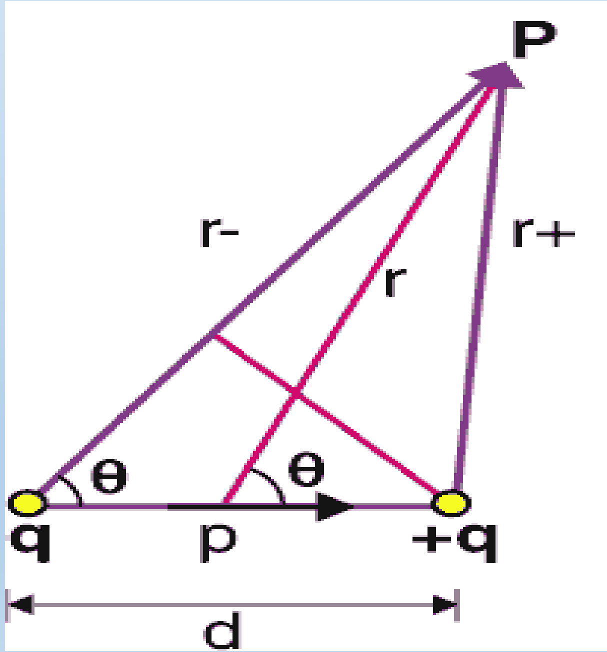


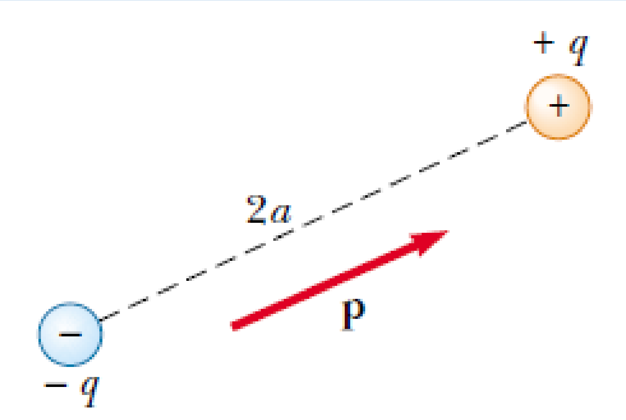
The field line is always oriented in the direction of decreasing potentials.

1-5 Electric Dipole

- **An electrostatic dipole is a system of two point electric charges, +q and –q located at a distance d=2a very small compared to the distance r.**
- **Molecules such as HCL, CO, H_20, CO_2 constitute examples of electrostatic dipoles.**





