University of Djilali Bounaama - Khemis Miliana
Faculty of Sciences and Technology
Department of Material Sciences

# Course of Physics 1: Mechanics

**Level :1st year ST**

**Academic year: 2023-2024**

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# Semestre 1

| Unité d'enseignement | Matières Intitulé | Crédits | Coefficient | Volume horaire hebdomadaire Cours | TD | TP | Volume Horaire Semestriel (15 semaines) | Travail Complémentaire en Consultation (15 semaines) | Mode d'évaluation Contrôle Continu | Examen |
|---|---|---|---|---|---|---|---|---|---|---|
| UE Fondamentale Code : UEF 1.1 Crédits : 18 Coefficients : 9 | Mathématiques 1 | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| | Physique 1 | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| | Structure de la matière | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| UE Méthodologique Code : UEM 1.1 Crédits : 9 Coefficients : 5 | TP Physique 1 | 2 | 1 | | | 1h30 | 22h30 | 27h30 | 100% | |
| | TP Chimie 1 | 2 | 1 | | | 1h30 | 22h30 | 27h30 | 100% | |
| | Informatique 1 | 4 | 2 | 1h30 | | 1h30 | 45h00 | 55h00 | 40% | 60% |
| | Méthodologie de la rédaction | 1 | 1 | 1h00 | | | 15h00 | 10h00 | | 100% |
| UE Découverte Code : UED 1.1 Crédits : 1 Coefficients : 1 | Les métiers en sciences et technologies 1 | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| UE Transversale Code : UET 1.1 Crédits : 2 Coefficients : 2 | Langue étrangère 1 Français | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| | Langue étrangère 2 anglais | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| **Total semestre 1** | | **30** | **17** | **16h00** | **4h30** | **4h30** | **375h00** | **375h00** | | |

# Semestre 2

| Unité d'enseignement | Matières | Crédits | Coefficient | Volume horaire hebdomadaire | | | Volume Horaire Semestriel (15 semaines) | Travail Complémentaire en Consultation (15 semaines) | Mode d'évaluation | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Intitulé | | | **Cours** | **TD** | **TP** | | | **Contrôle Continu** | **Examen** |
| UE Fondamentale<br>Code : UEF 1.2<br>Crédits : 18<br>Coefficients : 9 | Mathématiques 2 | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| | Physique 2 | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| | Thermodynamique | 6 | 3 | 3h00 | 1h30 | | 67h30 | 82h30 | 40% | 60% |
| UE Méthodologique<br>Code : UEM 1.2<br>Crédits : 9<br>Coefficients : 5 | TP Physique 2 | 2 | 1 | | | 1h30 | 22h30 | 27h30 | 100% | |
| | TP Chimie 2 | 2 | 1 | | | 1h30 | 22h30 | 27h30 | 100% | |
| | Informatique 2 | 4 | 2 | 1h30 | | 1h30 | 45h00 | 55h00 | 40% | 60% |
| | Méthodologie de la présentation | 1 | 1 | 1h00 | | | 15h00 | 10h00 | | 100% |
| UE Découverte<br>Code : UED 1.2<br>Crédits : 1<br>Coefficients : 1 | Les métiers en sciences et technologies 2 | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| UE Transversale<br>Code : UET 1.2<br>Crédits : 2<br>Coefficients : 2 | Langue étrangère 1 Français | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| | Langue étrangère 2 anglais | 1 | 1 | 1h30 | | | 22h30 | 02h30 | | 100% |
| **Total semestre 2** | | **30** | **17** | **16h00** | **4h30** | **4h30** | **375h00** | **375h00** | | |

## PROGRAMME "Physique1 » Chapitre 0.

**Rappels mathématiques :**

- Les équations aux dimensions.
- Calcul vectoriel

**Chapitre I. Cinématique :**

1- Vecteur position dans les systèmes de coordonnées (cartésiennes, cylindrique…)- loi de mouvement - Trajectoire.

2- Vitesse et accélération dans les systèmes de coordonnées.

3- Applications : Mouvement du point matériel dans les différents systèmes de coordonnées

4- Mouvement relatif.

