Chapter III : Work and Energy

3.1 Introduction:

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportationand heating, electricity for lights and appliances.

We first introduce the concept of work. Work is done by a force acting on an object when the point of application of that force moves through some distance and the force has a component along the line of motion.

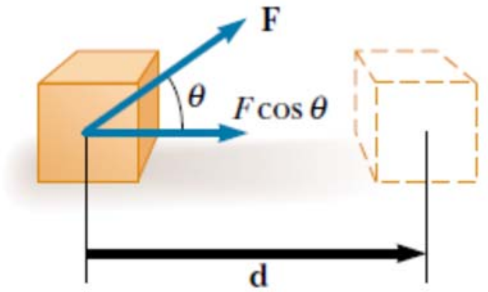

Figure 3.1

The work W done on an object by an agent exerting a constant force on the object is the product of the component of the force in the direction of the displacement and the magnitude of the displacement:

$$W = \vec{F}.\vec{d} = F.d\cos\theta \qquad 3.1$$

It is important to note that work is an energy transfer; if energy is transferred to the system (object), W is positive; if energy is transferred from the system, W is negative.
Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the newton_meter (N_m). This combination of units is used so frequently that it has been given a name of its own: the joule (J).

3.2 KINETIC ENERGY AND THE WORK – KINETIC ENERGY THEOREM

We have seen in In Chapter 2 that using Newton's second law of motion; we can relate the linear momentum of a particle to the resultant force acting on the particle: The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle:

$$\overrightarrow{\Delta p} = \vec{F}.\Delta t$$

If is Δt small $\overrightarrow{dp} = \vec{F}.dt$ then

$$\overrightarrow{dp} \equiv d(m\vec{v}) = \vec{F}.dt$$

Or $dW = \vec{F}.\overrightarrow{dx}$

If f is parallel to dx and the same direction

$$dW = F.dx = m\frac{dv}{dt}dx = mdv.v$$

So we have:

$$dW = \vec{F}.\overrightarrow{dx} = d\left(\frac{m}{2}v^2\right)$$

The quantity represents $\frac{1}{2}mv^2$ the energy associated with the motion of the particle. This quantity is so important that it has been given a special name kinetic energy. The net work done on a particle by a constant net $\vec{F}$ force acting on it equals the change in kinetic energy of the particle.
In general, the kinetic energy E_K of a particle of mass m moving with a speed v is defined as

$$E_K = \tfrac{1}{2}mv^2$$

TABLE 7.1 Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^{4}	2.65×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^{3}	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^{4}	3.14×10^{10}
Automobile at 55 mi/h	2 000	25	6.3×10^{5}
Running athlete	70	10	3.5×10^{3}
Stone dropped from 10 m	1.0	14	9.8×10^{1}
Golf ball at terminal speed	0.046	44	4.5×10^{1}
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

From the above equations for dW, this gives

$$\sum W = \int_{x_i}^{x_f} mv\frac{dv}{dx}\,dx = \int_{v_i}^{v_f} mv\,dv$$

$$\sum W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

In taking the ratio of the amount of work done to the time taken to do it, we have a way of quantifying this concept. The time rate of doing work is called power. It is given by:

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

The symbol W (not italic) for watt should not be confused with the symbol *W* (italic) for work. A unit of power in the British engineering system is the porsepower (hp):

$$1 \text{ hp} = 746 \text{ W}$$

3.3 Gravitational Potential Energy

As an object falls toward the Earth, the Earth exerts a gravitational force on the object, with the direction of the force being the same as the direction of the object's motion.

Let us calculate the Work related to the gravitational force:

$$\vec{F} = -\frac{GMm}{r^2}\vec{u}$$

Consider the displacement $\overrightarrow{dl}$

$$dW = \vec{F}.\overrightarrow{dl} = -\frac{GM_T m}{r^2}\vec{u}.\overrightarrow{dl}$$

Where $\vec{u}.\overrightarrow{dl}$ represent dr

$$dW = -\frac{GM_T m}{r^2}dr$$

$$W_{A\to B} = -\int_{r_B}^{r_A} -\frac{GM_T m}{r^2}dr = \frac{GM_T m}{r_B} - \frac{GM_T m}{r_A} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$\Rightarrow \frac{1}{2}mv_B^2 - \frac{GM_T m}{r_B} = \frac{1}{2}mv_A^2 - \frac{GM_T m}{r_A}$$

We remark that the quantity $\frac{1}{2}mv^2 - \frac{GM_T m}{r}$ is conserved

When

$E_K \nearrow \Rightarrow -\frac{GM_T m}{r} \searrow$

When

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The quantity $-\frac{GM_T m}{r}$ plays the role of energy reservoir

It is named the gravitational energy

$$U = -\frac{GM_T m}{r} + \text{constant}$$

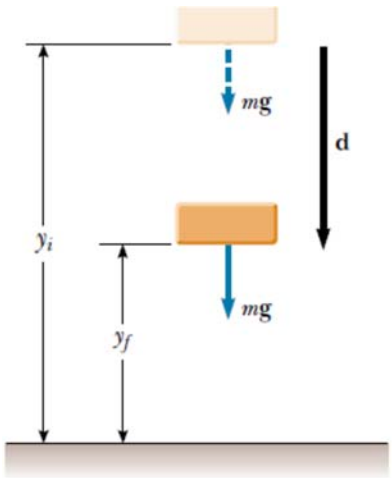
The quantity $E_T = \frac{1}{2}mv^2 - \frac{GM_T m}{r}$

Is named the total energy or mechanical energy
Note that the total mechanical energy of a system remains constant in any isolated system of objects that interact only through conservative forces. Because the total mechanical energy E of a system is defined as the sum of the kinetic and potential energies, we can write

$$E \equiv K + U$$

Let us now directly relate the work done on an object by the gravitational force to the gravitational potential energy of the object–Earth system.