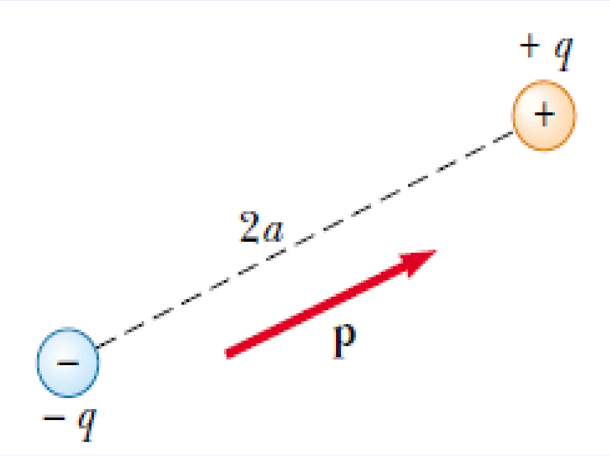


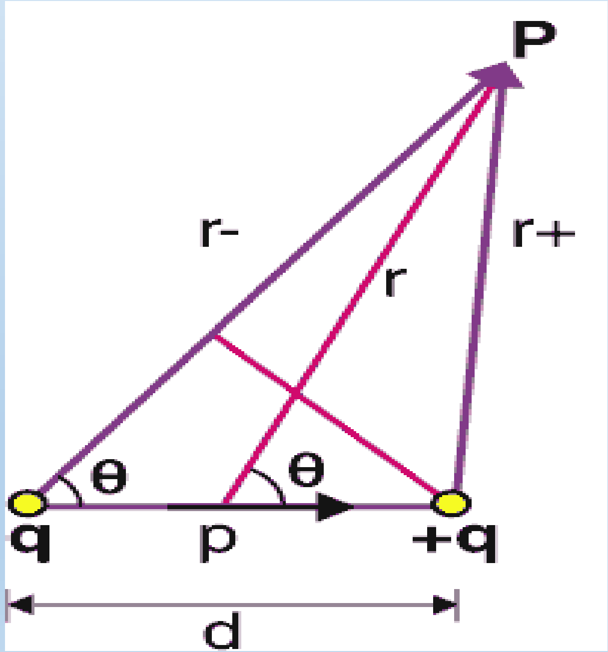
1-5 Electric Dipole

- **Definition: we call the electric dipole moment the quantity**

$$\vec{p} = qd\,\vec{i} = 2aq\,\vec{i}$$

$$[P] = C.m$$





1-5 Electric Dipole

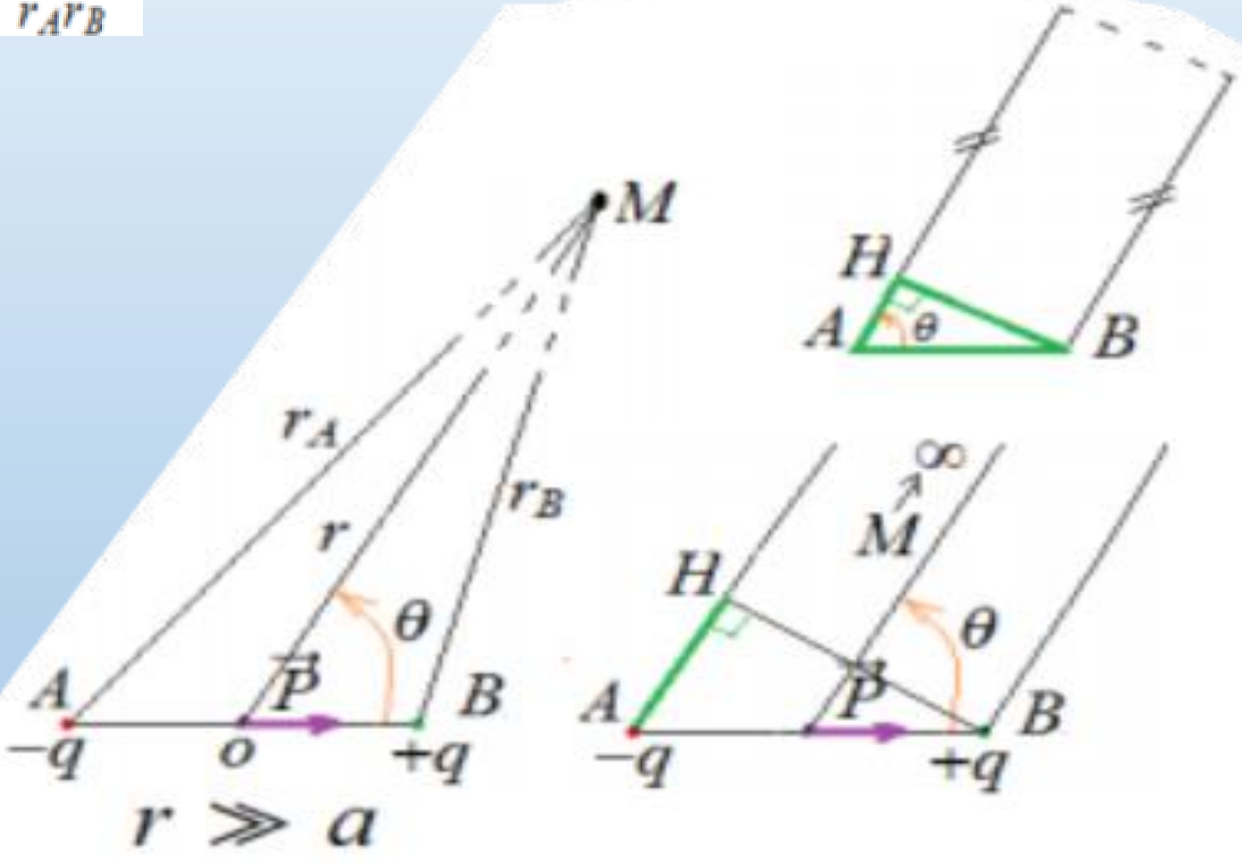
1-5-1Potential of a dipole

$$V_M = V_- + V_+ = Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \Rightarrow V_M = Kq\frac{r_A - r_B}{r_A r_B}$$

$$V_- = K\frac{-q}{r_A} \qquad V_+ = K\frac{q}{r_B}$$

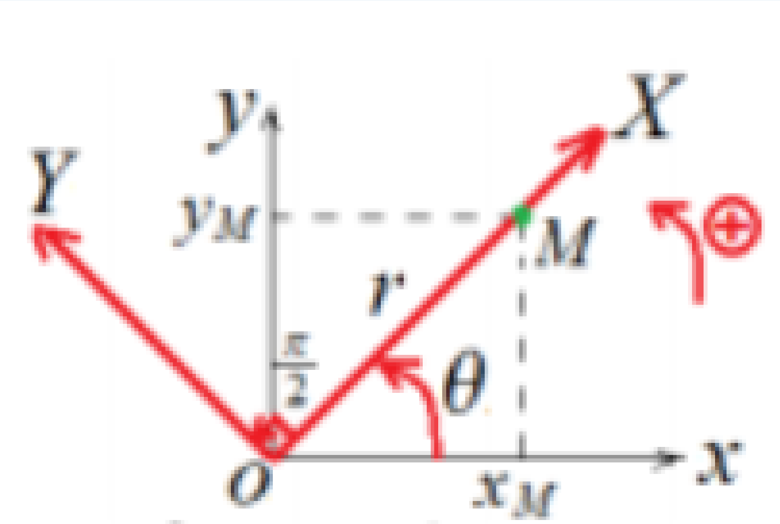
We have $\Rightarrow \begin{cases} r_A - r_B = AH = 2a\cos\theta \\ r_A \times r_B \simeq r^2 \end{cases}$

$$\Rightarrow V_M = \frac{K2aq\cos\theta}{r^2} \equiv \frac{KP\cos\theta}{r^2} = K\frac{\vec{P}\vec{r}}{r^2}$$



1-5 Electric Dipole

In polar coordinats $\vec{E} = -\vec{\nabla} V \equiv -\overrightarrow{\text{grad}} V$ $\qquad \vec{E} = \vec{E_r} + \vec{E_\theta}$



$$\vec{E} = -\vec{\nabla} V \Rightarrow \vec{E} = \begin{pmatrix} -\frac{\partial V}{\partial r} \\ -\frac{1}{r}\frac{\partial V}{\partial \theta} \end{pmatrix} = \begin{pmatrix} E_r \\ E_\theta \end{pmatrix}$$