**Chapitre II. Dynamique :**

1- Généralité : Masse - Force - Moment de force –Référentiel Absolu et Gallilien 2- Les lois de Newton

3- Principe de la conservation de la quantité de mouvement. 4- Equation différentielle du mouvement 5- Moment cinétique

6- Applications de la loi fondamentale pour des forces (constante, dépendant du temps, dépendant de la vitesse, force centrale, etc).

**Chapitre III. Travail et énergie :**

1- Travail d'une force 2- Energie Cinétique

3- Energie potentiel – Exemples d'énergie potentielle (pesanteur, gravitationnelle, élastique) 4- Forces conservatives et non conservatives - Théorème de l'énergie totale

# *Chapter 0: Mathematical reminders*

## *I.1. Generalities on Physical quantities* (المقادير الفيزيائية)

• A physical quantity [A] is a quantity wich can be measured, with instruments or even by using our senses, and wich that reports a physical property.

• **For example**: length, mass, time, temperature, electric current, light intensity, volume…. etc

•Physical quantities [A] have **numerical magnitude "a"** and **unit** $\{A\}$

$$a = \frac{[A]}{\{A\}}$$

$[A]$ → **Physical quantity**

$\{A\}$ → **Unit**

**numerical magnitude** → $a$

**Example: The Velocity V = 10 m/s**

## ➢ There are two types of measurable quantities

- **Scalar quantities:**

  **Length, mass, time, energy……….**

- **Vector quantities:**

  **Velocity, Acceleration, Electric and magnetic field……**

# ***International System of Units**(Called « SI » System)*

- This system is composed of the following fundamental units:

| Unit | Physical quantuty |
|---|---|
| Meter (m) | Length |
| Kilogram (Kg) | Mass |
| Second (S) | Time |
| Ampere (A) | Electric current intensity |
| Kelvin (K) | Temperature |
| Candela (Cd) | Luminous intensity |
| Mole | Quantity of matter |

- The First Fourth units form the system **MKSA**

## ***Derived quantities***

- These quantities are expressed as a combination of fundamental quantities.
- The units of all quantities other than fundamental units is called **derived unit**.
- Derived units are obtained in terms of fundamental quantities.

**Exemple :**

**Area :** $m^2$

**velocity :** $m.s^{-1}$ .

**Force :** Newton (N) = $Kg.m.s^{-2}$ .

**Energy :** Joule (J) =$Kg\ m^2.s^{-2}$

# I.2. Equation for dimensions (Dimensional Equations)

- **Determine derived units based on fundamental units nombres réels**

$$[A] = M^{\alpha} L^{\beta} T^{\gamma} I^{\lambda}$$

$\boldsymbol{\alpha, \beta, \gamma, \lambda}$: $\boldsymbol{real\ numberd}$

- **This equation consists of the equation for dimensions of a quantity A, with:**

  **M : Mass, L : Length, T : Time, I : Current intensity**

## Examples:

- **Velocity :** $[V] = L.T^{-1}(m/s)$
- **Acceleration:** $[a] = L.T^{-2}(m/s^2)$
- **Force :** $\vec{F} = m\vec{a} \quad \Rightarrow [F] = ML.T^{-2}(kg.m.s^{-2} = Newton)$
- **Work :** $W = \int \vec{F}.\vec{dl} \quad \Rightarrow [W] = [F][dl] = MLT^{-2}L = ML^2T^{-2}(Kg.m^2/s^2 = Joule)$

## Remark :

- The dimensional equation is used to check the homogeneity of the physical formulas.

## Example :

The period of oscillation of a simple pendulum of length L is it given by:

$T = 2\pi\sqrt{\frac{g}{L}}$ .................$(I)$ **Ou par** $T = 2\pi\sqrt{\frac{L}{g}}$ ................$(II)$

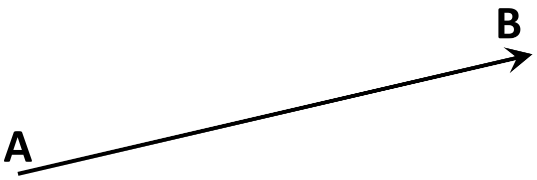
- $(I) \Rightarrow T = 2\pi g^{1/2}L^{-1/2} \Rightarrow [T] = (LT^{-2})^{1/2}L^{-1/2} = T^{-1} \Rightarrow [T] = T^{-1}$ $False$
- $(II) \Rightarrow T = 2\pi L^{1/2}g^{-1/2} \Rightarrow [T] = L^{1/2}(LT^{-2})^{-1/2} = T \Rightarrow [T] = T$ $right$

## II. Reminder on vectors

- Mathematical entity defined by multiple numeric values.