$r = R_T + h$ and $h \lll R_T$



To do this, let us consider a brick of mass m at an initial height yi above the ground, as shown in Figure3.1. If we neglect air resistance, then the only force that does work on the brick as it falls is the gravitational force exerted on the brick mg. The work W_g done by the gravitational force as the brick undergoes a downward displacement d is

$$W_g = (m\mathbf{g}) \cdot \mathbf{d} = (-mg\mathbf{j}) \cdot (y_f - y_i)\,\mathbf{j} = mgy_i - mgy_f$$

We just learned that the quantity mgy is the gravitational potential energy of the system U_g , and so we have

$$W_g = U_i - U_f = -(U_f - U_i) = -\Delta U_g$$

We remark that the quantity is a conserved value $U = mgh$

We just learned that the quantity is the gravitational potential energy of the system U_g , and so we have

$$W_g = U_i - U_f = -(U_f - U_i) = -\Delta U_g$$

From this result, we see that the work done on any object by the gravitational force is equal to the negative of the change in the system's gravitational potential energy.
The unit of gravitational potential energy is the same as that of work—the joule. Potential energy, like work and kinetic energy, is a scalar quantity.

Elastic Potential Energy
Now consider a system consisting of a block plus a spring, as shown in Figure 3.2. The force that the spring exerts on the block is given by

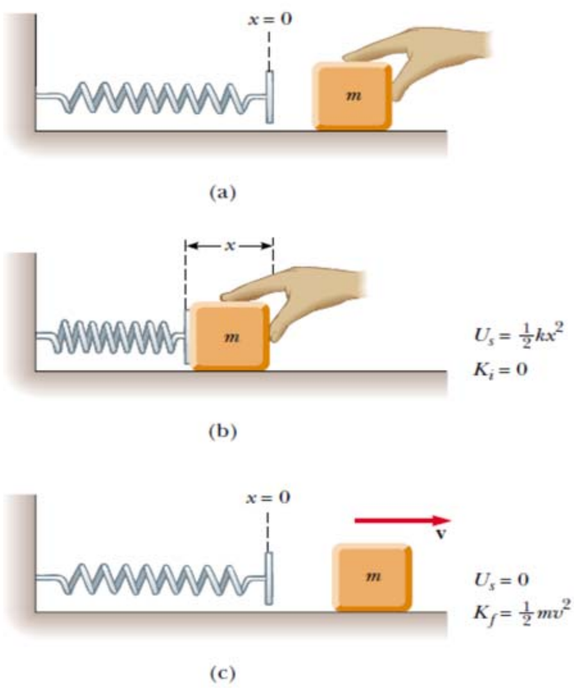
$$\vec{F}_s = -kx.$$

The work done by the spring force on a block connected to the spring is given by

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

In this situation, the initial and final x coordinates of the block are measured from its equilibrium position, x = 0. Again we see that W_s depends only on the initial and final x coordinates of the object and is zero for any closed path. The elastic potential energy function associated with the system is defined by

$$U_s \equiv \tfrac{1}{2}kx^2$$



CONSERVATIVE AND NONCONSERVATIVE FORCES

The work done by the gravitational force does not depend on whether an object falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. On the other hand, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to frictional forces. We can use this varying dependence on path to classify forces as either conservative or nonconservative.Of the two forces just mentioned, the gravitational force is conservative and the frictional force is nonconservative.

Conservative Forces

Conservative forces have two important properties:

1. A force is conservative if the work it does on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another. As we learned in the preceding section, the work done by the gravitational force on an object moving between any two points near the Earth's surface is

$$W_g = mgy_i - mgy_f.$$

From this equation we see that Wg depends only on the initial and final y coordi nates of the object and hence is independent of the path. Furthermore, Wg is zero when the object moves over any closed path.

For the case of the object–spring system, the work Ws done by the spring force

is given by $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Again, we see that the spring force is conservative because Ws depends only on the initial and final x coordinates of the object and is zero for any closed path.
We can associate a potential energy with any conservative force and can do this only for conservative forces. In the previous section, the potential energy associated with the gravitational force was defined as

$$U_g \equiv mgy.$$

In general, the work Wc done on an object by a conservative force is equal to the initial value of the potential energy associated with the object minus the final value:

$$W_c = U_i - U_f = -\Delta U$$

We have also
The relationship between conservative forces and potential energy

$$\vec{F}_c = -\overrightarrow{\text{grad}\,U}$$

Nonconservative Forces

A force is nonconservative if it causes a change in mechanical energy E, which we define as the sum of kinetic and potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases. As a result of the frictional force, the temperatures of the book and surface increase. The type of energy associated with temperature is internal energy,

WORK DONE BY NONCONSERVATIVE FORCES
As we have seen, if the forces acting on objects within a system are conservative,then the mechanical energy of the system remains constant. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system does not remain constant.