For the dipole, The component of the electrical field are

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{KP\cos\theta}{r^2}\right) = \frac{2KP\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{KP\cos\theta}{r^2}\right) = \frac{KP\sin\theta}{r^3}$$

E_r is the radial component whereas E_ϑ is the angular component

$$\vec{E} = \vec{E_r} + \vec{E_\theta} \Rightarrow E = \sqrt{E_r^2 + E_\theta^2} = \frac{KP\sqrt{1+3\cos^2\theta}}{r^3}$$

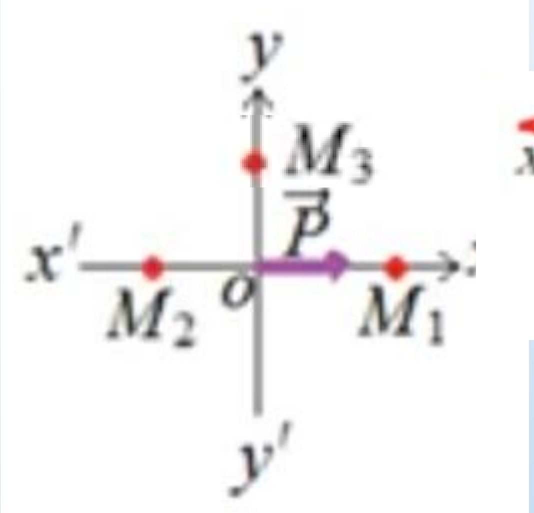
Gauss Positions

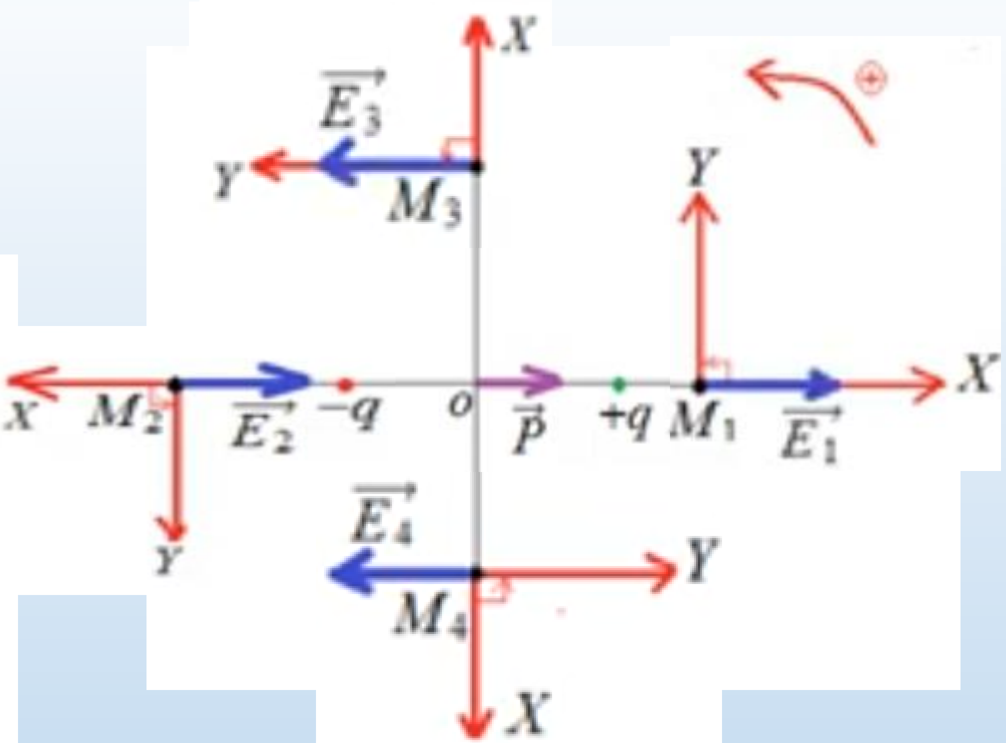
It is defined in the xx' and yy' axes

$$(\overrightarrow{ox}, \overrightarrow{oM}) = \theta$$

$$E_r = \frac{2KP\cos\theta}{r^3}$$

$$E_\theta = \frac{KP\sin\theta}{r^3}$$



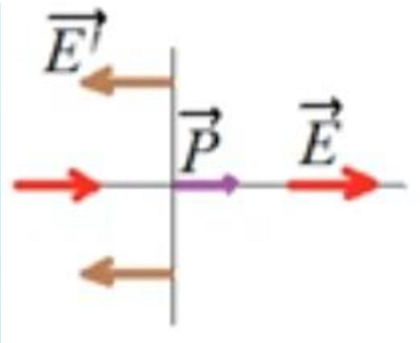


$M_1 \rightsquigarrow \theta = 0 \Rightarrow E_r = \frac{2KP}{r^3}; E_\theta = 0 \Rightarrow \overrightarrow{E_1} = \overrightarrow{E_r}$

$M_2 \rightsquigarrow \theta = \pi \Rightarrow E_r = -\frac{2KP}{r^3}; E_\theta = 0 \Rightarrow \overrightarrow{E_2} = \overrightarrow{E_r}$

$M_3 \rightsquigarrow \theta = \frac{\pi}{2} \Rightarrow E_r = 0; E_\theta = \frac{KP}{r^3} \Rightarrow \overrightarrow{E_3} = \overrightarrow{E_\theta}$