- These values describe the magnitude and orientation of the vector.

- A vector $\overrightarrow{AB}$ is characterized by:




  - Its origin or point of application.

  - Its direction, which is the direction of movement of a mobile having from point A to point B.

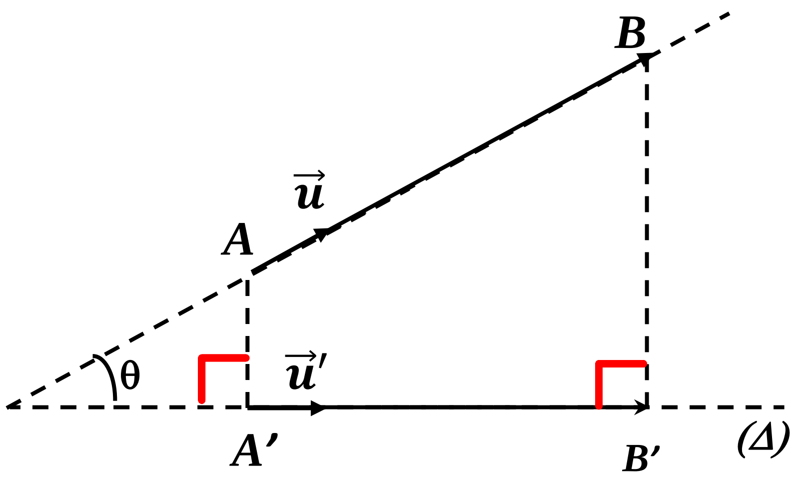
  - Its magnitude which presents the length AB. It is noted $\|\overrightarrow{AB}\|$

- **Unit vector** or **orth** is a vector whose length is equal to one.

## *II.1- Projecting a vector onto an axis:*

$$\overrightarrow{AB} = \|\overrightarrow{AB}\|\,\vec{u}$$

$\vec{u}$ represent unit vector, with $\|\vec{u}\| = 1$

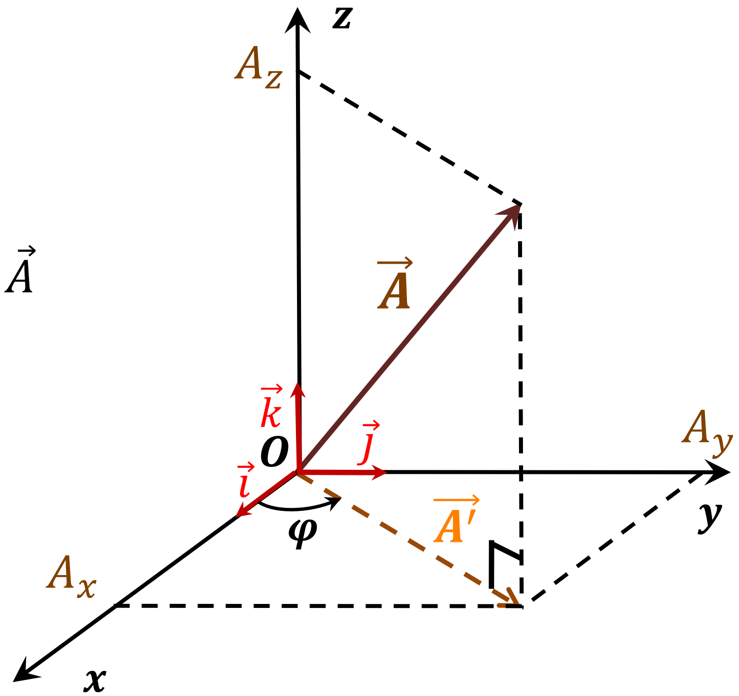



A' and B' are perpendicular projections of A and B on the axis *(Δ)*

$$\overrightarrow{A'B'} = \left\|\overrightarrow{A'B'}\right\|\,\overrightarrow{u'}$$

$$\boldsymbol{\overrightarrow{\|A'B'\|} = \overrightarrow{\|AB\|}\cos\theta}$$

## II.2- The components of a vector:




$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ Or $\vec{A}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$