$M_4 \rightsquigarrow \theta = \frac{3\pi}{2} \Rightarrow E_r = 0; E_\theta = -\frac{KP}{r^3} \Rightarrow \overrightarrow{E_4} = \overrightarrow{E_\theta}$



$$E = \frac{2KP}{r^3}$$

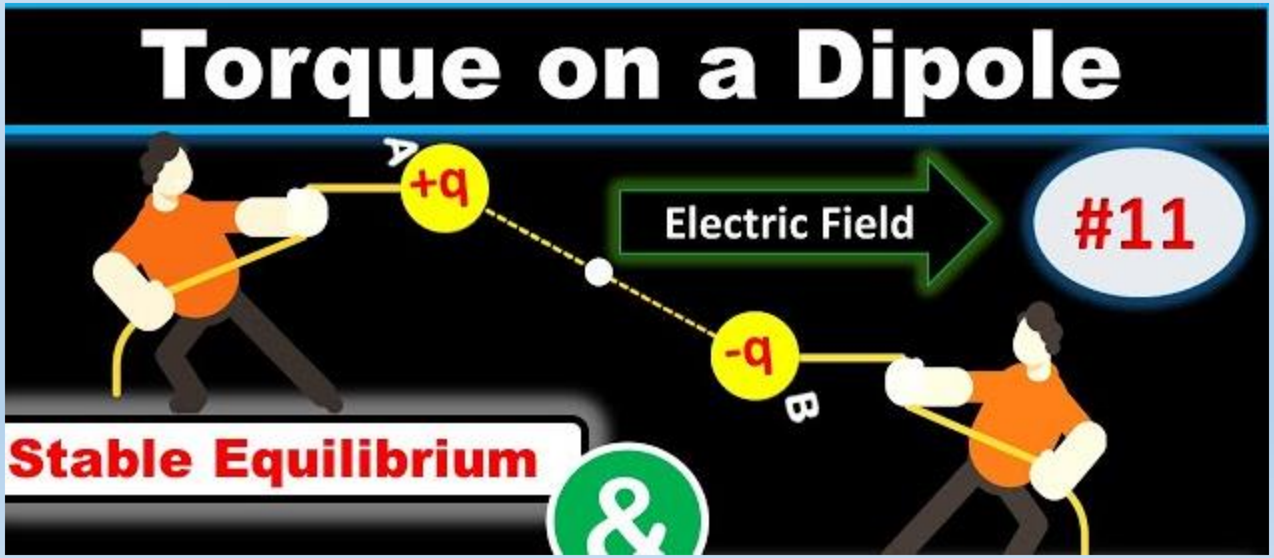
$$E' = \frac{KP}{r^3}$$

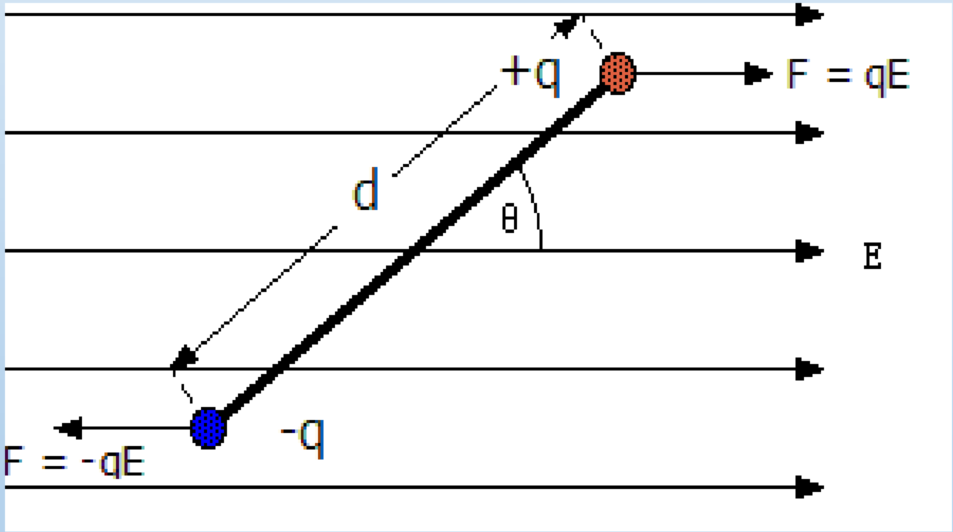
1-5 Electric Dipole

Interaction between an external field and the dipole

If we place a dipole, with electric moment p, in an external uniform field *E*, the charges, which constitute it, are subjected to equal forces and opposite.

The dipole is subjected to a torque with respect to o.





https://www.youtube.com/watch?v=aFfmZjXDOCM

1-5 Electric Dipole

Interaction energy:

The potential energy of a dipole, placed in an E field, is calculated by adding the potential energies of each charge: $E_p = E_{p(-q)} + E_{p(q)} = -\vec{P}\vec{E}$

Demonstration

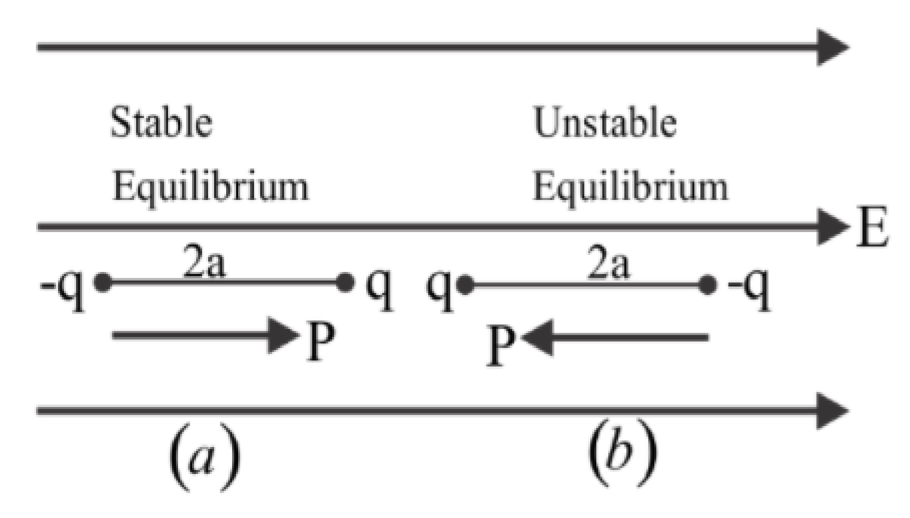
V_A is the potential at the point A : The charge *(-q)* has the potential energy $-q\ V_A$

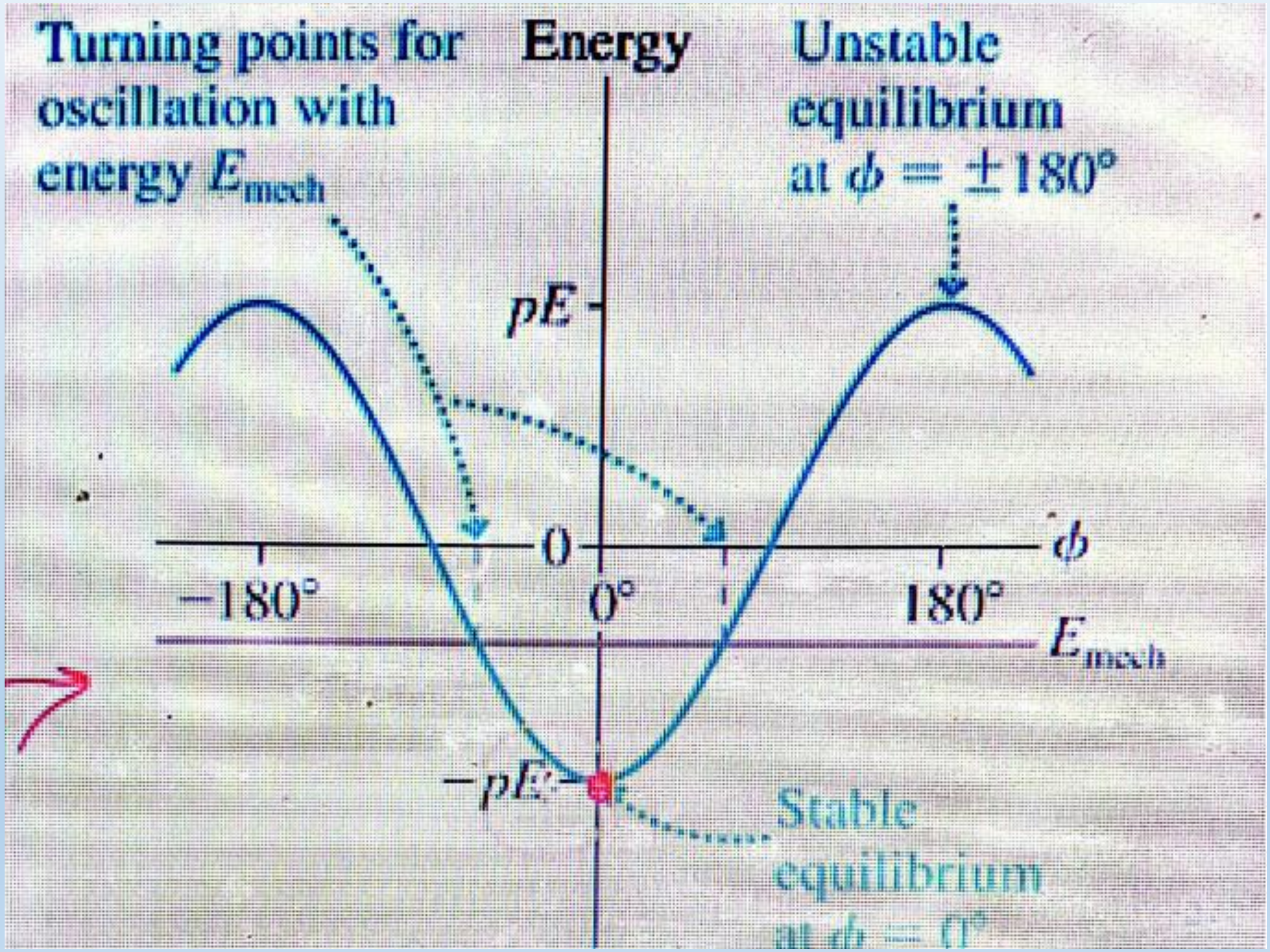
V_B is the potential at the point B : The charge *(+q)* has the potential energy $+q\ V_B$

$$E_p = E_{p(-q)} + E_{p(q)} = qV_B - qV_A = W_{B\to A}(\vec{F_B}) = -q\vec{E}.\overrightarrow{AB} = -\vec{P}\vec{E}$$

1-5 Electric Dipole

For $\theta=0$ The energy is minimum : Stable equilibrium position.
For $\theta=\Pi$ The energy is maximum : instable equilibrium position.