Tel that $\|\vec{A}\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ is the magnitude of $\vec{A}$

- If a vector $\overrightarrow{AB}$ set by the coordinates of the points $A(Ax\,; A_y; A_z)$ and $A(B_x; B_y; B_z)$ can be found using the following formula:

$$\overrightarrow{AB} = (B_x - A_x)\vec{i} + (B_y - A_y)\vec{j} + (B_z - A_z)\vec{k}$$

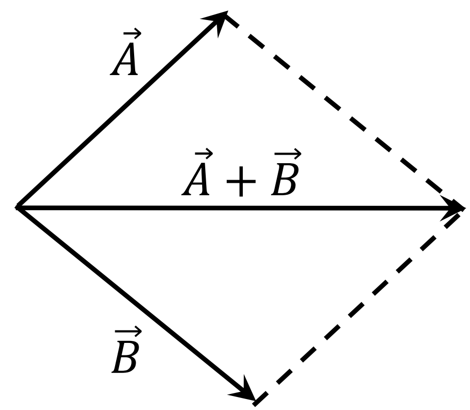
## Vector operations:

### I. Addition

$$\vec{A}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \text{ et } \vec{B}\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

**Analytically :** $$(\vec{A} \pm \vec{B})\begin{pmatrix} A_x \pm B_x \\ A_y \pm B_y \\ A_z \pm B_z \end{pmatrix}$$

**Geometrically**



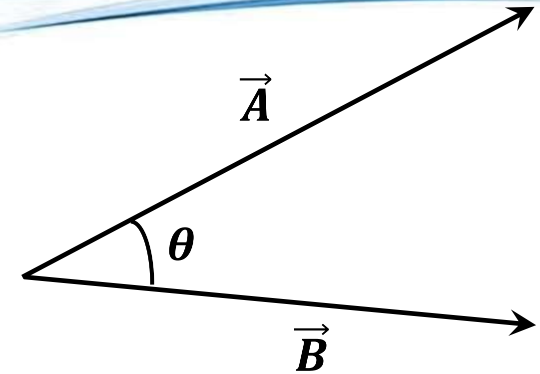

### Properties :

- $$\sum_{i=1}^{n} \overrightarrow{A_i} = \sum_{i=1}^{n} A_{x_i}\vec{i} + \sum_{i=1}^{n} A_{y_i}\vec{j} + \sum_{i=1}^{n} A_{z_i}\vec{k}$$
- $(\vec{A} + \vec{B}) = (\vec{B} + \vec{A})$
- $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
- $\|\vec{A} + \vec{B}\| \neq \|\vec{A}\| + \|\vec{B}\|$
- $$\vec{A}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \Rightarrow -\vec{A}\begin{pmatrix} -A_x \\ -A_y \\ -A_z \end{pmatrix}$$

## *II. multiplication of two vectors:*

### *II.1 Scalar multiplication:*

$$\vec{A}.\vec{B} = \|\vec{A}\|\|\vec{B}\| \cos\left(\widehat{\vec{A},\vec{B}}\right)$$




❑ **In Cartesian coordinates:**

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

❑ **The angle θ between $\vec{A}$ and $\vec{B}$ is given by:**

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}.\sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2}}$$

## Properties :

- $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

- $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

- $\vec{A}.(\vec{B}.\vec{C}) = (\vec{A}.\vec{B}).\vec{C}$

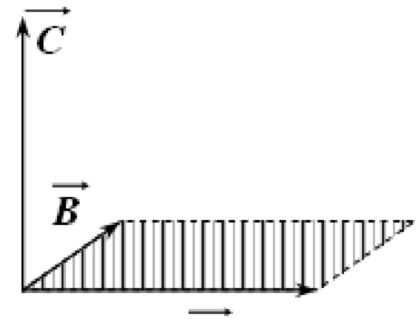
- $\vec{A}.\vec{A} = \|\vec{A}\|^2$

- $(\lambda\vec{A}).\vec{B} = \lambda(\vec{A}.\vec{B}) = \vec{A}.(\lambda\vec{B})$

- $\vec{A} \perp \vec{B} \Rightarrow \vec{A}.\vec{B} = 0$ ($\vec{A}$ and $\vec{B}$ are ortogonal )

## II.2. Vector multiplication :

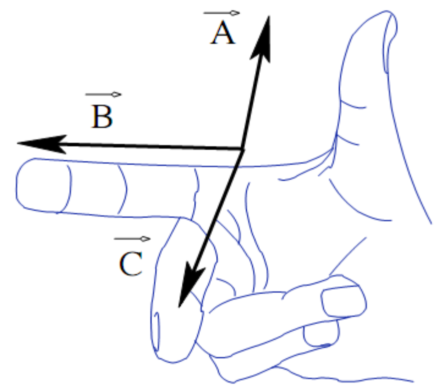
**The vector *multiplication* of vectors $\vec{A}$ and $\vec{B}$ , denoted $\vec{A} \wedge \vec{B}$ , is a vector $\vec{C}$ with:**



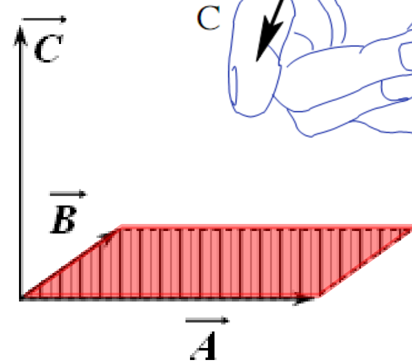

- $\vec{C}$ is perpendicular to the plane formed by the vectors $\vec{A}$ and $\vec{B}$

- The direction is given by using the right-hand rule.

$\Rightarrow$ $(\vec{A}\vec{B}\vec{C})$ **make a direct trihedron (ثلاثية مباشرة).**




- the magnitude of $\vec{C}$ corresponds to the area of the parallelogram constructed on $\vec{A}$ and $\vec{B}$




***Analytically :***

$$\|\vec{C}\| = \|\vec{A} \wedge \vec{B}\| = \|\vec{A}\|.\|\vec{B}\|.\left|\sin\left(\widehat{\vec{A},\vec{B}}\right)\right|$$

## ❑ Cartesian coordinates of $\vec{C}$ :

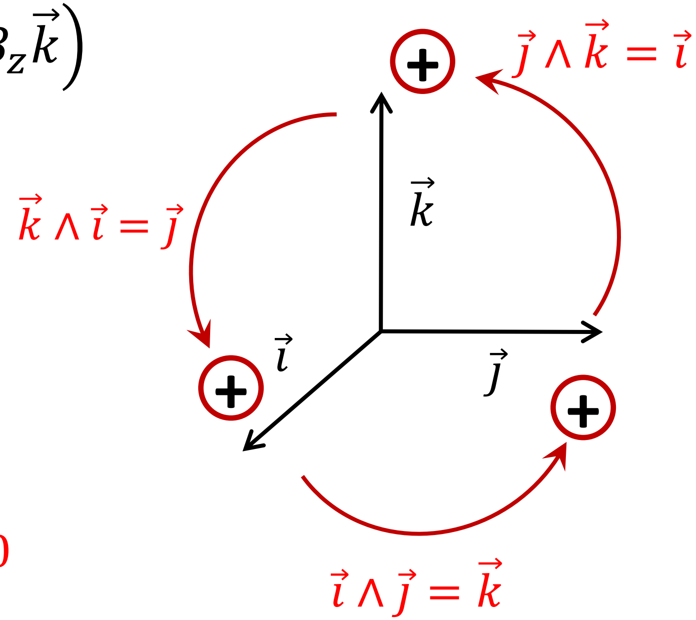
$$\vec{A}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad \text{et} \quad \vec{B}\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

$$\vec{C} = \vec{A} \wedge \vec{B} = \left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right) \wedge \left(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}\right)$$

$$= A_xB_x\vec{i} \wedge \vec{i} + A_xB_y\vec{i} \wedge \vec{j} + A_xB_z\vec{i} \wedge \vec{k}$$

$$+A_yB_x\vec{j} \wedge \vec{i} + A_yB_y\vec{j} \wedge \vec{j} + A_yB_z\vec{j} \wedge \vec{k}$$

$$+A_zB_x\vec{k} \wedge \vec{i} + A_zB_y\vec{k} \wedge \vec{j} + A_zB_z\vec{k} \wedge \vec{k}$$