1-5 Electric Dipole

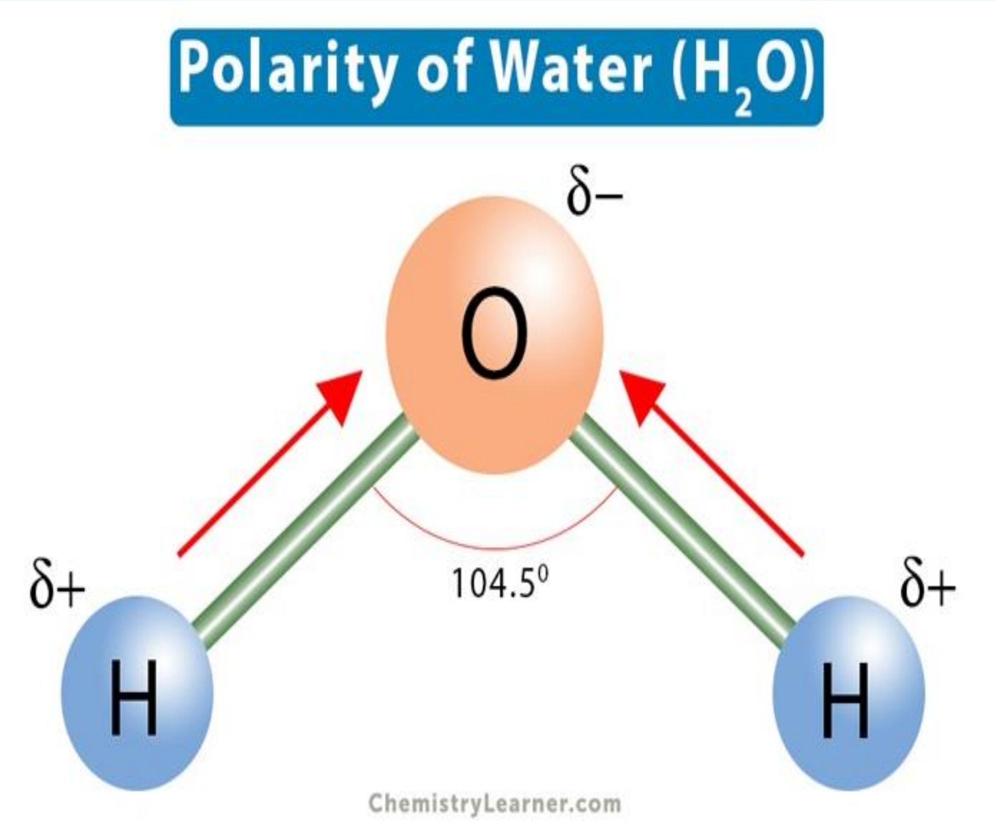
Dipole of the water molecule

The center N of the 10 negative charges and the center P of the 10 positive charges are distinct.

The electric dipole of the water molecule is:

$P = 6.2 \times 10^{-30} C.m$

$d = 9{,}65\ 10^{-11} m$



Exercise of dipole

The electric dipole associated with the water molecule has a dipole moment of P = 6.2×10^{-30} C·m.
It is placed in the electric field created by another water molecule. Their centers are separated by 3.44 Å.

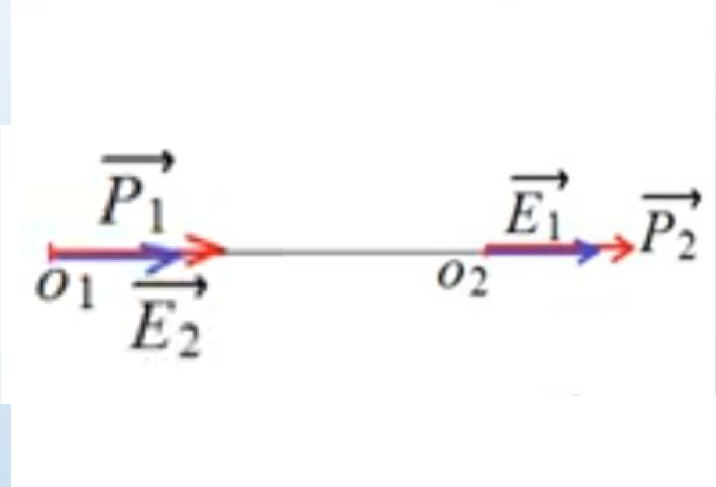
1) Assuming that their centers are fixed, how do they orient themselves?
What is then the potential energy of each molecule? What does it represent?

2) To evaporate 1 g of water, 2260 J of energy are required. Deduce the number of bonds that one water molecule forms with its neighbors. Then explain the phenomenon of body heat elimination through perspiration

Answer

$$\vec{E} = \frac{2K\vec{P}}{r^3} \qquad \vec{E'} = -\frac{K\vec{P}}{r^3}$$

The first Gauss position



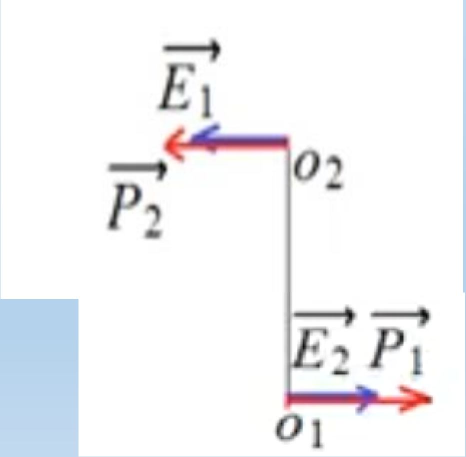
$$E_{p_1} = -\vec{P_1}\vec{E_2} = -\vec{P_1}\frac{2K\vec{P_2}}{r^3} = -2K\frac{\vec{P_1}\vec{P_2}}{r^3}$$

$$E_{p_2} = -\vec{P_2}\vec{E_1} = -\vec{P_2}\frac{2K\vec{P_1}}{r^3} = -2K\frac{\vec{P_1}\vec{P_2}}{r^3}$$

$$\vec{P_1} = \vec{P_2}$$

$$E_{p_1} = E_{p_2} = -1.7 \times 10^{-20} J$$

The second Gauss position



$$E_{p_1} = -\vec{P_1}\vec{E_2} = -\vec{P_1}\left(-\frac{K\vec{P_2}}{r^3}\right) = K\frac{\vec{P_1}\vec{P_2}}{r^3}$$

$$E_{p_2} = -\vec{P_2}\vec{E_1} = -\vec{P_2}\left(-\frac{K\vec{P_1}}{r^3}\right) = K\frac{\vec{P_1}\vec{P_2}}{r^3}$$

$$\vec{P_1} = -\vec{P_2}$$

$$E_{p_1} = E_{p_2} = -0.85 \times 10^{-20} J$$

Remark:

If $\vec{P_1}$ and $\vec{P_2}$ have the same direction ⇒ potential energy E_p is minimal ⇒ stable equilibrium.
A negative E_p represents a bonding energy between the two molecules.
$-E_p$ corresponds to the energy required to break this bond.

2) To evaporate 1 g of water:
→ 18 g corresponds to N_A molecules
→ Therefore, 1 g corresponds to $(N_A / 18) = 0.334 \times 10^{23}$ molecules

$$18g \rightarrow N_A$$
$$1g \rightarrow n$$

Since 2260 J is needed for 1 g of water, the energy per molecule is:

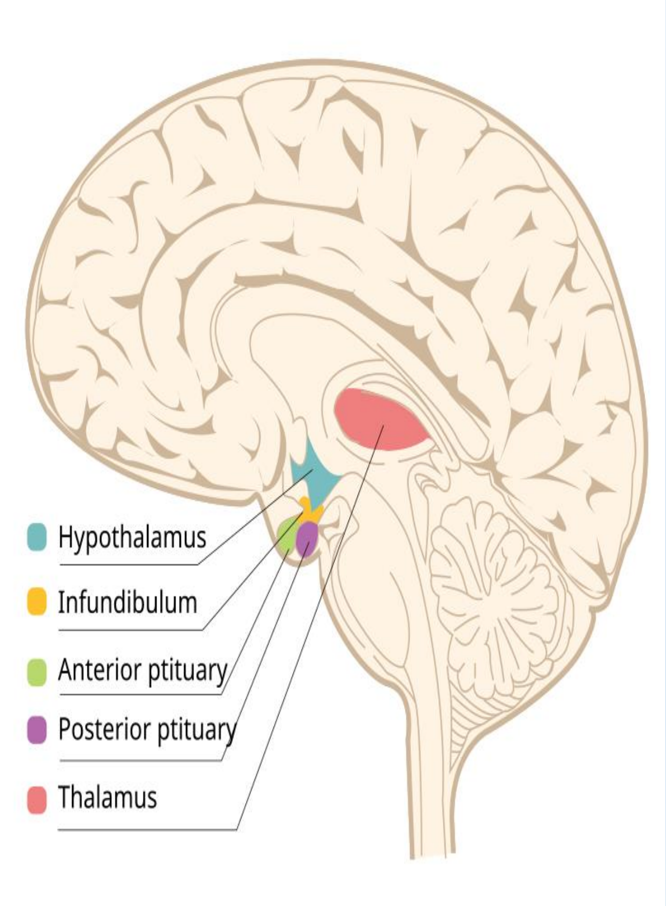
$$2260 J \rightarrow 0.334 \times 10^{23} \text{ molécules}$$
$$E \rightarrow 1 \text{ molécule.}$$

$E = 2260 / (0.334 \times 10^{23}) = 6.76 \times 10^{-20}$ J per molecule

Given that the energy required to break one bond is 1.7×10^{-20} J, the number of bonds per molecule is: $x = (6.76 \times 10^{-20}) / (1.7 \times 10^{-20}) = 4$ bonds.

Explanation

To function, the muscle consumes O_2 and glucose, which react together to release CO_2 and energy. Part of this energy is used by the muscle, and the excess is released as heat.
This heat is sensed by the skin and the hypothalamus.
At the skin level, this heat breaks the hydrogen bonds between water molecules, causing water to evaporate as sweat — a mechanism that eliminates body heat through perspiration.



The hypothalamus is a small structure in the brain located below the thalamus, which acts as a major control center for many vital functions. It regulates functions such as body temperature, hunger, thirst, sleep, and emotions..

1-5 Gauss Law

The **total electric flux is the** Integral of the electric field (E) over a closed surface (S) and equal the total enclosed charge (Q) divided by the permittivity of free space (ε_0).

$$\Phi = \oiint_S \vec{E} \cdot \vec{dS} = \frac{Q_{\text{int}}}{\varepsilon_0}$$

-Gauss' theorem is used to calculate the electric field for charge distributions.

-General method:

-Finding the closed surface.

-Writing the definition of the flow.

-Applying the Gauss theory.