- $\vec{i} \wedge \vec{i} = \|\vec{i}\|\|\vec{i}\| \sin(\vec{i},\vec{i}) = \vec{0}$ Also $\vec{j} \wedge \vec{j} = \vec{k} \wedge \vec{k} = 0$

$$\Rightarrow \vec{A} \wedge \vec{B} = \left(A_yB_z - A_zB_y\right)\vec{i} - \left(A_xB_z - A_zB_x\right)\vec{j} + \left(A_xB_y - A_yB_x\right)\vec{k}$$

## ❖ Determinant method:

$$\vec{A}\wedge\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=+\vec{i}\left(A_yB_z-A_zB_y\right)-\vec{j}(A_xB_z-A_zB_x)+\vec{k}\left(A_xB_y-A_yB_x\right)$$

$$\Rightarrow \vec{A}\wedge\vec{B}=\left(A_yB_z-A_zB_y\right)\vec{i}-(A_xB_z-A_zB_x)\vec{j}+\left(A_xB_y-A_yB_x\right)\vec{k}$$

## ❑ Properties:

1. $\vec{A}\wedge\vec{B}=-\left(\vec{B}\wedge\vec{A}\right)$

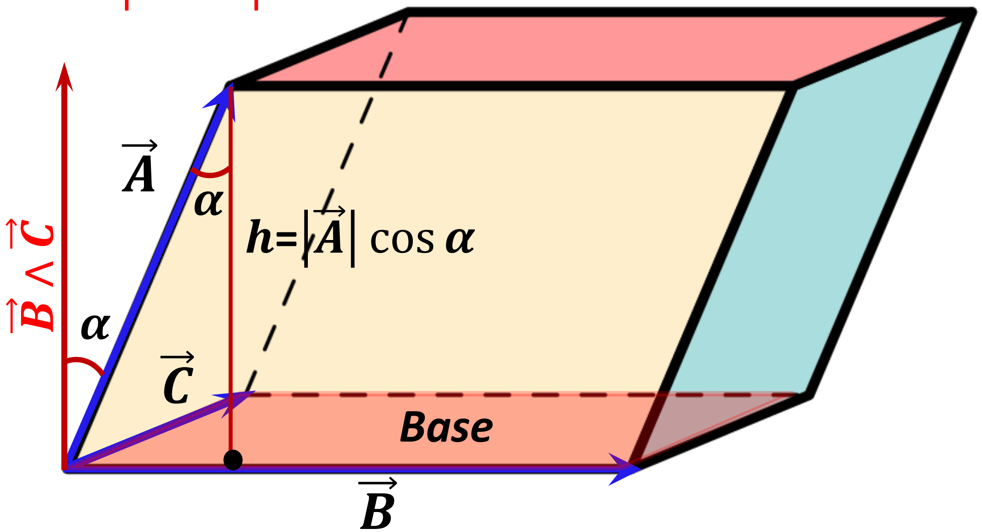
2. $\vec{A}//\vec{B}\Rightarrow\vec{A}\wedge\vec{B}=\vec{0}$

## *II.3. Mixed product:*

Mixed product is a triple vector product that combines the concept of *scalar* and *vectorial* products to yield a scalar value: $m = \vec{A}.(\vec{B} \wedge \vec{C})$

### *Geometric interpretation:*

- The absolute value $m$ of the mixed product is the volume of the parallelepiped formed by the vectors $\vec{A}, \vec{B}$ *and* $\vec{C}$.
- The vector $\vec{B} \wedge \vec{C}$ is perpendicular of the base of The parallelepiped and its magnitude equal the area of the base: $b = |\vec{B} \wedge \vec{C}|$
- The altitude of the parallelepiped $h$ is given by: $h = |\vec{A}| \cos \alpha$




➢ **Therefor, the volume is given by :**

$$V = Base(b) \times h$$

$$\vec{A}.(\vec{B} \wedge \vec{C}) = \underbrace{|\vec{B} \wedge \vec{C}|}_{b} . \underbrace{|\vec{A}| \cos \alpha}_{h}$$

## ***Mixed product properties :***

- $\vec{A}.(\vec{B}\wedge\vec{C}) = \vec{B}.(\vec{C}\wedge\vec{A}) = \vec{C}.(\vec{A}\wedge\vec{B}) = (\vec{B}\wedge\vec{C}).\vec{A} = (\vec{C}\wedge\vec{A}).\vec{B} = (\vec{A}\wedge\vec{B}).\vec{C}$
- $\vec{A}.(\vec{B}\wedge\vec{C}) = -\vec{A}.(\vec{C}\wedge\vec{B}) = -\vec{B}.(\vec{A}\wedge\vec{C}) = -\vec{C}.(\vec{B}\wedge\vec{A})$
- If any two of vectors $\vec{A}$, $\vec{B}$ and $\overrightarrow{C}$ are parallel, or if $\vec{A}$, $\vec{B}$ and $\overrightarrow{C}$ are **Coplanar**, then:

$$\vec{A}.(\vec{B}\wedge\vec{C}) = 0$$

- **Analytically, if**: $\vec{A}\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}$, $\vec{B}\begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}$ and $\vec{C}\begin{Bmatrix} c_x \\ c_y \\ c_z \end{Bmatrix}$:

$$\vec{A}.(\vec{B}\wedge\vec{C}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

$$= a_x(b_y c_z - b_z c_y) - a_y(b_x c_z - b_z c_x) + a_z(b_x c_y - b_y c_x)$$

## *II.4. Vector triple product*

$$\vec{A} \wedge (\vec{B} \wedge \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C} = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$$

***Properties:***

- **Non-Associativity:** $\vec{A} \wedge (\vec{B} \wedge \vec{C}) \neq (\vec{A} \wedge \vec{B}) \wedge \vec{C}$
  - $\vec{A} \wedge (\vec{B} \wedge \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$
  - $(\vec{A} \wedge \vec{B}) \wedge \vec{C} = (\vec{A}.\vec{C})\vec{B} - (\vec{B}.\vec{C})\vec{A}$

- The vector $\vec{A} \wedge (\vec{B} \wedge \vec{C})$ is in the plane defined by $\vec{B}$ and $\vec{C}$

- The vector $(\vec{A} \wedge \vec{B}) \wedge \vec{C}$ is in the plane defined by $\vec{A}$ and $\vec{B}$

## *II.5. Differential Operators :*

- ***Operator Nabla :***

$$\vec{\nabla}=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k} \quad \textbf{Ou} \quad \vec{\nabla}=\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}$$

- ***Gradient operator :***

The gradient operator is a differential operator that applies to a scalar function dependent on space and time and transforms it into a vector dependent on space and time. It is read "gradient f" or "nabla f" and is noted :

$$\overrightarrow{\boldsymbol{grad}}\boldsymbol{f} \quad \textit{or} \quad \vec{\boldsymbol{\nabla}}\boldsymbol{f}$$

In the Cartesian coordinate system the gradient is expressed as follows:

$$\vec{\nabla}f(x,y,z,t)=\frac{\partial f(x,y,z,t)}{\partial x}\vec{i}+\frac{\partial f(x,y,z,t)}{\partial y}\vec{j}+\frac{\partial f(x,y,z,t)}{\partial z}\vec{k}$$

**Properties:**

- ❑ $\vec{\nabla}(\alpha f + \beta g) = \alpha\vec{\nabla}f + \beta\vec{\nabla}g$ (with $(\alpha, \beta) \in \mathbb{R}^2$
- ❑ $\vec{\nabla}(f.g) = f\vec{\nabla}g + g\vec{\nabla}f$

**Example:**

$$f(x, y, z) = 3x^2y + z$$

- Calculate $\overrightarrow{grad} f(x, y, z)$ in point M(1, 2, -2)

**Sol:**

$$\overrightarrow{grad} f(x, y, z) = \left.\frac{\partial f}{\partial x}\right|_{(y,z)=Cts} \vec{\imath} + \left.\frac{\partial f}{\partial y}\right|_{(x,z)=Cts} \vec{\jmath} + \left.\frac{\partial f}{\partial z}\right|_{(x,y)=Cts} \vec{k}$$

$$\Longrightarrow \overrightarrow{grad} f(x, y, z) = \vec{\nabla} f(x, y, z) = 6xy\,\vec{\imath} + 3x^2\,\vec{\jmath} + \vec{k}$$

$$\Longrightarrow \overrightarrow{grad} f(1, 2, -2) = \vec{\nabla} f(1, 2, -2) = \mathbf{2\vec{\imath} + 3\vec{\jmath} + \vec{k}}$$

## ❑ ***Divergence operator:***

The divergence operator is a differential operator that applies to a vector field and returns a scalar field. It reads divergence and is noted:

$$div\vec{A} \text{ or } \vec{\nabla}.\vec{A}$$

In the Cartesian coordinate system the Divergence of $\vec{A}$ is expressed as follows:

$$div\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

**Properties:**

- ❑ $div(\vec{A} + \vec{B}) = div\vec{A} + div\vec{B}$
- ❑ $div(\alpha\vec{A}) = \alpha\, div\vec{A}$
- ❑ $div(f\vec{A}) = f\, div\vec{A} + \overrightarrow{grad}f.\vec{A}$ (with $f$ is scalar function)

**Demonstration : Home work (واجب منزلي)**

## ❑ *Rotational operator:*

The rotational operator is a differential operator that transforms a vector field into another vector field. It reads rotational of $\vec{A}$ and is noted: $\overrightarrow{rot}\vec{A}$ Or $\vec{\nabla} \wedge \vec{A}$

➢ In the Cartesian coordinate system the Rotational of $\vec{A}$ is expressed as follows:

$$\vec{\nabla} \wedge \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}$$

➢ ***Properties:***

❑ $div\left(\overrightarrow{rot}\vec{A}\right) = \vec{\nabla}.\left(\vec{\nabla} \wedge \vec{A}\right) = 0$

❑ $\overrightarrow{rot}\left(\alpha\vec{A} + \beta\vec{B}\right) = \alpha\overrightarrow{rot}\vec{A} + \beta\overrightarrow{rot}\vec{B}$

❑ $\overrightarrow{rot}\,\overrightarrow{grad}f = \vec{\nabla} \wedge \vec{\nabla}f = 0$

❑ $\overrightarrow{rot}\left(f\vec{A}\right) = \vec{\nabla} \wedge \left(f\vec{A}\right) = \vec{\nabla}f \wedge \vec{A} + f\vec{\nabla}f \wedge \vec{A}$

**Demonstration : Home work (واجب منزلي)**

❑ ***Laplacian operator***

**Pierre-Simon Laplace**
(1749 - 1827)

## *1. The Scalar Laplacian*

The scalar Laplacian operator is a differential operator of order two that transforms a scalar function into another scalar function. The scalar Laplacian is obtained by taking the divergence of the gradient and denoted:

$$\boldsymbol{\Delta f = div\left(\overrightarrow{grad} f\right) = \nabla^2 f}$$

- In the Cartesian coordinate: $\boldsymbol{\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}}$

- ***Properties :***

  - ❑ $\Delta(\alpha f + \beta g) = \alpha \Delta f + \beta \Delta g$

  - ❑ $\Delta(fg) = (\Delta f)g + 2(\nabla f).(\nabla g) + f(\Delta g)$

## 2. ***The Vector Laplacian:***

Laplacian also applies to a vector field. In this case it returns another vector field

and denotes: $\triangle \vec{A}$

By definition, the vector Laplacian is obtained using the identity (***Vector triple product )***:

$$\overrightarrow{rot}\,\overrightarrow{rot}\,\vec{A} = \vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A} = \overrightarrow{grad}(div\vec{A}) - \triangle\,\vec{A}$$

❑ ***Properties :***

I. $\overrightarrow{rot}\left(\overrightarrow{grad} f\right) = \vec{0}$

II. $div\left(rot\vec{A}\right) = 0$