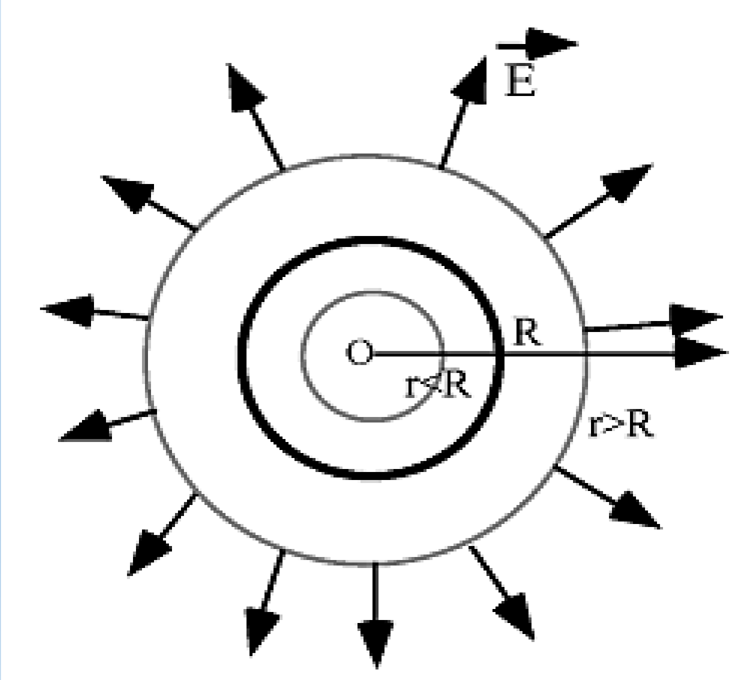
Karl Friedrich Gauss German mathematician and astronomer

$$\begin{aligned} Sphere\ V &= \frac{4}{3}\pi R^3,\ S = 4\pi R^2 \\ Cylinder\ V &= \pi R^2 h,\ S = 2\pi R^2 + 2\pi R h \end{aligned}$$

1-5 Gauss Law

- Example :

We consider a sphere with center O and radius R, charged with a surface distribution of charges **σ**. This distribution having a spherical symmetry Calculate the resulting electrostatic field and potential in all space. We take V(∞)=0. draw the graph E(r) and V(r).



1-5 Gauss Law

Solution:

1-The field :

From the Gauss theory, we have :

$$\Phi = \oiint_S \vec{E}\cdot\vec{dS} = \oiint_S E(r)dS = E(r)\,4\pi\,r^2 = \frac{Q_{\text{int}}}{\varepsilon_0}$$

Case r<R

$$Q_{int} = 0 \Rightarrow \boldsymbol{E_1} = \mathbf{0}$$

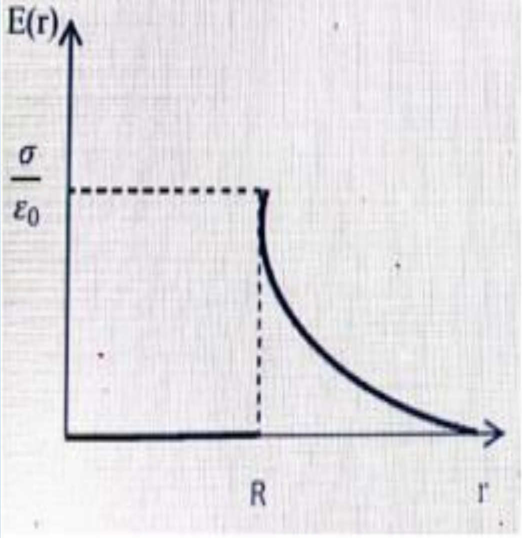
Case r>R

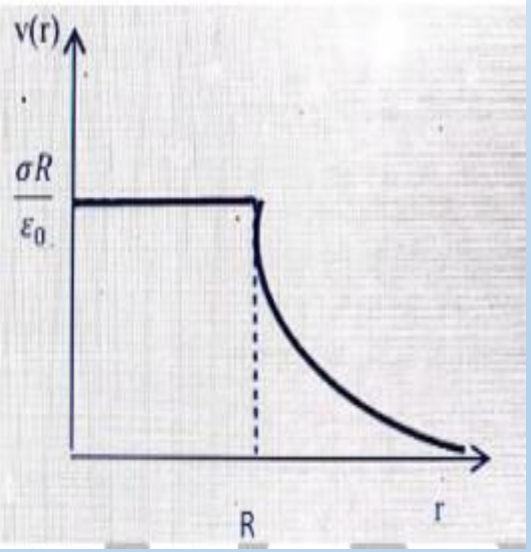
$$dq = \sigma ds \Rightarrow\ Q_{int} = \sigma s = \sigma 4\pi R^2$$

Therefor: $$E_2 4\pi r^2 = \frac{\sigma 4\pi R^2}{\varepsilon_0} \Rightarrow \boldsymbol{E_2} = \frac{\boldsymbol{\sigma R^2}}{\boldsymbol{\varepsilon_0 r^2}}$$

Case r<R $$E_1 = 0 \Rightarrow v_1 = C_1$$

Case r>R $$E_2 = \frac{\sigma R^2}{\varepsilon_0 r^2} \Rightarrow\ v_2 = -\frac{\sigma R^2}{\varepsilon_0}\int \frac{dr}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r} + C_2$$





1-5 Gauss Theory

TABLE Typical Electric Field Calculations Using Gauss's LawCharge

Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$K\frac{Q}{r^2}$	$r > R$
	$K\frac{Q}{R^3}r$	$r < R$
Thin spherical shell of radius R and total charge Q	$K\frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2K\frac{\lambda}{r}$	Outside the line
Nonconducting, infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\frac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor